

RADIATION ABSORPTION AND CHEMICAL REACTION EFFECTS ON RIVLIN-ERICKSEN FLOW PAST A VERTICAL MOVING POROUS PLATE

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An analysis has been carried out to study the combined effects of radiation absorption and chemical reaction on an incompressible, electrically conducting and radiating flow of a Rivlin-Ericksen fluid along a semi-infinite vertical permeable moving plate in the presence of a transverse applied magnetic field. It is assumed that the suction velocity, the temperature and the concentration at the wall are exponentially varying with time. The dimensionless governing equations for this investigation are solved analytically using two-term harmonic and non-harmonic functions. A comparison is made with the available results in the literature for a special case and our results are in very good agreement with the known results. A parametric study of the physical parameters is made and results are presented through graphs and tables. The results indicate that the fluid velocity and temperature could be controlled by varying the radiation absorption.

Key words: Rivlin-Ericksen fluid, radiation absorption, chemical reaction, variable temperature, concentration and suction.

1. Introduction

Fluids with the elasticity property of solids as well as viscous property of liquids are called visco-elastic fluids. These fluid flows have numerous and wide-ranging applications in various fields, e.g., petrochemical, biomedical and environmental engineering including polypropylene coalescence sintering, dynamically -loaded journal bearings, blood flow and geological flows [1-4].

Visco-elastic fluid flow through porous media has applications in the field of petroleum engineering in the field of chemical engineering for the purification and filtration processes, in studies on drug permeation through human skin, and in the field of hydrology for the study of migration of underground water. Many researchers [5-10] have paid their attention to-the study of visco-elastic fluid flow through a porous medium.

There are many visco-elastic fluids that cannot be characterised by Maxwell's or Oldroyd's constitutive relations. One such class of visco-elastic fluids is the Rivlin-Ericksen fluid. Rivlin and Ericksen [11] proposed a theoretical model for such elastic-viscous fluids. The Rivlin-Ericksen visco-elastic fluid has applications in geophysical fluid dynamics, chemical technology and industry. Srivastava and Singh [12] studied the unsteady flow of a dusty elastico-viscous Rivlin-Ericksen fluid through channels of different cross-sections in the presence of a time-dependent pressure gradient. Garg *et al.* [13] studied the rectilinear

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oscillations of a sphere along its diameter in a conducting dusty Rivlin-Ericksen fluid in the presence of uniform magnetic field. Sharma and Kango [14] investigated the thermal convection in a Rivlin-Ericksen elastico-viscous fluid in a porous medium in the presence of a uniform magnetic field. Many articles on the Rivlin-Ericksen fluid flow through a porous medium are available in the literature [15-19].

Kim [20] investigated an unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. In this study, the fluid considered was Newtonian. Ravikumar *et al.* [21] extended the study of Kim [20], by considering the Rivlin-Ericksen fluid for the case of a semi-infinite moving porous plate in the presence of pressure gradient and constant velocity in the flow direction when the magnetic field is imposed transversely to the plate.

No attempt has been made so far to analyze the boundary layer flow of unsteady MHD mixed convective heat and mass transfer flow of a Rivlin-Ericksen fluid along a vertical permeable moving porous plate with chemical reaction and radiation absorption with variable suction, temperature and concentration. The present work aims to fill the gap in the existing literature. The governing boundary layer equations are solved analytically using two-term harmonic and non-harmonic functions. The effects of different emerging parameters are demonstrated through graphical representations and discussed at length.

2. Formulation of problem

Consider an unsteady two-dimensional laminar flow of an incompressible electrically conducting Rivlin-Ericksen fluid past a permeable semi infinite vertical moving plate embedded in a porous medium. The x^* - axis is taken along the vertical plate and the y^* - axis is normal to the plate (see Fig.1). A uniform magnetic field of strength B_0 is applied in the y^* -direction. The magnetic Reynolds number is assumed to be very small so the induced magnetic field is neglected. In addition, as there is no applied electric field Joule heating and Hall effects are also neglected. In the present analysis, thermal radiation, radiation absorption and first order chemical reaction effects are taken into account. Due to the assumption that the plate is semi-infinite, the flow variables are functions of normal distance y^* and t^* only. Under the above assumptions, the governing equations are given by [21], [22]

$$\frac{\partial v^*}{\partial y^*} = 0, \quad (2.1)$$

$$\begin{aligned} \frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = & -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + v \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_T (T^* - T_\infty^*) + \\ & + g\beta_C (C^* - C_\infty^*) - v \frac{u^*}{k^*} - \frac{\sigma B_0^2 u^*}{\rho} - \beta_I \left(\frac{\partial^3 u^*}{\partial t^* \partial y^{*2}} + v^* \frac{\partial^3 u^*}{\partial y^{*3}} \right), \end{aligned} \quad (2.2)$$

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \alpha \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho c_p} \frac{\partial q_r^*}{\partial y^*} + Q_c^* (C^* - C_\infty^*), \quad (2.3)$$

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - K_I (C^* - C_\infty^*), \quad (2.4)$$

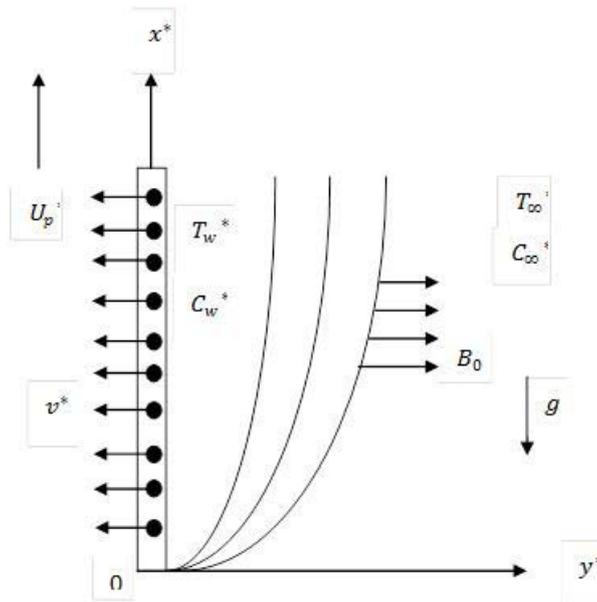


Fig.1. Schematic diagram of the physical model.

It is assumed that the permeable plate moves with a constant velocity in the direction of fluid flow and the free stream velocity follows the exponentially increasing small perturbation law. In addition, it is also assumed that the temperature and the concentration at the wall as well as the suction velocity are exponentially varying with time. With these assumptions the boundary conditions for the velocity, temperature and concentration fields are [21]

$$u^* = U_p^*, \quad T^* = T_w^* + \varepsilon(T_w^* - T_\infty^*)e^{n^*t^*}, \quad C^* = C_w^* + \varepsilon(C_w^* - C_\infty^*)e^{n^*t^*} \quad \text{at } y^* = 0, \quad (2.5)$$

$$u^* = U_\infty^* = U_0 \left(1 + \varepsilon e^{n^*t^*}\right), \quad T^* \rightarrow T_\infty^*, \quad C^* \rightarrow C_\infty^* \quad \text{as } y^* \rightarrow \infty.$$

It is clear from Eq.(2.1) that the suction velocity at the plate is either a constant or a function of time. Hence, the suction velocity normal to the plate is assumed in the form

$$v^* = -v_0 \left(1 + \varepsilon A e^{n^*t^*}\right) \quad (2.6)$$

where v_0 is the scale of suction velocity which is a non-zero positive constant. The negative sign indicates that the suction is towards the plate.

Outside the boundary layer, Eq.(2.2) gives

$$-\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} = \frac{dU_\infty^*}{dt^*} + \frac{\nu}{K^*} U_\infty^* + \frac{\sigma B_0^2 U_\infty^*}{\rho}. \quad (2.7)$$

For the case of an optically thin gray gas, the radiative heat flux in energy Eq.(2.5) in the spirit of Vincenti and Kruger [23] is expressed as

$$\frac{\partial q_r^*}{\partial y^*} = -4a^* \sigma^* (T_\infty^{*4} - T^{*4}). \quad (2.8)$$

It is assumed that the temperature differences within the flow are sufficiently small such that T^{*4} may be expressed as a linear function of the temperature T^* . This is accomplished by expanding T^{*4} in Taylor series about T_∞^* and neglecting higher order terms, thus

$$T^{*4} \cong 4T_\infty^{*3}T^* - 3T_\infty^{*4}. \quad (2.9)$$

By substituting Eq.(2.9) in Eq.(2.8) we obtain

$$\frac{\partial q_r^*}{\partial y^*} = 16a^* \sigma^* T_\infty^{*3} (T^* - T_\infty^*). \quad (2.10)$$

In order to write the governing equations and boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$U = \frac{u^*}{U_0}, \quad \eta = \frac{V_0 y^*}{\nu}, \quad U_\infty = \frac{U_\infty^*}{U_0}, \quad U_p = \frac{U_p^*}{U_0}, \quad t = \frac{t^* V_0^2}{\nu}, \quad \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*},$$

$$R = \frac{16\sigma^* a^* T_\infty^{*3} \nu}{\rho c_p V_0^2}, \quad Rm = \frac{\beta_l V_0^2}{\nu^2}, \quad C = \frac{C_w^* - C_\infty^*}{C_w^* - C_\infty^*}, \quad K = \frac{k^* V_0^2}{\nu^2}, \quad n = \frac{n^* \nu}{V_0^2},$$
(2.11)

$$Pr = \frac{\nu}{\alpha} = \frac{\nu \rho c_p}{k}, \quad Qc = \frac{Q_C^* \nu (C_w^* - C_\infty^*)}{V_0^2 (T_w^* - T_\infty^*)}, \quad kr = \frac{K_l \nu}{V_0^2},$$

$$M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, \quad Gr = \frac{\nu g \beta_T (T_w^* - T_\infty^*)}{U_0 V_0^2}, \quad Gc = \frac{\nu g \beta_C (C_w^* - C_\infty^*)}{U_0 V_0^2}, \quad Sc = \frac{\nu}{D}.$$

In view of Eqs (2.6)-(2.11), Eqs (2.2)-(2.4) reduce to the following dimensionless form.

$$\frac{\partial U}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial U}{\partial \eta} = \frac{dU_\infty}{dt} + N(U_\infty - U) + \frac{\partial^2 U}{\partial \eta^2} +$$

$$+ Gr\theta + GcC - Rm \left(\frac{\partial^3 U}{\partial t \partial \eta^2} - (1 + \varepsilon A e^{nt}) \frac{\partial^3 U}{\partial \eta^3} \right), \quad (2.12)$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial \theta}{\partial \eta} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial \eta^2} - R\theta + QcC, \quad (2.13)$$

$$\frac{\partial C}{\partial t} - \left(1 + \varepsilon A e^{nt}\right) \frac{\partial C}{\partial \eta} = \frac{1}{Sc} \frac{\partial^2 C}{\partial \eta^2} - KrC \quad (2.14)$$

where $N = M + \frac{1}{K}$.

The corresponding boundary conditions are

$$\begin{aligned} U = U_p, \quad T = 1 + \varepsilon e^{nt}, \quad C = 1 + \varepsilon e^{nt} \quad \text{at } \eta = 0, \\ U = U_\infty = 1 + \varepsilon e^{nt}, \quad T \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } \eta \rightarrow \infty. \end{aligned} \quad (2.15)$$

2. Solution of the problem

Equations (2.12)-(2.14) are coupled, non-linear partial differential equations and these cannot be solved in closed-form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the plate as

$$\begin{aligned} U &= f_0(\eta) + \varepsilon e^{nt} f_1(\eta) + O(\varepsilon^2) + \dots, \\ \theta &= g_0(\eta) + \varepsilon e^{nt} g_1(\eta) + O(\varepsilon^2) + \dots, \\ C &= h_0(\eta) + \varepsilon e^{nt} h_1(\eta) + O(\varepsilon^2) + \dots \end{aligned} \quad (3.1)$$

Substituting Eq.(3.1) in Eqs (2.12)-(2.14) and equating harmonic and non-harmonic terms, and neglecting the higher order terms of $O(\varepsilon^2)$, we obtain

$$Rmf_0''' + f_0'' + f_0' - Nf_0 = -Gr g_0 - Gch_0 - N, \quad (3.2)$$

$$Rmf_1''' + (1 - nR_m)f_1'' + f_1' - (N + n)f_1 = -n - N - Af_0' - Grg_1 - Gch_1 - RmAf_0''', \quad (3.3)$$

$$g_0'' + Pr g_0' - RPr g_0 = -QcPr h_0, \quad (3.4)$$

$$g_1'' + Pr g_1' - (R + n)Pr g_1 = -QcPr h_1 - g_0'Pr A, \quad (3.5)$$

$$h_0'' + Sc h_0' - KrSc h_0 = 0, \quad (3.6)$$

$$h_1'' + Sc h_1' - (Kr + n)Sc h_1 = -h_0' Sc A \quad (3.7)$$

where prime denotes ordinary differentiation with respect to η . The corresponding boundary conditions can be written as

$$f_0 = U_p, \quad f_1 = 0, \quad g_0 = 1, \quad g_1 = 1, \quad h_0 = 1, \quad h_1 = 1 \quad \text{at} \quad \eta = 0, \quad (3.8)$$

$$f_0 = 1, \quad f_1 = 1, \quad g_0 \rightarrow 1, \quad g_1 \rightarrow 1, \quad h_0 \rightarrow 1, \quad h_1 \rightarrow 1 \quad \text{as} \quad \eta \rightarrow \infty.$$

The solutions of Eqs (3.4) - (3.7) subject to the boundary conditions (3.8) are

$$g_0 = k_4 e^{-m_6 \eta} + k_3 e^{-m_2 \eta}, \quad (3.9)$$

$$g_1 = k_8 e^{-m_8 \eta} + k_5 e^{-m_6 \eta} + k_6 e^{-m_4 \eta} + k_7 e^{-m_2 \eta}, \quad (3.10)$$

$$h_0 = e^{-m_2 \eta}, \quad (3.11)$$

$$h_1 = k_2 e^{-m_4 \eta} + k_1 e^{-m_2 \eta}. \quad (3.12)$$

Equations (3.2) and (3.3) are third order differential equations when $Rm \neq 0$ and we have two boundary conditions, so we follow Beard and Walters [24] as

$$f_0 = f_{01} + Rmf_{02} + O(Rm^2), \quad (3.13)$$

$$f_1 = f_{11} + Rmf_{12} + O(Rm^2). \quad (3.14)$$

Substituting Eqs (3.13) and (3.14) into Eqs (3.2) and (3.3), equating different powers of Rm and neglecting $O(Rm^2)$, we obtain

$$f_{01}'' + f_{01}' - Nf_{01} = -Gr k_4 e^{-m_6 \eta} - (Gr k_3 + Gc) e^{-m_2 \eta} - N, \quad (3.15)$$

$$f_{02}'' + f_{02}' - Nf_{02} = -f_{01}''', \quad (3.16)$$

$$f_{11}'' + f_{11}' - (N+n)f_{11} = -n - N - Af_{01}' - Gr g_1 - Gc h_1, \quad (3.17)$$

$$f_{12}''' + f_{12}'' - nf_{11}'' + f_{12}' - (N+n)f_{12} = -Af_{02}' - Af_{01}''', \quad (3.18)$$

and the corresponding boundary conditions are

$$f_{01} = U_p, \quad f_{02} = 0, \quad f_{11} = 0, \quad f_{12} = 0 \quad \text{at} \quad \eta = 0, \quad (3.19)$$

$$f_{01} = 1, \quad f_{02} = 0, \quad f_{11} = 1, \quad f_{12} = 0 \quad \text{as} \quad \eta \rightarrow \infty.$$

Solving Eqs (3.15) - (3.18) under the boundary conditions (3.19), we obtain solutions as

$$f_{01} = k_{11} e^{-m_{10} \eta} + k_9 e^{-m_6 \eta} + k_{10} e^{-m_2 \eta} + 1, \quad (3.20)$$

$$f_{02} = (k_{15} + k_{12})e^{-m_{10}\eta} + k_{13}e^{-m_6\eta} + k_{14}e^{-m_2\eta}, \tag{3.21}$$

$$f_{11} = k_{21}e^{-m_{12}\eta} + k_{16}e^{-m_{10}\eta} + k_{17}e^{-m_6\eta} + k_{18}e^{-m_2\eta} + k_{19}e^{-m_8\eta} + k_{20}e^{-m_4\eta} + I, \tag{3.22}$$

$$f_{12} = (k_{2815m} + k_{22})e^{-m_{12}\eta} + k_{23}e^{-m_{10}\eta} + k_{24}e^{-m_6\eta} + k_{25}e^{-m_2\eta} + k_{1826}e^{-m_8\eta} + k_{1827}e^{-m_4\eta}. \tag{3.23}$$

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become

$$C(\eta, t) = h_0(\eta) + \varepsilon e^{nt} h_1(\eta) = e^{-m_2\eta} + \varepsilon e^{nt} (k_2 e^{-m_4\eta} + k_1 e^{-m_2\eta}), \tag{3.24}$$

$$\begin{aligned} \theta(\eta, t) &= g_0(\eta) + \varepsilon e^{nt} g_1(\eta) = k_4 e^{-m_6\eta} + k_3 e^{-m_2\eta} + \\ &+ \varepsilon e^{nt} (k_8 e^{-m_8\eta} + k_5 e^{-m_6\eta} + k_6 e^{-m_4\eta} + k_7 e^{-m_2\eta}), \end{aligned} \tag{3.25}$$

$$\begin{aligned} U(\eta, t) &= f_0(\eta) + \varepsilon e^{nt} f_1(\eta) = (f_{01} + Rmf_{02}) + \varepsilon e^{nt} (f_{11} + Rmf_{12}) = \\ &= (k_{11}e^{-m_{10}\eta} + k_9e^{-m_6\eta} + k_{10}e^{-m_2\eta} + I) + Rm((k_{15} + k_{12})e^{-m_{10}\eta} + k_{13}e^{-m_6\eta} + k_{14}e^{-m_2\eta}) + \\ &+ \varepsilon e^{nt} ((k_{21}e^{-m_{12}\eta} + k_{16}e^{-m_{10}\eta} + k_{17}e^{-m_6\eta} + k_{18}e^{-m_2\eta} + k_{19}e^{-m_8\eta} + k_{20}e^{-m_4\eta} + I) + \\ &+ Rm((k_{2815m} + k_{22})e^{-m_{12}\eta} + k_{23}e^{-m_{10}\eta} + k_{24}e^{-m_6\eta} + k_{25}e^{-m_2\eta} + k_{1826}e^{-m_8\eta} + k_{1827}e^{-m_4\eta})) \end{aligned} \tag{3.26}$$

where the expressions for the constants are given in the Appendix.

The physical quantities of engineering interest such as skin-friction, Nusselt number and Sherwood number for this type of boundary layer flow can be defined and determined as follows

$$\begin{aligned} \tau &= \left(\frac{\partial U}{\partial \eta} \right)_{\eta=0} = -[m_{10}k_{11} + m_6k_9 + m_2k_{10} + Rm((k_{15} + k_{12})m_{10} + k_{13}m_6 + k_{14}m_2)] + \\ &- \varepsilon e^{nt} [k_{21}m_{12} + k_{16}m_{10} + k_{17}m_6 + k_{18}m_2 + k_{19}m_8 + k_{20}m_4 + \\ &+ Rm((k_{2815m} + k_{22})m_{12} + k_{23}m_{10} + k_{24}m_6 + k_{25}m_2 + k_{1826}m_8 + k_{1827}m_4)], \end{aligned} \tag{3.27}$$

$$Nu = \left(\frac{\partial \theta}{\partial \eta} \right)_{\eta=0} = -m_6k_4 - m_2k_3 + \varepsilon e^{nt} (-m_8k_8 - m_6k_5 - m_4k_6 - m_2k_7), \tag{3.28}$$

$$Sh = \left(\frac{\partial C}{\partial \eta} \right)_{\eta=0} = -m_6k_4 - m_2k_3 + \varepsilon e^{nt} (-m_8k_8 - m_6k_5 - m_4k_6 - m_2k_7). \tag{3.28}$$

3. Results and discussion

In order to get physical insight into the problem, the effects of various parameters encountered in the equations of the problem are analyzed with the help of Figs 2 -14, while the values of some of the physical

parameters are taken as constants such as $Sc=0.62$ (water), $\varepsilon=0.2$, $t=1.0$, $kr=0.5$, $R=0.1$, $n=1$, $Pr=0.72$ (air), $M=2.0$, $k=100$, $Gr=2.0$, $Gc=2.0$, $Up=1.0$, $A=0.5$, $Qc=0.5$, $Rm=0.05$ in all the graphs and tables. All the graphs and tables therefore correspond to these values unless specifically indicated in the appropriate graph or table.

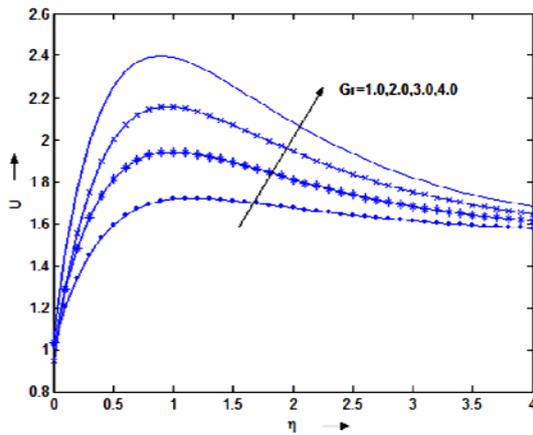


Fig.2. Effect of Gr on the velocity.

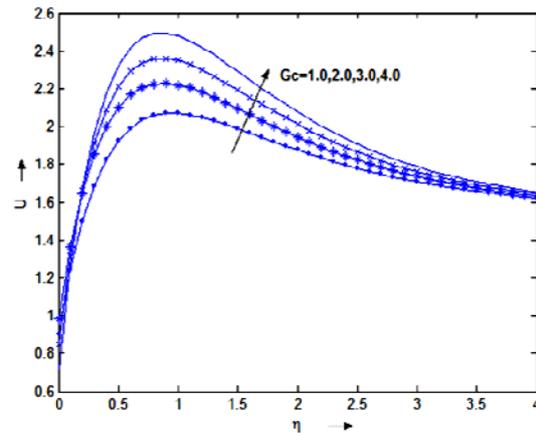


Fig.3. Effect of Gc on the velocity.

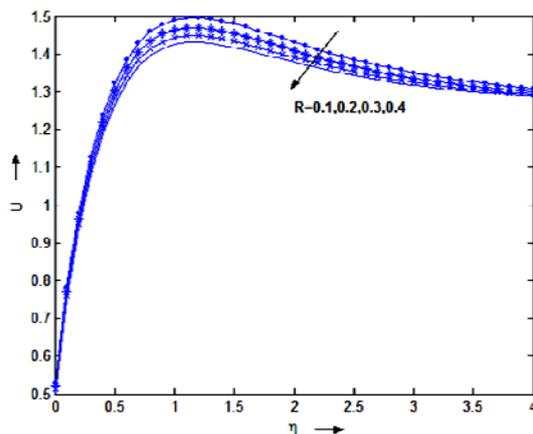


Fig.4. Effect of R on the velocity.

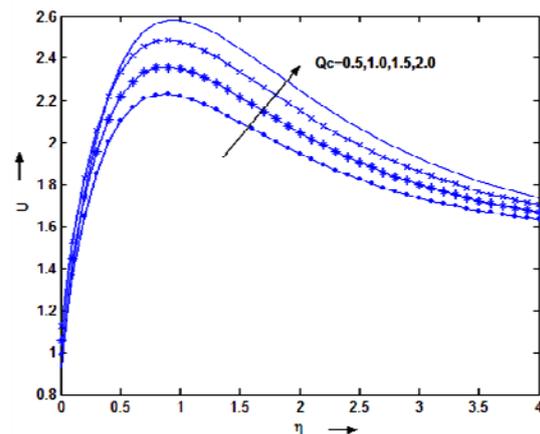


Fig.5. Effect of Qc on the velocity.

Figure 2 illustrates the effect of thermal Grashof number Gr on the velocity profiles. It is noticed from the figure that an increase in Gr leads to a rise in the velocity due to the enhancement in buoyancy force. Here, the positive values of Gr correspond to cooling of the surface. The similar behavior is observed with increasing values of solutal Grashof number Gc (Fig.3) and positive values of Gc indicates that the chemical species concentration in the free stream region is less than the concentration at the boundary. The effect of the radiation parameter R on the velocity field is depicted in Fig.4. It is observed that the velocity profiles decreases with an increasing values of the radiation parameter R . This is because of the fact that an increase in R implies less interaction of radiation with the momentum boundary layer. Figure 5 depicts the effect of the radiation absorption parameter Qc on the fluid velocity and it is observed that the fluid velocity increases as the Qc increases. This is due to the fact when heat is absorbed, it increases the buoyancy force thereby increasing the fluid velocity.

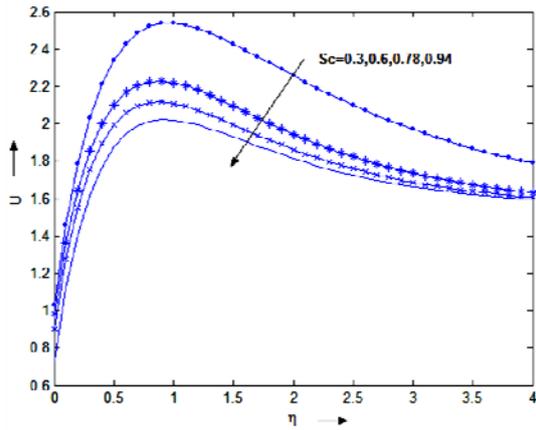


Fig.6. Effect of Sc on the velocity.

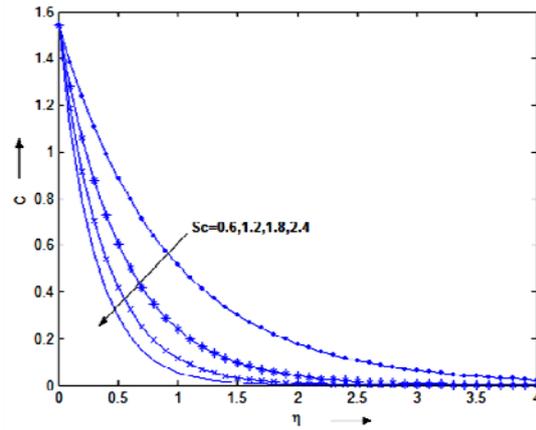


Fig.7. Effect of Sc on the concentration.

The effect of the Schmidt number Sc on the velocity and concentration are shown in Figs 6-7, respectively. The Schmidt number embodies the ratio of momentum to mass diffusivity. The Schmidt number quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reduction in the velocity and concentration boundary layer thicknesses.

Figure 8 illustrates the variation in velocity function for several values of the visco-elasticity parameter Rm . It is found that an increase in Rm leads to a decrease in the velocity distribution across the boundary layer. This can be explained by the fact that, as the viscoelastic parameter increases, the hydromagnetic boundary layer adheres strongly to the surface, which in turn retards the flow.

Figure 9 illustrates the variation of velocity distribution across the boundary layer for several values of plate moving velocity U_p in the direction of the fluid flow. From the figure it is interesting to note that, for different values of plate moving velocities, the velocities started at different points, but they reached to single value. That is, although we have different initial plate moving velocities, the velocity decreases to the constant value for given material parameters.

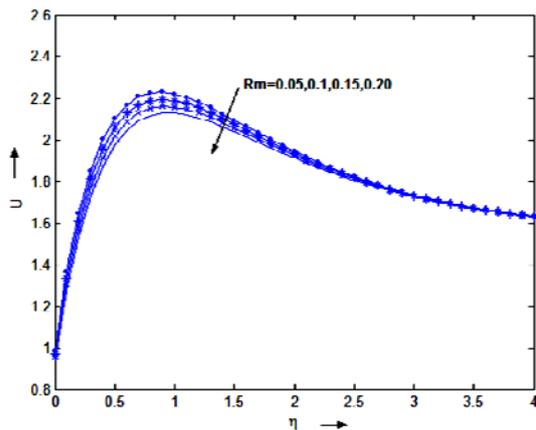


Fig.8. Effect of Rm on the velocity.

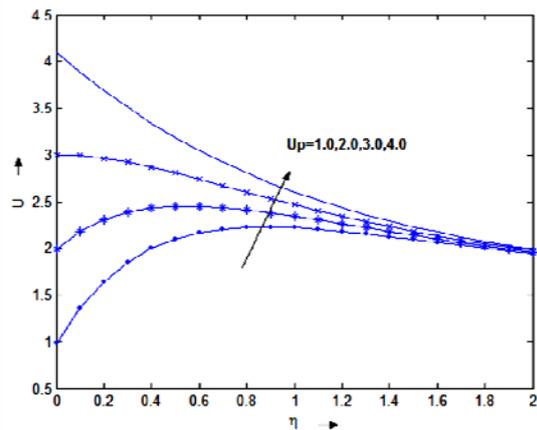


Fig.9. :Effect of U_p on the velocity.

From Fig.10, it follows that an increase in the Prandtl number results in decrease in the temperature. This is, because for small values of the Pr, there will be more thermal conductivity of the fluid. Therefore, for large values of Pr, heat is able to diffuse away from the heated surface more rapidly. Hence, in the case of smaller Pr numbers the thermal boundary layer is thicker and the rate of heat transfer is reduced. Figure 11 illustrates the variation in temperature function for several values of radiation absorption parameter Q_c . It is found that an increase in Q_c leads to an increase in the temperature distribution across the boundary layer. Figure 12 shows the effect of the radiation parameter R on the temperature. A rise in R causes a significant fall in the temperature values from the highest value at the wall across the boundary layer to the free stream. Thus, a greater value of R corresponds to smaller radiation flux and the minimum temperature is observed. Radiation thereby reduces the rate of energy transport to the fluid.

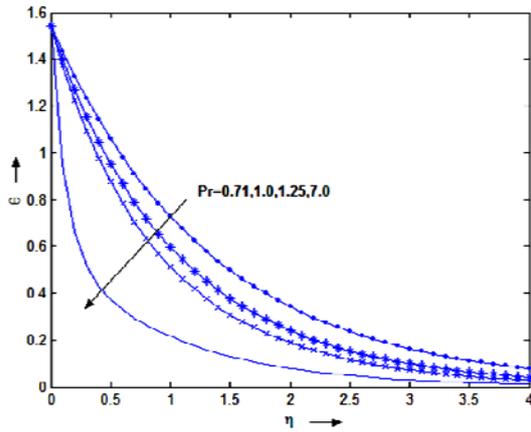


Fig.10. Effect of Pr on the temperature.

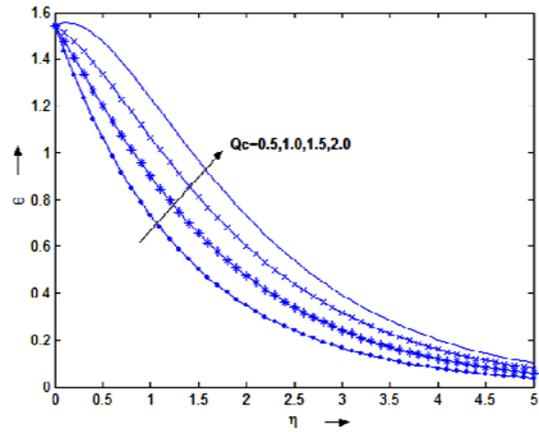


Fig.11. Effect of Q_c on the temperature.

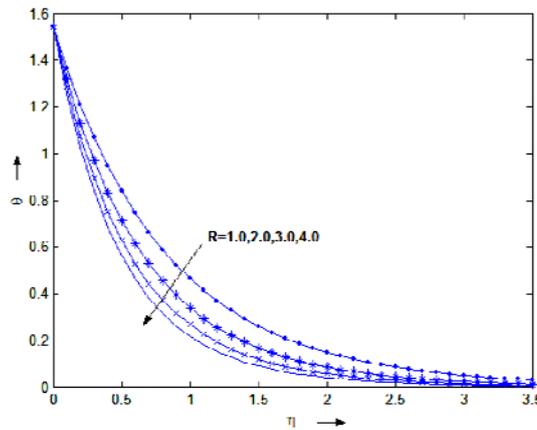


Fig.12. Effect of R on the temperature.

Figure 13 shows the effect of the chemical reaction parameter Kr on concentration. It is noticed that species concentration and concentration boundary layer are decreasing, as the values of the chemical reaction parameter are increasing. It follows from Fig.14 that an increase in either the chemical reaction (Kr) or Schmidt number (Sc) decreases the Sherwood number (Sh) (This is in agreement with the result of Ref. [18]).

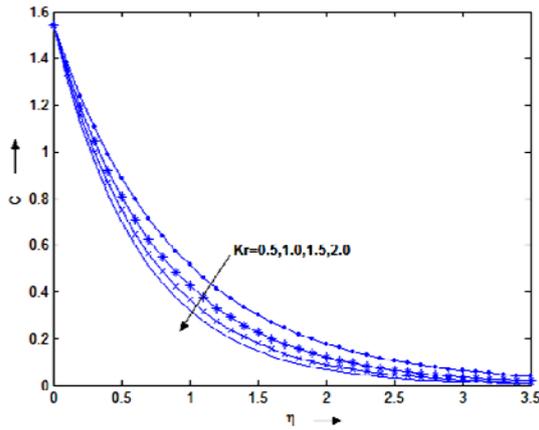


Fig.13. Effect of Kr on the concentration.

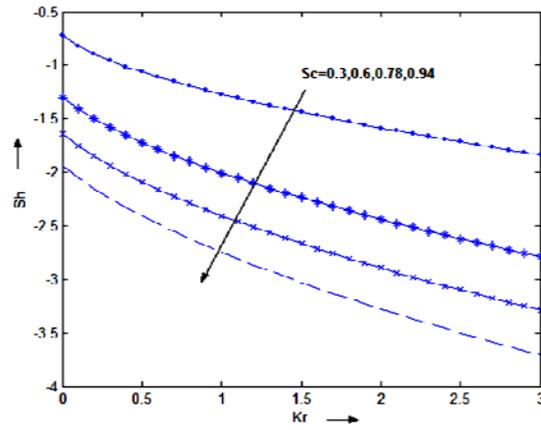


Fig.14. Effect of Sc on Sh against Kr .

Table 1 represents a comparison of the numerical values of skin-friction τ obtained in the present case with those of Ravikumar *et al.* [21], Kim [20] for different values of non dimensional exponential index n . It is found that there is a good agreement. From Tab.2, it is of interest to note that the Nusselt number (Nu) increases with increasing intensity of the radiation absorption Q_c , however, an increase in the Schmidt number Sc , chemical reaction parameter Kr , scalar constant ϵ , non-dimensional exponential index n , suction velocity parameter A , radiation parameter R causes an decrease in the Nusselt number. In particular, the Nusselt number decreases with an increase in the Prandtl number Pr (this is in agreement with the result of Ref.[21])

Table 1. Comparison of the skin friction (τ) of present case with those of Ravikumar *et al.* [21], Kim [20] for different values of n .

n	Kim [20] ($Rm=0, Kr=0,$ $R=0, Gc=0, Qc=0$)	Ravikumar <i>et al.</i> [21] ($Rm=0.1, Kr=0,$ $R=0, Gc=0, Qc=0$)	Present work ($Rm=0.1, Kr=0.1,$ $R=0.1, Gc=0.1, Qc=0.1$)
1.0	3.9475	3.9508	3.9519
0.1	2.8780	2.8671	2.8682
0.01	2.8172	2.7653	2.7615

Table 2. Effect of $Sc, Kr, \epsilon, n, A, Pr, R$ and Q_c on the Nussult number Nu ($t=1.0$).

Sc	Kr	ϵ	n	A	Pr	R	Q_c	Nu
0.6	0.5	0.2	1.0	0.5	0.71	0.1	0.01	-1.6109
								-1.6124
	1.0							-1.6135
		0.3						-2.0232
			2.0					-4.7603
					1.0			-6.1524
						0.2		-6.3146
				0.7				-6.5464
							0.02	-6.5314

From Tab.3, it can be seen that the skin friction τ on the porous plate decreases with increasing values of magnetic field parameter M , Schimdt number Sc and radiation absorption parameter Q_c , whereas it

increases with an increase in all other parameters, in particular it increases with increasing the strength of suction velocity (A) as well as the thermal buoyancy (Gr) (this is in agreement with the result of Ref.[21]).

Table 3. Effect of Gr , Gc , M , Up , Sc , ε , Kr , n , A , Pr , R , Qc and K on skin friction τ ($t=1.0$).

Gr	Gc	M	Up	Sc	ε	Kr	n	A	Pr	R	Qc	K	τ
1.0	1.0	2.0	0.5	0.6	0.2	0.5	1.0	0.5	0.71	0.1	0.01	100	4.0445
	2.0												4.5498
		2.0											5.5323
			3.0										4.6761
				0.6									5.4559
					0.78								4.3238
						0.3							5.1244
							0.6						5.1426
								2.0					11.7657
									0.6				12.1926
										1.0			15.3086
											0.02		15.2970
												0.2	15.4582
												200	16.2547

4. Conclusion

This paper studied the combined effects of radiation absorption and chemical reaction on a boundary layer flow of a Rivlin-Ericksen fluid with heat and mass transfer over a vertical moving porous plate. The thermal radiation, radiation absorption, transversely applied magnetic field and first order chemical reaction effects are taken into consideration. It is assumed that the temperature and the concentration at the wall as well as the suction velocity are exponentially varying with time. The governing equations of the problem were solved analytically by using the perturbation technique. A comprehensive set of graphical results for the velocity, temperature and concentration is presented and their dependence on physical parameters is discussed. Our results revealed that the velocity, temperature and Nusselt number increase and the skin friction decreases with an increase in radiation absorption. Furthermore, an increase in chemical reaction enhances the skin friction and reduces the Sherwood number and Nusselt number.

Nomenclature

- A – real positive constant
- a^* – mean absorption coefficient
- B_o – magnetic induction
- C^* – species concentration in the boundary layer
- C_∞^* – species concentration in the fluid far away from the plate
- c_p – specific heat at constant pressure
- D – molecular diffusivity
- Gc – solutal Grashof number
- Gr – thermal Grashof number
- g – acceleration due to gravity
- Kr – chemical reaction parameter
- K_l – chemical reaction parameter
- k^* – permeability of the porous medium

- M – magnetic field parameter
 Pr – Prandtl number
 p^* – pressure
 R – radiation parameter
 Rm – viscoelasticity parameter
 Sc – Schmidt number
 T^* – temperature of the fluid in the boundary layer
 T_∞^* – temperature of the fluid far away from the plate
 T_w^*, C_w^* – wall temperature and concentration
 T_∞^*, C_∞^* – temperature and concentration
 t^* – dimensional time
 U_p^* – plate velocity
 U_∞^* – free stream velocity
 U_0, n^* – constants
 u^*, v^* – dimensional velocity components in x^*, y^* directions
 Qc – radiation absorption parameter
 Q_c^* – coefficient of proportionality for the absorption of radiation
 q_r^* – radiative heat flux
 x^*, y^* – dimensional distance along and perpendicular to the plate
 ρ – fluid density
 ν – kinematic viscosity
 α – thermal diffusivity
 β_I – kinematic visco-elasticity
 β_T, β_C – thermal and concentration expansion coefficients
 $\varepsilon, \varepsilon A$ – small values less than unity
 σ – fluid electrical conductivity
 σ^* – Stefan-Boltzmann constant

APPENDIX

$$m_2 = \frac{Sc + \sqrt{Sc^2 + 4KrSc}}{2}, \quad m_4 = \frac{Sc + \sqrt{Sc^2 + 4(Kr+n)Sc}}{2}, \quad m_6 = \frac{Pr + \sqrt{Pr^2 + 4RPr}}{2},$$

$$m_8 = \frac{Pr + \sqrt{Pr^2 + 4(R+n)Pr}}{2}, \quad m_{10} = \frac{I + \sqrt{I + 4N}}{2}, \quad m_{12} = \frac{I + \sqrt{I + 4(N+n)}}{2},$$

$$k_1 = \frac{AScm_2}{m_2^2 - Scm_2 - (Kr+n)Sc}, \quad k_2 = I - k_1, \quad k_3 = \frac{-QcPr}{m_2^2 - Prm_2 - RPr}, \quad k_4 = I - k_3,$$

$$k_5 = \frac{Prm_6k_4A}{m_6^2 - Prm_6 - (R+n)Pr}, \quad k_6 = \frac{-QcPrk_2}{m_4^2 - Prm_4 - (R+n)Pr}, \quad k_7 = \frac{Pr(m_2k_3A - k_1Qc)}{m_2^2 - Prm_2 - (R+n)Pr},$$

$$k_8 = I - k_5 - k_6 - k_7, \quad k_9 = \frac{-Gr k_4}{m_6^2 - m_6 - N}, \quad k_{10} = \frac{-(Gr k_3 + Gc)}{m_2^2 - m_2 - N},$$

$$k_{11} = U_p - k_9 - k_{10} - I, \quad k_{12} = \frac{k_{11} m_{10}^3}{m_{10}^2 - m_{10} - N}, \quad k_{13} = \frac{k_9 m_6^3}{m_6^2 - m_6 - N},$$

$$k_{14} = \frac{k_{10} m_2^3}{m_2^2 - m_2 - N}, \quad k_{15} = -k_{12} - k_{13} - k_{14}, \quad k_{16} = \frac{A k_{11} m_{10}}{m_{10}^2 - m_{10} - (N + n)},$$

$$k_{17} = \frac{A k_9 m_6 - Gr k_5}{m_6^2 - m_6 - (N + n)}, \quad k_{18} = \frac{A k_{10} m_2 - Gr k_7 - Gc k_1}{m_2^2 - m_2 - (N + n)}, \quad k_{19} = \frac{-Gr k_8}{m_8^2 - m_8 - (N + n)},$$

$$k_{20} = \frac{-Gr k_6 - Gc k_2}{m_4^2 - m_4 - (N + n)}, \quad k_{21} = -I - k_{16} - k_{17} - k_{18} - k_{19} - k_{20}, \quad k_{22} = \frac{k_{21} m_{12}^3 + n k_{21} m_{12}^2}{m_{12}^2 - m_{12} - (N + n)},$$

$$k_{23} = \frac{k_{16} m_{10}^3 + n k_{16} m_{10}^2 + A(k_{15} + k_{12}) m_{10} + A k_{11} m_{10}^3}{m_{10}^2 - m_{10} - (N + n)}, \quad k_{24} = \frac{k_{17} m_6^3 + n k_{17} m_6^2 + A k_{13} m_6 + A k_9 m_6^3}{m_6^2 - m_6 - (N + n)},$$

$$k_{25} = \frac{k_{18} m_2^3 + n k_{18} m_2^2 + A k_{14} m_2 + A k_{10} m_2^3}{m_2^2 - m_2 - (N + n)}, \quad k_{26} = \frac{k_{19} m_8^3 + n k_{19} m_8^2}{m_8^2 - m_8 - (N + n)},$$

$$k_{27} = \frac{k_{20} m_4^3 + n k_{20} m_4^2}{m_4^2 - m_4 - (N + n)}, \quad k_{28} = -k_{22} - k_{23} - k_{24} - k_{25} - k_{26} - k_{27}.$$

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