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MODEL ORDER REDUCTION TECHNIQUE APPLIED ON HARMONIC ANALYSIS OF A SUBMERGED VIBRATING BLADE

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As part of an ongoing study into hydropower runner failure, a submerged, vibrating blade is investigated both experimentally and numerically. The numerical simulations performed are fully coupled acoustic-structural simulations in ANSYS Mechanical. In order to speed up the simulations, a model order reduction technique based on Krylov subspaces is implemented. This paper presents a comparison between the full ANSYS harmonic response and the reduced order model, and shows excellent agreement. The speedup factor obtained by using the reduced order model is shown to be between one and two orders of magnitude. The number of dimensions in the reduced subspace needed for accurate results is investigated, and confirms what is found in other studies on similar model order reduction applications. In addition, experimental results are available for validation, and show good match when not too far from the resonance peak.

Key words: model order reduction, FSI, vibration, harmonic response.

1. Introduction

The quality and precision in the manufacturing industry have improved massively during the last couple of decades, due to automation and CNC machining. Even still, there have been several failures in new high head Francis turbines lately [1, 2]. This suggests that there is a problem in the design process. The dominating periodic load on the runner is known to be the forces due to the pressure field created by the flow passing the stationary components interacting with the pressure field following the rotating runner (known as Rotor-Stator Interaction (RSI)) [3]. When engineering a turbine, the RSI frequency is known in advance, and the design aims to have natural frequencies of components and assembly far away from the RSI frequencies to avoid any resonance issues. The runner however, is submerged in water, which is known to change its structural behavior [4]. The surrounding fluid complicates the structural calculations, as the added mass of water will lower the natural frequencies of the structure, and dampen the amplitude of the deflections. Furthermore, moving water will affect the structure differently from water standing still. It is also observed that the presence of water can change the order of the structural modes [5]. An acoustic-structural simulation will account for the presence of the surrounding fluid. Before such simulations were available, the industry

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used empirical estimates to approximate the reduction of the natural frequency. Today however, studies show that it is not possible to obtain an all-purpose rule [4-6]. To conclude, if the goal is to investigate dynamic response of a turbine by performing a harmonic sweep of a Francis turbine with surrounding water, a coupled acoustic-structural simulation is needed.

The calculation of the coupled acoustic-structural harmonic response of a submerged structure is computationally expensive [7] and not applicable for all industries. This article will implement a Krylov-subspace based model order reduction method for rapid calculation of harmonic analyses, based on the methodology presented by Rudnyi [8]. The structure in question is a submerged vibrating hydrofoil, a geometry studied in a research project investigating Francis runner failures at the Norwegian University of Science and Technology [9]. Experimental data on the same geometry is available for validation. This data is publicly available from the Francis99 project website [10].

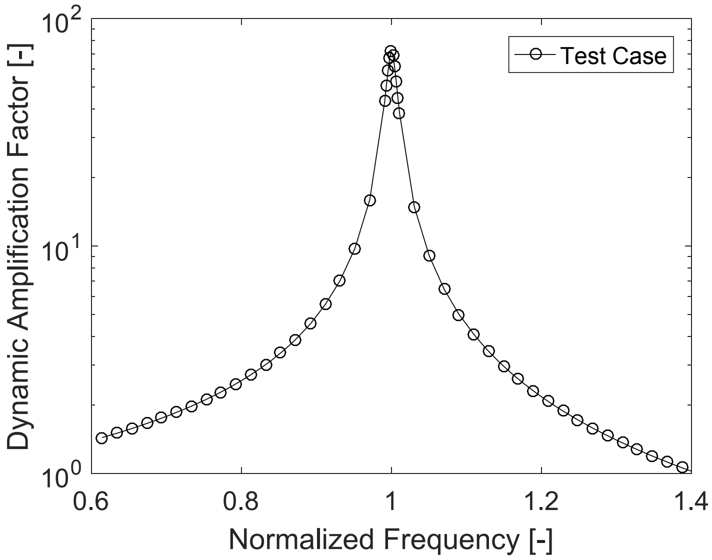

Fig.1. Dynamic amplification factor on generic blade.

2. Theory and methods

All structures have several natural frequencies and corresponding vibrational modes. If a structure is loaded at a frequency close to its natural frequency, the structural response A will be magnified compared with the static load magnitude A_0. This is referred to as resonance and could in the worst case cause violent structural failure [11]. Figure 1 shows an example of the first mode Dynamic Amplification Factor, DAF = A/A_0, on a generic blade. The DAF is defined as deformation normalized by the steady response of the applied harmonic load. Equivalently, normalized with the response as the frequency goes to zero, $f \rightarrow 0$ [11].

From a design point of view, this graph is very interesting. In a design process, you will always try to avoid the natural frequencies and resonance. What Fig.1 shows is how the structure responds, not only at resonance, but at off-resonance conditions. The exact shape of the response graph is dependent of many factors; the damping; closeness to next natural frequency, etc., however let us use this figure to illustrate a design issue; If one assumes a linear material, and linear force-deflection relationship, then the amplification factor can be directly translated into a multiplication factor for the applied load. In the above figure we can see that even loading as far away as *25%* from the natural frequency will be multiplied by a factor of 2, whereas loading at the natural frequency will be multiplied with about 75. The danger is the following; there will always be an uncertainty in the calculation of the natural frequency of your component. Especially submerged structures can be difficult to perfectly predict, and the response away from the natural frequency then becomes even more important. Let us assume that you design to be 15% from resonance, but the error in natural frequency is in the order of *10*%. The multiplication factor during operation will then be anywhere between 2 and 10. Clearly this is not acceptable for subsequent operation of the investigated component.

This underlines the need for the construction of harmonic response and amplification factor plots, the computational cost of removing all uncertainty in the calculation of the natural frequency is extreme, and maybe impossible. It is therefore desirable to obtain the dynamic response early in a design process, to identify risks, find sufficient safety margins, and perform design changes accordingly. To obtain this you need to solve a set of time-consuming harmonic equations. The simulation time will in this case be reduced by the use of Krylov subspaces. Model order reduction based on Krylov subspaces started in the electrical community [12, 13], and later in other industries with good results [14, 15].

The following sections will describe the governing equations of coupled structural-acoustic problems, as well as some of the theory behind a Krylov subspace model order reduction technique.

2.1. Second order, dynamic structural systems

Much of the theory in the following sections is adapted from [16], please refer here for more information. Most dynamic systems are second order. A general second order system can be modeled as follows

$$M\ddot{x}(t)+C\dot{x}(t)+Kx(t)=Fy(t) \tag{2.1}$$

where $x \in R^N$ is the state variable (typically displacement), and $y \in R^N$ is the force vector. The matrices $M,C,K \in R^{NxN}$ are the usual mass, damping and stiffness matrices respectively, and N is the degrees of freedom. F controls the distribution of the input force. In the case of a harmonic excitation and response we have

$$Fy(t)=\{F\}e^{i\omega t}, \tag{2.2}$$

$$x(t)=x_{max}e^{i\varphi}e^{i\omega t}=\{x\}e^{i\omega t} \tag{2.3}$$

where ω denotes the angular frequency, and φ a potential phase shift. By using Eqs (2.2) and (2.3) and removing the time dependency, Eq.(2.1) can be rewritten as

$$\left[-\omega^2 M+i\omega C+K\right]\{x\}=\{F\}. \tag{2.4}$$

Equation (2.4) is the equation solved when performing a harmonic analysis, and the one implemented in most commercial codes, including ANSYS Mechanical, used in this paper. However, if solved as is, the effects of added mass of the surrounding fluid is not accounted for. The structural natural frequencies will be wrong, and useless in a design phase. Therefore, we have to expand this equation to include the acoustic domain.

2.2. Coupled acoustic-structural systems

Acoustics denotes the science of mechanical waves in fluids and structures. In terms of the fluid, no advection terms are modelled, only the pressure propagation is resolved. The pure harmonic motion of the sound pressure inside a fluid domain can be modelled by the Helmholtz equation (time-independent wave equation) [17]

$$\nabla^2 p + k^2 p = 0 \tag{2.5}$$

where p is the acoustic pressure, $k = \omega / c$ is the wave number, and c is the speed of sound in the fluid. A structure submerged in water will change characteristics due to the density of water. Especially eigenfrequencies and harmonic response are significantly altered by a surrounding heavy fluid. The above Eq.(2.5) can therefore be rewritten for harmonic motions as done in the previous section, and combined with the structural response, Eq.(2.4), to obtain a coupled acoustic-structural system, referred to as the Eulerian displacement-pressure formulation [16, 18]

$$\left(-\omega^2 \begin{bmatrix} M_s & 0 \\ M_{fs} & M_a \end{bmatrix} + i\omega \begin{bmatrix} C_s & 0 \\ 0 & C_a \end{bmatrix} + \begin{bmatrix} K_s & K_{fs} \\ 0 & K_a \end{bmatrix} \right) \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} F_s \\ F_a \end{Bmatrix} \tag{2.6}$$

where u is the structural displacement, and p is the acoustic pressure. The subscripts: s, a, fs denotes structure, acoustic, and fluid-structure respectively. The cross-multiplication matrices (M_{fs}, K_{fs}) are obtained by enforcing boundary conditions on the fluid-structure interface. This way information will cross the domain interfaces in a consistent way. This second order coupled formulation allows for accurate harmonic analysis of submerged structures. A major drawback is the increased computational expense of solving the above acoustic/structural system.

2.3. Model order reduction

In engineering problems, the number of degrees of freedom could be extremely large. When considering acoustic elements as well, the coefficient matrices become unsymmetric (see Eq.(2.6)) [19, 20]. The added complexity from the acoustic-structural coupling makes the above system in many cases too expensive to solve, especially if a large frequency range is to be covered with satisfactory resolution [21]. The reasoning behind the Model Order Reduction (MOR) is to find a lower dimensional subspace $V \in R^{Nxq}$ such that

$$\begin{Bmatrix} u \\ p \end{Bmatrix} = \{x\} \approx Vz + \varepsilon \tag{2.7}$$

where $z \in R^q$ and $q << N$. The symbol ε denotes a small error introduced by utilizing the reduced model. If one assumes that the subspace V is available, Eq.(2.6) can be rewritten as

$$\left[-\omega^2 M_r + i\omega C_r + K_r \right]\{z\} = \{F_r\} \tag{2.8}$$

where the subscript r denotes a reduced quantity, and the reduced matrices are defined as follows

$$M_r = V^T MV \ ; \quad C_r = V^T CV \ ; \quad K_r = V^T KV \ ; \quad F_r = V^T FV \ . \tag{2.9}$$

The matrices in Eq.(2.8) are reduced to order R^{qxq}, an enormous improvement from the original system in Eq.(2.6), where the coefficients were R^{NxN}. For a subspace of order $q=30$ or similar, the new system is solved in seconds.

The problem is to obtain the subspace V. In this article, V is chosen to be a Krylov subspace, created using the Arnoldi algorithm. This subspace satisfies the moment-matching property to resemble the original system, see [22]. The details of the model reduction procedure will not be explained here, interested readers can find more in [8, 16, 23, 24]. In the process of creating the reduced model, the number of dimensions, q, must be chosen. In general, the larger the q, the higher the accuracy, but at a computational cost.

2.3.1. Application of model order reduction

The application of the model order reduction process outlined in the previous sections is shown in Fig.2. The commercial software ANSYS Mechanical is used to set up the system, define loads, constraints and more, and to create the coefficient matrices used in the reduction process. Then the reduction process is performed with the main parameter being the number of dimensions of the reduced system. Finally, the reduced system is solved.

2.4. Experimental setup

This study is a part of a larger research project at the Norwegian University of Science and Technology (NTNU), where the goal is to understand why

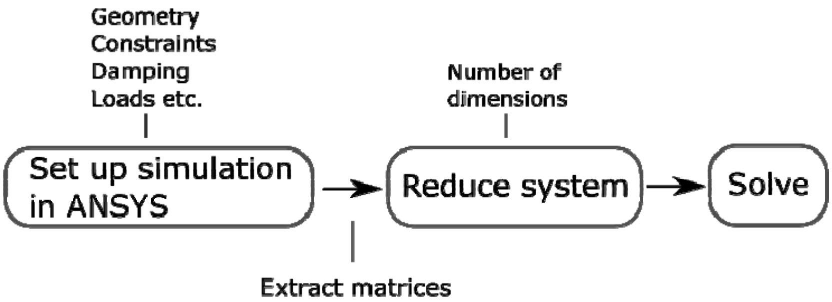

Fig.2. The process of performing the model order reduction.

high head Francis runners experience cracks [9]. Experiments have been performed on both an unsymmetrical hydrofoil (to resemble a Francis turbine runner blade), and a symmetric hydrofoil. The goal of these experiments is to study the damping characteristics of the fluid-structure system, and importantly, the relationship between the flow velocity and the damping. The experimental setup and results from the unsymmetrical hydrofoil can be found in [25], as well as on the Francis99 project homepage [10]. The same setup is used for the symmetric hydrofoil which will be studied here.

In short, an aluminum hydrofoil is excited by electric muscles (Piezoelectric Macrofiber composite actuators from PI Ceramic) to vibrate in a harmonic motion. Laser Doppler Vibrometry and strain gauges is used to measure the vibrating trailing edge motion. The frequency response is obtained for several different flow velocities, and used to calculate the damping characteristics of the system. The hydrodynamic damping ratio, ξ, obtained at $v = [2.5, 10, 20][m/s]$ in the experiments, is used in all the simulations presented here.

2.5. Numerical setup

The goal of this article is to present a model order reduction method. Experimental data is in this case strictly not needed, as a comparison with the assumed correct ANSYS solution would determine the accuracy of the MOR approximation and the speedup of the method. However, it is chosen to use the same geometry as in the aforementioned experiments as well as some of the

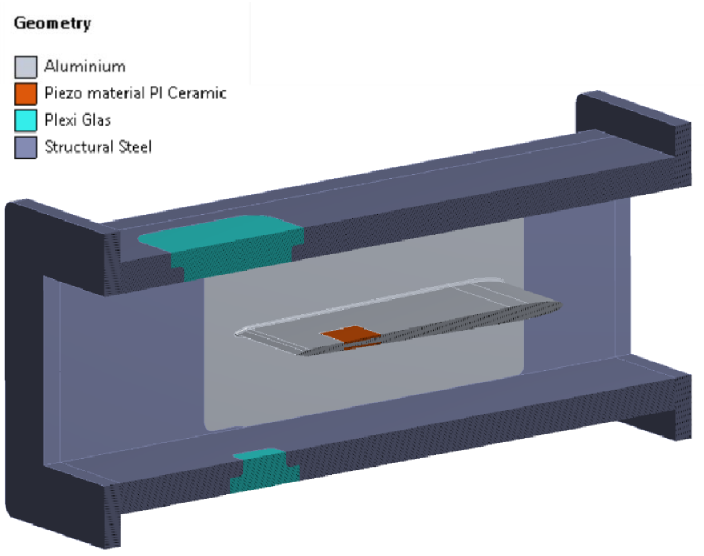

Fig.3. Numerical geometry.

results from the damping measurements, so that the current study is relevant to the overall project. The numerical domain is therefore as shown in Fig.3, and Tab.1 lists a summary of the most important simulation settings used in this article.

The simulations performed in ANSYS Mechanical will henceforth be referred to as the "full" solution. A constant load is used in all the simulations. This load was obtained from a CFD simulation on the same geometry, where the blade was vibrating at its natural frequency [27]. The fluid load (pressure) is imported to the blade in the harmonic analysis. A harmonic sweep is then performed in the range *300-750* Hz, divided into 100 equally spaced frequencies. The damping from the experiments, ξ, is used, from which the numerical damping is defined as $\beta = 2\xi/\omega_n$, [11] with ω_n being the natural frequency of the structure. There are two things to note about this procedure. First, the pressure load is extracted from a blade vibrating at a

Table 1. Numerical settings.

Parameter	Value
Software	ANSYS Mechanical
Analysis type	Full Damped, MOR
Damping	$\beta = [4.43e^{-6}, 8.16e^{-6}, 2.17e^{-5}]^{*}$
Frequency range	*300-750* Hz
Number of frequencies	100
Mesh	500.000 nodes **
Acoustic domain	Water: $c = 1482\ m/s$, $\rho = 998 kg/m^3$
Dimensions in MOR	q=[*10*, *30*, *50*, *100*]

* Corresponding to flow velocity of *2.5*,*10* and *20m/s* respectively [25]
** Discretization error estimated to be *0.2*% using GCI method [26]

given mode shape. The corresponding load distribution and magnitude is therefore "locked" to the given mode. If one wants to investigate a range of frequencies where more than one bending mode is excited, more load distributions should be included. Second, note that the damping ratio is also valid for the first bending mode only. The same argument as above can be used for the damping, if more modes are to be investigated. The assumptions made in this paper are a simplification, but should be valid as the focus is on one mode at the time.

In addition to the full simulation, reduced simulations were performed. The simulation was set up as above, and the model reduction was performed on the equation system extracted from ANSYS, such that a direct comparison of the reduced versus the full solution is possible. In the reduced models, the number of dimensions were set to q=[*10*, *30*, *50*, *100*], to investigate both the accuracy of the reduced model, as well as the computational cost. Many papers report the use of $q=30$, however this is from different industries, and chosen somewhat arbitrary, and may not be applicable here [8].

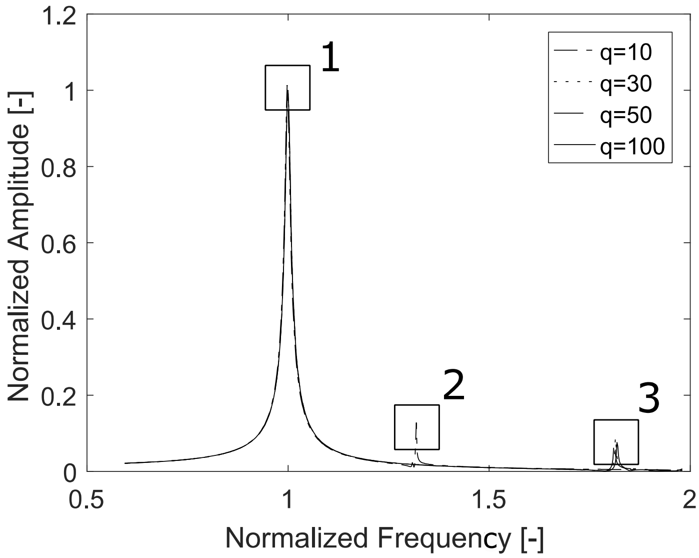

Fig.4. Comparison of the different number of dimensions in the reduced model.

3. Results

This first section will investigate the effect of changing the number of dimensions of the reduced order model, recall Eq.(2.8). The objective is to evaluate how many dimensions are needed to fully capture the behavior of the original system. The test case of $v=2.5m/s$ and corresponding damping is used. Figure 4 compares the different reduced models, with q=[*10*, *30*, *50*, *100*]. The amplitude and frequency is normalized with the simulation using $q=100$ as it is assumed to be the most accurate. As the models perform very similarly, three boxes are marked in Fig.4, to be further investigated in Fig.5.

Figure 5 shows a zoomed view of the boxes marked in Fig.4. From Figs 5a, b it can be seen that only $q=10$ dimensions show some discrepancy compared to the rest of the simulations. In Fig.5c it is seen that the accuracy drops for $q=30$ and $q=50$ as well. Based on Fig.5 it is concluded that 30 dimensions are sufficient in terms of accuracy for this case. Therefore, all reduced order models will from this point on use $q=30$, as was reported in the literature to be sufficient.

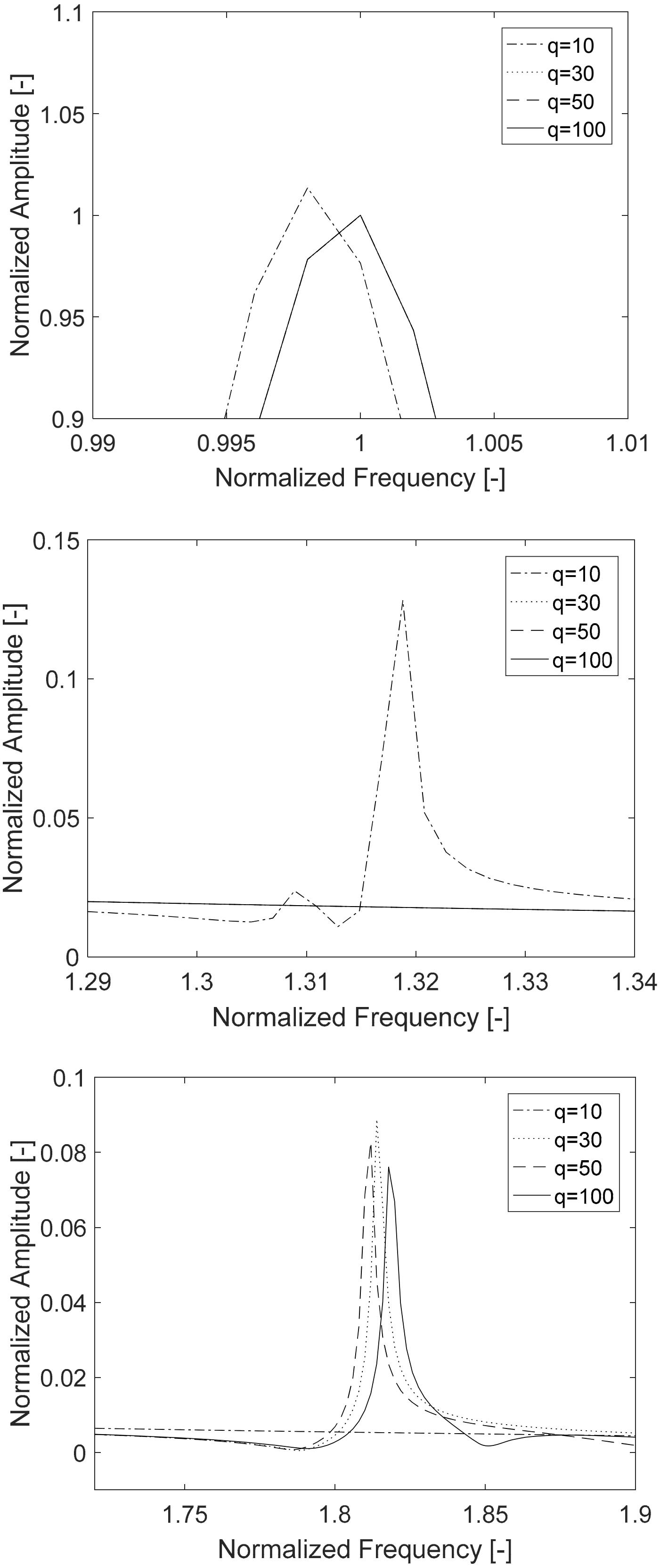

Fig.5. Detailed view of the effect of changing the number of dimensions in the MOR.

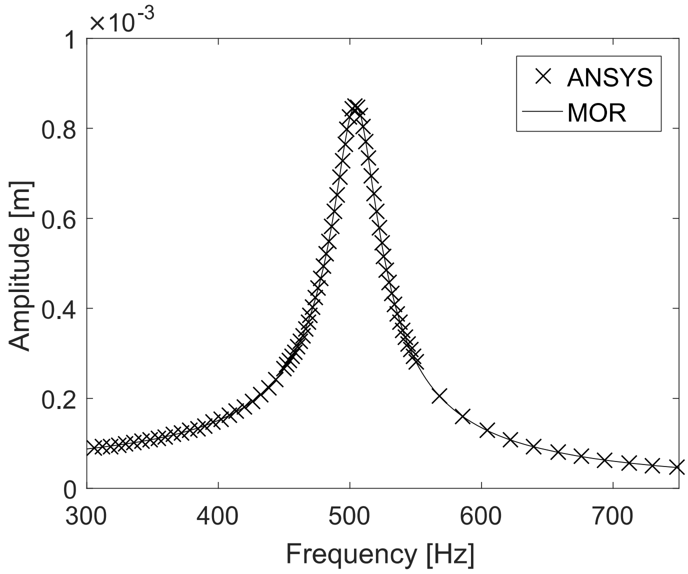

Fig.6. Comparison between full ANSYS solution and reduced order model.

3.1. Verification

In the following section, the MOR results using *q=30* will be compared with the full ANSYS solution. Figure 6 shows a comparison using the damping corresponding to a flow velocity of *v=20m/s* ($\beta = 2.17e^{-5}$). There is an excellent match in the complete range. Similar results are seen for the other flow velocities.

In addition to accuracy with respect to the full solution, the obtained speedup is the second crucial metric used to evaluate the MOR method. The speedup factor is calculated as follows; 100 evenly spaced frequencies were simulated with the full ANSYS solution. The MOR was performed on the same problem, and the total simulation time was compared. The results are shown in Tab.2. The speedup is case-dependent, mesh-dependent, etc., however Tab.2 will give a qualitative indication of the gain in simulation time.

It is clear that both the accuracy and simulation time are excellent. If we return to the dynamic amplification factor in Fig.1, creating such a plot is now possible in a reasonable time frame due to the speedup documented here.

Table 2. Speedup factor.

Method	Speedup*
Full ANSYS solution	1
MOR 10 dimensions	56
MOR 30 dimensions	40
MOR 50 dimensions	31
MOR 100 dimensions	18

*Per 100 frequencies

3.2. Validation

This section will compare the numerical frequency response with the experimental one. Figure 7 shows the scaled harmonic response of the vibrating blade obtained in experiments and in the simulations for flow velocities *v=20m/s*.

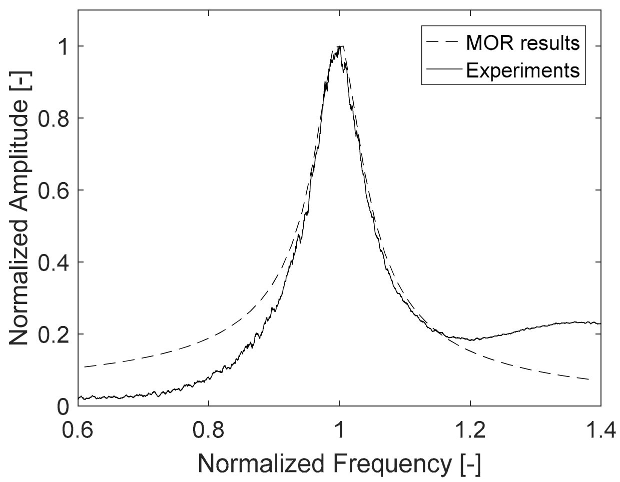

Fig.7. Comparison between experimental results and MOR solution.

It is observed that the overall match close to the response peak is very good, and that the accuracy decreases when moving further away from it.

4. Discussion

As in other studies, a reduced system of order 30 was deemed sufficient. This may be by chance, in the literature this number was chosen more or less arbitrarily [8], however it does indicate that a fairly low number of vectors/dimensions is needed to properly describe the original system. One thing that we can observe is that the method used for reducing the original system will be most accurate around a pre-determined point, i.e. the mid-point of the domain (think Taylor expansion about a point). In essence, the accuracy of the approximation will decrease in the ends of the investigated domain, this is seen in, i.e., Fig.5c, where $q=30$, 50 starts to deviate from the solution of $q=100$. This may imply that fewer dimensions are needed if the interesting frequencies are known in advance, and conversely, more dimensions are needed if a large sweep is to be performed with no prior knowledge of the location of the natural frequencies.

Observe that in the numerics, only one bending mode is excited in the frequency sweep (bottom right corner of Fig.7 indicates new mode in the experiments). This is due to the fact that the load distribution imported from CFD is linked to the first bending mode only. For resonance to occur, both the frequency and the spatial load distribution have to match with the mode in question. This is only satisfied for the first bending mode in this case. If a more generic load was applied, specifically one where the spatial distribution does not limit which bending modes are possible to obtain, a larger frequency range and more modes could be investigated.

Another factor is the damping from the experiments. The damping factor is strictly only valid at the natural frequency, not when moving away from the resonance peak. This may explain why the accuracy decreases when moving away from the peak in Fig.7. Yet another factor is the point at which the amplitude is measured in the simulations and the experiments. It is unlikely that exactly the same location is tracked, and this will therefore possibly introduce an uncertainty.

5. Conclusion

Solving complex engineering problems involving submerged structures require a coupled acoustic-structural simulation. This is computationally expensive, but this article shows that the simulation time can be reduced by an order of magnitude of one to two, without reducing the accuracy. A Krylov subspace method is used in the model order reduction process. Using this method can allow the designers to obtain dynamic amplification plots early in a design process, and can give valuable information regarding product design. The results are also compared with ongoing experiments, and show overall good results.

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Nomenclature

A	– amplitude of deflection
A_0	– amplitude at static load
β	– proportional/ Rayleigh damping
c	– speed of sound
DAF	– Dynamic Amplification Factor
f	– frequency
k	– wave number
MOR	– Model Order Reduction
M,C,K,F	– mass, damping, stiffness and force matrices
N	– degrees of freedom in original system
p	– pressure
q	– degrees of freedom in reduced system
RSI	– Rotor Stator Interaction
s, a, fs	– as subscripts; structure, acoustic, fluid-structure
V	– Krylov subspace
v	– flow velocity
$x, \dot{x}, \ddot{x}$	– deflection, velocity and acceleration
y	– force vector
ε	– error
ξ	– damping ratio
ρ	– density
ϕ	– phase shift between load and response
ω	– angular frequency
ω_n	– natural frequency

References

[1] Kobro E. (2010): *Measurement of Pressure Pulsations in Francis Turbines*. – Ph.D. Thesis, Norwegian University of Science and Technology.

[2] Liu X., Luo Y. and Wang Z. (2016): *A review on fatigue damage mechanism in hydro turbines*. – Renewable and Sustainable Energy Reviews, vol.54, pp.1–14.

[3] Seidel U., Hübner B., Löfflad J. and Faigle P. (2012):, *Evaluation of RSI-induced stresses in Francis runners*. – In: IOP Conference Series: Earth and Environmental Science, vol.15, IOP Publishing, pp.052010.

[4] Dörfler P., Sick M. and Coutu A. (2012):, *Flow-induced pulsation and vibration in hydroelectric machinery: engineers guidebook for planning, design and troubleshooting*. – Springer Science and Business Media.

[5] Liang Q., Rodríguez C.G., Egusquiza E., Escaler X., Farhat M. and Avellan F. (2007): *Numerical simulation of fluid added mass effect on a francis turbine runner*. Computers and Fluids, vol.36, No.6, pp.1106–1118.

[6] Tanaka H. (2011): *Vibration behavior and dynamic stress of runners of very high head reversible pump-turbines*. – International Journal of Fluid Machinery and Systems, vol.4, No.2, pp.289–306.

[7] Pirk R., Desmet W., Pluymers B., Sas P. and Goes L.C. (2002): *Vibro-acoustic analysis of the brazilian vehicle satellite launcher (vls) fairing*. – In: Proceedings of the International Conference on Noise and Vibration Engineering, ISMA, Leuven, Belgium.

[8] Rudnyi E.B. (2013): *Mor for ansys*. – System-Level Modeling of MEMS, pp.425–438.

[9] The hifrancis research project, https://www.ntnu.edu/web/norwegian-hydropower-center/hifrancis, accessed: 2018-05-07.

[10] The francis99 research project, https://www.ntnu.edu/nvks/francis-99, accessed: 2018-05-23.

[11] Craig R.R. and Kurdila A.J. (2006): *Fundamentals of Structural Dynamics*. – John Wiley and Sons.

[12] Bai Z. (2002): *Krylov subspace techniques for reduced-order modeling of largescale dynamical systems*. – Applied Numerical Mathematics, vol.43, No.1-2, pp.9–44.

[13] Freund R.W. (2000): *Krylov-subspace methods for reduced-order modeling in circuit simulation*. – Journal of Computational and Applied Mathematics, vol.123, No.1-2, pp.395–421.

[14] Willcox K., Peraire J. and White J. (2002): *An arnoldi approach for generation of reduced-order models for turbomachiner*y. – Computers and Fluids, vol.31, No.3, pp.369–389.

[15] Lassaux G. (2002): *High-fidelity reduced-order aerodynamic models: Application to active control of engine inlets*. – Ph.D. Thesis, Massachusetts Institute of Technology.

[16] Puri R.S., Morrey D., Bell A.J., Durodola J.F., Rudnyi E.B. and Korvink J.G. (2009): *Reduced order fully coupled structural–acoustic analysis via implicit moment matching*. – Applied Mathematical Modelling, vol.33, No.11, pp.4097–4119.

[17] Fahy F.J. (2000): *Foundations of Engineering Acoustics*. – Elsevier.

[18] Everstine G. (1997): *Finite element formulations of structural acoustics problems*. – Computers and Structures, vol.65, No.3, pp.307–321.

[19] Zienkiewicz O.C. (1969): *Coupled vibrations of a structure submerged in a compressible fluid*. – In: Proc. of Symposium on Finite Element Techniques Held at the University of Stuttgart.

[20] Craggs A. (1971): *The transient response of a coupled plate-acoustic system using plate and acoustic finite elements*. – Journal of Sound and Vibration, vol.15, No.4, pp.509–528.

[21] Atalla N. and Bernhard R. (1994): *Review of numerical solutions for low-frequency structural-acoustic problems*. – Applied Acoustics, vol.43, No.3, pp.271–294.

[22] Benner P., Feng L. and Rudnyi E.B. (2008): *Using the superposition property for model reduction of linear systems with a large number of inputs.* – In: Proceedings of the 18th International Symposium on Mathematical Theory of Networks and Systems.

[23] Rudnyi E.B. and Korvink J.G. (2004): *Model order reduction for large scale engineering models developed in ansys*. – In: International Workshop on Applied Parallel Computing, Springer, pp.349–356.

[24] Rudnyi E.B., Lienemann J., Greiner A. and Korvink J.G. (2004): *Mor4ansys: Generating compact models directly from Ansys models.* – In: Technical Proceedings of the 2004 Nanotechnology Conference and Trade Show, Nanotech, vol.2, pp.279–282.

[25] Bergan C., Solemslie B., Ostby P. and Dahlhaug O.G. (2018): *Hydrodynamic damping of a fluttering hydrofoil in high-speed flows*. – International Journal of Fluid Machinery and Systems, vol.11, No.2, pp.146–153.

[26] Celik I.B., Ghia U., Roache P.J., Freitas C.J., Coleman H. and Raad P.E. (2008): *Procedure for estimation and reporting of uncertainty due to discretization in {CFD} applications*. – Journal of Fluids {Engineering-Transactions} of the {ASME}, vol.130, No.7.

[27] Tengs E, Bergan C., Storli P.-T. and Jakobsen K.-R. (2018): *Numerical simulation of the hydrodynamic damping of a vibrating hydrofoil*. – In: Proceedings of the 29th IAHR Symposium on Hydraulic Machinery and Systems, *Not yet published*, 2018.

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