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INVESTIGATION OF WAVES GENERATED IN TRANSVERSELY ISOTROPIC MICROPOLAR GENERALIZED THERMOELASTIC HALF SPACE UNDER TEMPERATURE DEPENDENT PROPERTIES

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The present study deals with the propagation of waves in a transversely isotropic micropolar generalized thermoelastic material possessing temperature dependent elastic properties. After developing the solution for LS, GL and CT theory, the phase velocities and attenuation quality factor have been obtained. The expressions for amplitudes of stresses, displacements, microratation and temperature distribution have been derived and computed numerically. The numerically evaluated results have been plotted graphically. Some particular cases of interest have also been obtained.

Key words: micropolar, transversely isotropic, generalized thermoelastic, amplitude ratios.

1. Introduction

Eringen [1], purposed the concept of microcontinuum theory by considering the microstructure effects, although the theory itself was still a continuum formulation. At the initial stage, the micro-continuum theory consists of the micropolar, microstretch and micromorphic theories, depending on the number of microdegrees of freedom involved. These theories turn out to be a potential tool to model the behavior of the material with a complicated microstructure. For instance, in a foam composite the size of the reinforced phase is comparable to the intrinsic length scale of the foam, so in this situation the consideration of microstructure of the foam to some extent is must. Hence, a high order continuum model must be assigned for the foam matrix. A similar behavior is observed in the case of nanocomposites where the scale of the reinforced phase is very small and the surrounding matrix cannot be homogenized as a simple material (Cauchy medium).

It is predicted from the classical theory of heat conduction that the effect of thermal disturbance from a heat conducting material subjected to a thermal disturbance will be felt instantaneously at distances infinitely far from the point of its origin. This forecast seems unrealistic from a physical point of view, in particular, for problems involving sudden heat inputs. This inadequacy of the theory stems from the governing parabolic type partial differential equation of temperature distribution, which allows an infinite speed for thermal signals. During last three decades this drawback of non- classical theories has been overcome by considering the modified version of classical Fourier's law of heat conduction, which involves hyperbolic-type heat transport equation and admit finite speed for thermal signals.

Nowacki [2] and Eringen [3] included the thermal effects in the micropolar theory and developed the micropolar theory of thermoelasticity. The basic equations of linear theory of micropolar thermoelasticity were also derived by Tauchert *et al*. [4]. Dost and Tabarrok [5] obtained the equations for micropolar generalized thermoelasticity by considering the Green -Lindsay theory. Dhaliwal and Singh [6] presented a review on the theory of micropolar thermoelasticity. The micropolar theory including heat-flux among the constitutive variables was developed by Chandrasekhariah [7].

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The elastic modulus is an important physical property of materials reflecting the elastic deformation capacity of the material when subjected to an applied external load. Most of the investigations were made under the assumption of the temperature-independent material properties, which limit the applicability of the solutions obtained to certain ranges of temperature. Modern structural elements are often subjected to temperature change of such magnitude that their material properties may no longer be regarded as having constant values even in an approximate sense. Fernandes and Stouffer [8] gave the general theory for elastic solids with temperature dependent mechanical properties. Lomarkin [9] showed that at high temperature the material characteristics, such as the modulus of elasticity, coefficient of thermal expansion and thermal conductivity are no longer constants. The thermal and mechanical properties of the materials vary with temperature, so the temperature-dependence of the material properties must be taken into consideration in the thermal stress analysis of these elements.

Various authors investigated the problem of based on temperature dependent elastic modulus for the thermoelastic, thermo-viscoelastic, micropolar medium (Ezzat *et al*. [10], Othman [11], Aouadi [12], Othman *et al.* [13]).

In the present paper, we discussed the propagation of waves in a transversely isotropic micropolar generalized thermoelastic half space under temperature dependent properties. The phase velocities and attenuation quality factors are obtained and plotted numerically for different theories of thermoelasticity. The expressions for the amplitude ratio of components of displacement, microrotation, stresses and temperature distribution are also obtained. Some special cases of interest are also deduced.

2. Basic equations

The basic equations in the dynamic theory of plain strain of a homogeneous, transversely isotropic micropolar generalized thermoelastic solid in the absence of body forces, body couples and heat sources are given by:

Balance laws

$$t_{kl,k} = \rho \ddot{u}_l \, , \tag{2.1}$$

$$m_{kl,k} + \varepsilon_{lmn} t_{mn} = \rho j \ddot{\varphi}_l \, , \tag{2.2}$$

and the heat conduction equation is given by

$$K^*_{mn} T_{,mn} = \rho C^* \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \dot{T} + T_0 \left(\frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2} \right) \beta_{mn} u_{m,n} \, . \tag{2.3}$$

The constitutive relations can be given as

$$t_{kl} = A_{klmn} E_{mn} + G_{klmn} \Psi_{mn} - \beta_{kl} \left(1 + \tau_1 \frac{\partial}{\partial t} \right) T \, , \tag{2.4}$$

$$m_{kl} = B_{lkmn} E_{mn} + G_{mnlk} \Psi_{mn} \tag{2.5}$$

where the deformation and wryness tensor are defined by

$$E_{mn} = u_{n,m} + \varepsilon_{nmk} \varphi_k \, , \quad \Psi_{mn} = \varphi_{m,n} \, . \tag{2.6}$$

The list of symbols are given in the nomenclature.

3. Problem formulation and solution

Following Slaughter [14], appropriate transformations have been used on the set of Eqs (2.1)-(2.5) to derive equations for a transversely isotopic micropolar generalized thermoelastic medium.

We consider a homogeneous, transversely isotropic micropolar generalized thermoelastic half space initially in undeformed state and at uniform temperature T_0. We take the origin of the coordinate system on the top plane surface and the x_3 axis pointing normally into the half-space, which is thus represented by $x_3 \ge 0$. We choose the x_1-axis along the direction of wave propagation so that all particles on a line parallel to the x_2-axis are equally displaced. Therefore, all the field quantities will be independent of x_2 coordinate. Further, the disturbance is assumed to be confined to the neighborhood of the free surface and hence vanishes as $x_3 \to \infty$. So, we assume the components of the displacement and microrotation vector of the form

$$\boldsymbol{u} = (u_1, 0, u_3), \qquad \varphi = (0, \varphi_2, 0). \tag{3.1}$$

Thus, the field equations reduce to

$$A_{11}\frac{\partial^2 u_1}{\partial x_1^2} + A_{55}\frac{\partial^2 u_1}{\partial x_3^2} + (A_{13}+A_{56})\frac{\partial^2 u_3}{\partial x_1 \partial x_3} + K_1\frac{\partial \varphi_2}{\partial x_3} - \beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1} = \rho\frac{\partial^2 u_1}{\partial t^2}, \tag{3.2}$$

$$A_{66}\frac{\partial^2 u_3}{\partial x_1^2} + A_{33}\frac{\partial^2 u_3}{\partial x_3^2} + (A_{13}+A_{56})\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + K_2\frac{\partial \varphi_2}{\partial x_1} - \beta_3\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3} = \rho\frac{\partial^2 u_3}{\partial t^2}, \tag{3.3}$$

$$B_{77}\frac{\partial^2 \varphi_2}{\partial x_1^2} + B_{66}\frac{\partial^2 \varphi_2}{\partial x_3^2} - X\varphi_2 + K_1\frac{\partial u_1}{\partial x_3} + K_2\frac{\partial \varphi_2}{\partial x_3} = \rho\frac{\partial^2 \varphi_2}{\partial t^2}, \tag{3.4}$$

$$K_1^*\frac{\partial^2 T}{\partial x_1^2} + K_3^*\frac{\partial^2 T}{\partial x_3^2} = \rho c^*\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} + T_0\left(1+n_0\tau_0\frac{\partial}{\partial t}\right)\left(\beta_1\frac{\partial \dot{u}_1}{\partial x_1} + \beta_3\frac{\partial \dot{u}_3}{\partial x_3}\right), \tag{3.5}$$

$$t_{33} = A_{11}\frac{\partial u_1}{\partial x_1} + A_{33}\frac{\partial u_3}{\partial x_3} - \beta_3\left(1+\tau_1\frac{\partial}{\partial t}\right)T, \tag{3.6}$$

$$t_{31} = A_{65}\frac{\partial u_3}{\partial x_1} - K_1\varphi_2 + A_{55}\frac{\partial u_1}{\partial x_3}, \tag{3.7}$$

$$m_{32} = B_{66}\frac{\partial \varphi_2}{\partial x_3}, \tag{3.8}$$

where $\beta_1 = A_{11}\alpha_1 + A_{13}\alpha_3, \beta_3 = A_{31}\alpha_1 + A_{33}\alpha_3$, $K_1 = A_{56} - A_{55}, K_2 = A_{66} - A_{56}, X = K_2 - K_1$, and we have used the notations $11 \to 1, 33 \to 3, 12 \to 7, 13 \to 6, 23 \to 5$ for the material constants.

For the Lord and Shulman (L-S) theory we take $\tau_1 = 0, n_0 = 1$, for the Green and Lindsay (G-L) [15] theory we take $\tau_1 \ge \tau_0 \ge 0, n_0 = 0$, and for the coupled theory (CT) $\tau_1 = \tau_0 = 0, n_0 = 0$.

Our aim is to investigate the effect of temperature dependence of modulus of elasticity, keeping the other elastic and thermal parameters as constant. Therefore we assume

$$\begin{aligned}&\left(A_{11},A_{33},A_{13},A_{55},A_{66},A_{56},B_{77},B_{66},C^*,K_i^*,\beta_i\right)=\\&=\left(\bar{A}_{11},\bar{A}_{33},\bar{A}_{13},\bar{A}_{55},\bar{A}_{66},\bar{A}_{56},\bar{B}_{77},\bar{B}_{66},\bar{C}^*,\bar{K}_i^*,\bar{\beta}_i\right)f(T)\end{aligned} \tag{3.9}$$

where $i = 1, 3$, $f(T)=\left(1-\alpha^* T\right)$ and α^* is called the empirical material constant. Using Eq.(3.9) in Eqs (3.2)-(3.5) yields

$$\bar{A}_{11}\frac{\partial^2 u_1}{\partial x_1^2}+\bar{A}_{55}\frac{\partial^2 u_1}{\partial x_3^2}+\left(\bar{A}_{13}+\bar{A}_{56}\right)\frac{\partial^2 u_3}{\partial x_1\partial x_3}+\bar{K}_1\frac{\partial\varphi_2}{\partial x_3}-\bar{\beta}_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1}=\rho R\frac{\partial^2 u_1}{\partial t^2}, \tag{3.10}$$

$$\bar{A}_{66}\frac{\partial^2 u_3}{\partial x_1^2}+\bar{A}_{33}\frac{\partial^2 u_3}{\partial x_3^2}+\left(\bar{A}_{13}+\bar{A}_{56}\right)\frac{\partial^2 u_1}{\partial x_1\partial x_3}+\bar{K}_2\frac{\partial\varphi_2}{\partial x_1}-\bar{\beta}_3\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3}=\rho R\frac{\partial^2 u_3}{\partial t^2}, \tag{3.11}$$

$$\bar{B}_{77}\frac{\partial^2 \varphi_2}{\partial x_1^2}+\bar{B}_{66}\frac{\partial^2 \varphi_2}{\partial x_3^2}-\bar{X}\varphi_2+\bar{K}_1\frac{\partial u_1}{\partial x_3}+\bar{K}_2\frac{\partial\varphi_2}{\partial x_3}=\rho\, j\, R\frac{\partial^2 \varphi_2}{\partial t^2}, \tag{3.12}$$

$$\bar{K}_1^*\frac{\partial^2 T}{\partial x_1^2}+\bar{K}_3^*\frac{\partial^2 T}{\partial x_3^2}=\rho R C^*\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}+RT_0\left(1+n_0\tau_0\frac{\partial}{\partial t}\right)\left(\bar{\beta}_1\frac{\partial \dot{u}_1}{\partial x_1}+\bar{\beta}_3\frac{\partial \dot{u}_3}{\partial x_3}\right) \tag{3.13}$$

where $\qquad R=1\big/\left(1-\alpha^* T\right)$.

For further considerations, it is convenient to introduce the dimensionless variables defined by

$$\begin{gathered}(x_1',x_3')=\frac{\omega^*}{c_1}(x_1,x_3),\quad \left(u_1',u_3'\right)=\frac{\rho c_1\omega^*}{\bar{\beta}_1 T_0}\left(u_1,u_3\right),\quad t_{ij}'=\frac{t_{ij}}{\bar{\beta}_1 T_0},\quad m_{ij}'=\frac{m_{ij}\omega^*}{c_1\bar{\beta}_1 T_0},\\ \varphi_2'=\frac{\varphi_2\rho c_1^2}{\bar{\beta}_1 T_0},\quad T'=\frac{T}{T_0},\quad t'=\omega^* t,\quad \tau_0'=\omega^*\tau_0,\quad \tau_1'=\omega^*\tau_1,\quad \omega^{*2}=\frac{X}{\rho j},\quad c_1^2=\frac{A_{55}}{\rho}\end{gathered} \tag{3.14}$$

where ω^* is the characteristic frequency of the material and c_1 is the longitudinal wave velocity of the medium.

4. Plane wave propagation

Let $\boldsymbol{p}=\left(p_1,0,p_3\right)$ denote the unit propagation vector, c and ξ are respectively the phase velocity and the wave number of the plane waves propagating in the x_1-x_3 plane.

We assume plane wave solution of the form

$$(u_1,u_3,\varphi_2,T)=(1,\bar{u}_3,\bar{\varphi}_2,\bar{T})u_1e^{i\xi(x_1+mx_3-ct)} \tag{4.1}$$

where ξ is the wave number, $\omega=\xi c$ is the angular frequency and c is the phase velocity of the wave, m is the unknown parameter which signifies the penetration depth of the wave, $\bar{u}_3,\bar{\varphi}_2,\bar{T}$, are respectively, the amplitude ratios of the displacement u_3, microrotation φ_2 and temperature distribution to that of the displacement u_1.

With the help of Eqs (3.14) and (4.1) on the set of Eqs (3.10)-(3.13), and solving the resulting equations for non-trivial solution of $\bar{u}_1,\bar{u}_3,\bar{\varphi}_2$ and $\bar{T}$

$$\left|\begin{matrix} \left(\omega^2-\xi^2a_1\right) & \xi^2a_2 & \xi a_3 & \xi a_4 \\ \xi^2a_5 & \left(\omega^2-\xi^2a_6\right) & \xi a_7 & \xi a_8 \\ \xi a_9 & \xi a_{10} & \left(\omega^2-d_{11}-\xi^2a1_1\right) & 0 \\ \xi a_{12} & \xi a_{13} & 0 & \left(-\omega^2a_{15}+\xi^2a_{14}\right) \end{matrix}\right|=0 \tag{4.2}$$

where

$$a_1=-d_1\xi^2+\omega^2,\ a_2=-i\xi d_{13}\left(1-i\omega\tau_1\right),\ a_3=-d_4\xi^2+\omega^2d_7,\ a_4=-i\xi d_{14}\left(1-i\omega\tau_1\right),$$

$$a_5=-d_8\xi^2-d_{11}+\omega^2d_{12},\qquad a_6=\varepsilon_2\bar{\beta}\omega\xi\left(1-i\omega n_0\tau_0\right),\qquad a_7=a_6/\bar{\beta},$$

$$d_1=A_{11}/A_{55},\quad d_2=(A_{13}/+A_{56})/A_{55}=d_5,\quad d_3=K_1^2\big/A_{55}^2,\quad d_4=A_{66}/A_{33},$$

$$d_6=K_1K_2/A_{33}A_{55},\quad d_7=A_{55}/A_{33},\ d_8=B_{77}/B_{66},\quad d_9=-A_{55}c_1^2/B_{66}\omega^{*2}, \tag{4.3}$$

$$d_{10}=-K_2A_{55}c_1^2/K_1B_{66}\omega^{*2},\quad d_{11}=-Xc_1^2/B_{66}\omega^{*2},\quad d_{12}=A_{55}j/B_{66},$$

$$d_{13}=\beta_1T_0/A_{33},\quad d_{14}=\beta_3T_0/A_{33},\quad \varepsilon_1=\rho c^*c_1^2/K_3\omega^*,$$

$$\varepsilon_2=\beta_3c_1^2/K_3\omega^*,\quad \bar{K}=K_1/K_3,\quad \bar{\beta}=\beta_1/\beta_3,\quad d_{15}=K_1^2/A_{55}^2,\quad d_{16}=A_{65}/A_{55}.$$

Solving the determinant (4.2), a quartic equation in c^2 is obtained that can be written as

$$Ac^8+Bc^6+Cc^4+Dc^2+E=0 \tag{4.4}$$

where

$$A=-\xi^8,\qquad B=-\xi^4a_4a_7+\xi^6\left(a_3+a_5+a_1-a_8+d_3d_9\right)+\xi^8d_2d_5,$$

$$\begin{aligned}C=\xi^4&\left[a_2(d_5a_7-a_6)-a_3a_5+a_4d_2a_6+d_6d_{10}-(a_1-a_8)(a_3+a_5)+\right.\\ &\left.+a_1a_8-d_3d_9(a_7-a_8)\right]+\xi^2a_7(a_4a_1+a_5-d_3d_9)-\xi^6(d_2d_5a_5+d_2d_6d_9+d_3d_5d_{10}),\end{aligned}$$

$$D=\xi^2\left[a_2a_3a_6-a_2a_5(d_5a_7-a_6)+a_7d_6d_9a_2-a_4a_6(a_5d_2+d_3d_{10})+\right.$$
$$\left.+(a_1-a_8)a_3a_5-a_1a_8(a_3+a_5)-d_3d_9a_7a_8\right]-a_4a_5a_1a_7+$$
$$-\xi^4\left[(a_1-a_8)d_6d_{10}+d_2(d_5a_5a_8+d_6d_9d_8)+d_3d_5d_{10}d_8\right],$$

$$E=a_2\left(\xi^2d_6a_6d_{10}-a_5a_3a_6\right)+a_1a_8\left(a_3a_5-\xi^2d_6d_{10}\right).$$

Equation (4.4) is quartic in c^2, therefore, the roots of this equation give four values of c^2. Each value of c^2 corresponds to a velocity of propagation of four possible waves. The waves with velocities c_j (j = 1, 2, 3, 4) correspond to four types of quasi-coupled waves. The complex coefficients A, B, C, D in Eq.(4.4), imply that four roots of this equation may be complex. The complex velocity of a quasi-wave 'j', i.e., $c_j=c_R+ic_I$ j = 1, 2, 3, 4, defines the phase propagation velocity $V_j=\left(c_R^2+i\,c_I^2\right)\Big/c_R$ and attenuation quality factor $Q_j^{-1}=-2c_I\,/\,c_R$ for the corresponding wave. Therefore, the four waves propagating in such a medium are attenuating waves. Let us name these four waves: quasi-longitudinal displacement (qLD) wave, quasi-coupled transverse microrotational (qTM) wave, quasi-coupled transverse displacement(qTD) wave and quasi-thermal wave (qT), that are propagating with the descending phase velocities V_j, (j=1, 2, 3, 4), respectively.

5. Boundary condition

For a stress-free surface $x_3=0$, the boundary conditions are:

(i) Vanishing of the normal stress component, i.e., $t_{33}=0$, (5.1)

(ii) Vanishing of the tangential stress component, i.e., $t_{31}=0$, (5.2)

(iii) Vanishing of the tangential couple stress component, i.e., $m_{32}=0$, (5.3)

(iv) Vanishing of the temperature gradient field, i.e., $\dfrac{\partial T}{\partial x_3}+hT=0$ (5.4)

where h is the surface heat transfer coefficient;

$h\to 0$ corresponds to thermally insulated boundaries and

$h\to\infty$ refers to isothermal boundaries. (5.5)

6. Amplitudes of stresses and temperature distribution

In this section, the expression for the amplitude of the components of displacement, microrotation, stresses and temperature distribution for plane waves can be obtained as

$$t_{33}=\sum_{j=1}^{4}A_j a_j^{*} e^{i\xi(x_1-ct+im_i x_2)}, \qquad t_{31}=\sum_{j=1}^{4}A_j b_j^{*} e^{i\xi(x_1-ct+im_i x_2)},$$

$$m_{32}=\sum_{j=1}^{4}A_j s_j m_j e^{i\xi(x_1-ct+im_i x_2)}, \qquad t_{33}=\sum_{j=1}^{4}A_j a_j^{*} e^{i\xi(x_1-ct+im_i x_2)}$$

where,

$$a_j^{*}=R\left(i\xi d_1-d_7 r_j m_j \xi-d_{14}\left(1-i\omega\tau_1\right)t_j/d_7\right), \quad b_j^{*}=R\left(i\xi d_6 r_j-m_j\xi+s_j d_{15}^2\right).$$

7. Particular case

(i) Substituting $R=1$ in the above equations, we will obtain the resulting expressions for the micropolar generalized thermo elastic half-space.

(ii) Taking $R=1$, $A_{11}=A_{33}=\lambda+2\mu+K$, $A_{55}=A_{66}=\mu+K$, $A_{13}=\lambda$, $A_{56}=\mu$, $B_{66}=B_{77}=\gamma$, $\kappa_1=\kappa_2=\kappa$, $\beta_1=\beta_3=\beta$, with $-K_1=K_2=X/2=K$ we obtain the corresponding expressions for the isotropic micropolar generalized thermoelastic half space.

8. Numerical results and discussion

In order to illustrate theoretical results obtained in the preceding sections, we now present some numerical results. For numerical computation, we take the value for relevant parameters for the transversely isotropic micropolar thermoelastic solid as

$$A_{11}=17.8\times10^{10}\,Nm^{-2},\quad A_{33}=1.843\times10^{10}\,Nm^{-2},\quad A_{55}=4.357\times10^{10}\,Nm^{-2},\quad A_{66}=4.42\times10^{10}\,Nm^{-2},$$

$$A_{13}=7.59\times10^{10}\,Nm^{-2},\quad A_{56}=1.89\times10^{10}\,Nm^{-2},\quad B_{77}=2.63\times10^{9}\,N,\quad B_{66}=5.648\times10^{9}\,N.$$

$$\rho=1.74\,Kg/m^3,\quad j=0.02m^2,\quad K_1^{*}=1.7\,Cal/K,\quad C^{*}=1.04\,Cal/K,\quad T=298\,K,\quad \tau_0=0.4\,s,\quad \tau_1=0.8\,s.$$

Figures 1 and 2 show the variation of phase velocities V_i, i = $1,..4$, and attenuation quality factors Q_i, i = $1,..4$. In these figures the solid curve represents the case of a transversely isotropic micropolar generalized thermoelastic (MTITD) half space with temperature dependent properties, while the dotted curve represents the case of a micropolar transversely isotropic generalized thermoelastic (MTITI) half space without temperature dependent properties. The comparison of three theories of generalized thermoelasticity, viz, the coupled thermoelasticity (C-T), Lord and Shulman (L-S) theory and Green Lindsay (G-L) theory, have been shown in all the graphs. The solid lines and broken lines correspond to the variation for $\alpha^{*}=0$(TD), $\alpha^{*}=0.5$(TI), respectively. The solid and dotted lines without a center symbol correspond to the C-T theory, solid and dotted lines with a center symbol $(-o-o-)$ correspond to the L-S theory and the solid and dotted lines with a center symbol $(-\times-\times-)$ correspond to the case of G-L theory.

It can be seen from Fig.1a that the value of phase velocity V_1 starts with a sharp initial decrease within the interval (0, 5), then attains a constant value, for all the three theories of thermoelasticity. The variation pattern of MTITD and MTITI is similar with difference in their amplitudes. Figure b shows that the value of phase velocity V_2, for all the cases sharply increases within the interval (0, 1) and then sharply decreases to attain a constant value at the end. The amplitude of variation for MTITI is more as compared to MTITD. It is evident form Fig.c that the value of phase velocity V_3, for the case of MTITD and for all the theories of thermoelasticity, sharply decreases in a short interval and then attains a constant value. Similar variations are observed for the case of MTITI, when the case of L-S theory is concerned, while for the case of G-L and C-T theories, its value smoothly decreases to attain a constant value with an increase in frequency. The values of amplitudes for the case of MTITD are less as compared to those of MTITI. Figure d represents the variation in the value of phase velocity V_4 with frequency. It can be seen from this figure that the variation pattern is similar to the case of Fig.c except with difference that the variations for MTITD in the present figure are similar to those of MTITI of Fig.c and vice-versa, with difference in there amplitudes.

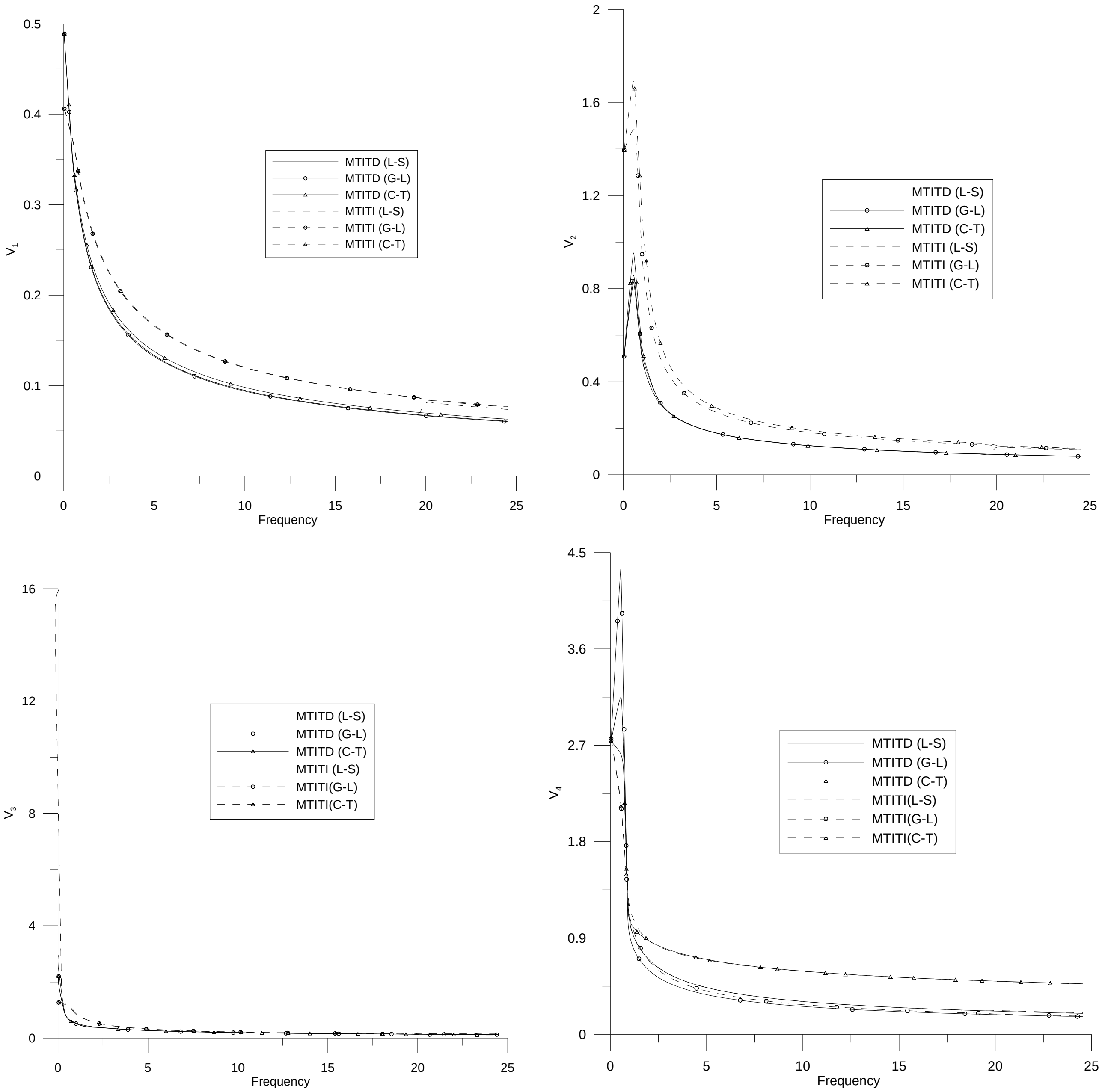

Fig.1. Variations in the phase velocity (a) V_1 (b) V_2 (c) V_3 (d) V_4 of waves with respect to frequency.

Figure 2 represents variations in the value of attenuation quality factors *Qi*, *i=1*, ... *4*. It is shown in Fig.a that the value of attenuation quality Q1 for the case of MTITD increases with an increase in frequency, for all the theories. For MTITI and for L-S theory its value increases with an increase in frequency and flattens out to become constant at the end, while for G-L and C-T theories its value initially decreases and then attains a constant value. Figures b and d represent the variation of attenuation quality factors Q2,Q4 with frequency. It can be seen from these Figures that for the case of MTITD the value of attenuation quality factor shows a small hump within the interval (*0*, *3*) and then attains a constant value for all the 3 theories, while for Q4 it shows a large hump and the remaining pattern of variation is similar to those depicted in earlier case. While for MTITI, its value initially decreases, then increases to attain a constant value with an increase in frequency. It can be seen from Fig.c that the value of attenuation quality factor Q3 sharply increases, then decreases and then attains a constant value for all the cases.

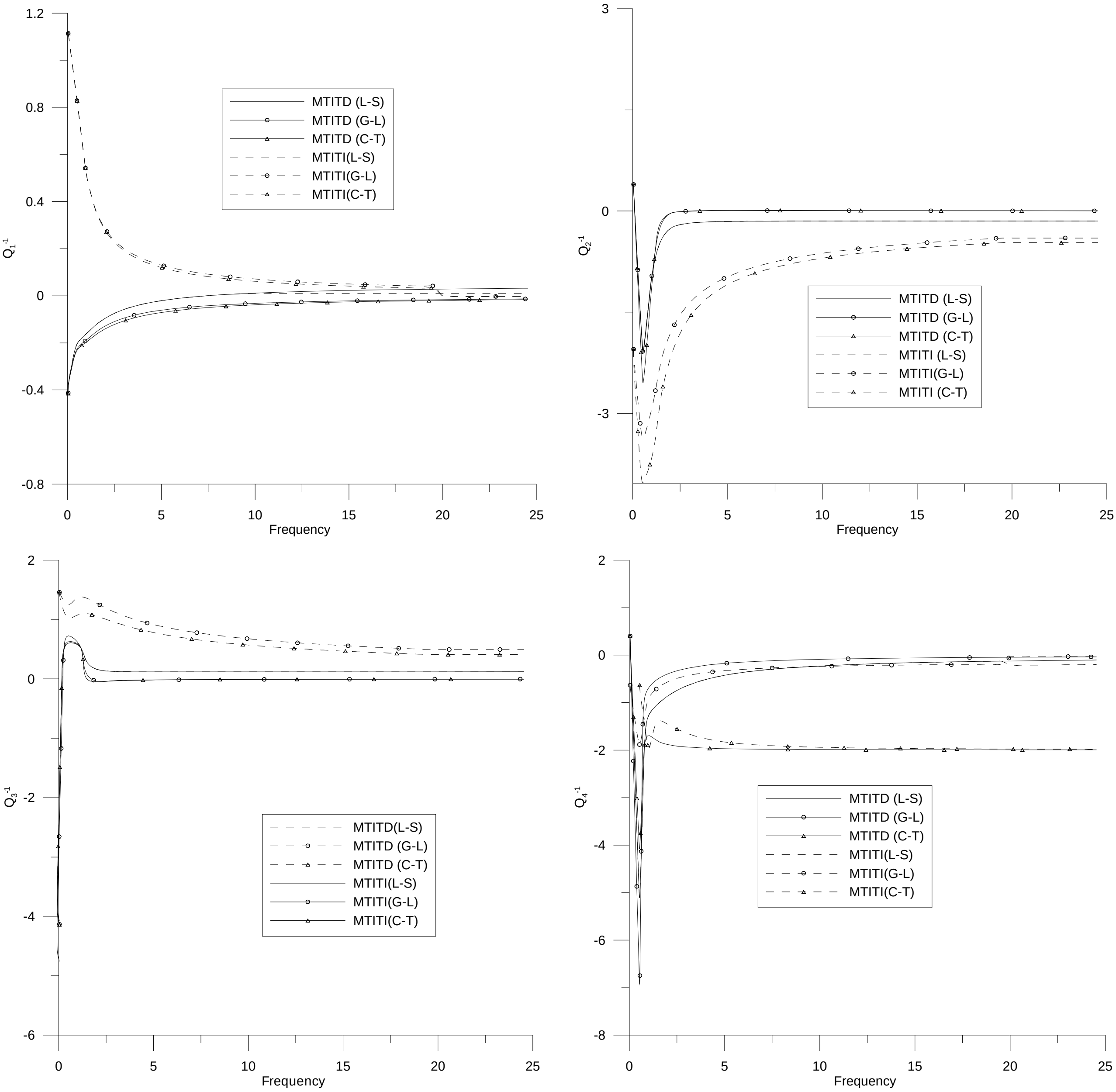

Fig.2. Variations in the attenuation quality factor (a) Q_1^{-1} (b) Q_2^{-1} (c) Q_3^{-1} (d) Q_4^{-1} of waves with respect to frequency.

Figures 3-9 show the variations of stress, temperature distribution and components of displacement and microrotation with distance. All numerical computations are carried out for a single fixed value of frequency and for two given values of wave number 0.25 and 0.35. The computations were carried out within the range $0 \le x_1 \le 10$. It follows from Figs 3 and 4 that the variation of stresses starts with initial increase within the range $0 \le x_3 \le 2$ and then decreases with an increase in distance from the surface $x1 = 0$. With an increase in wave number the value of stresses gets decreased. The variations for all the 3 theories are similar with slight difference in there amplitude.

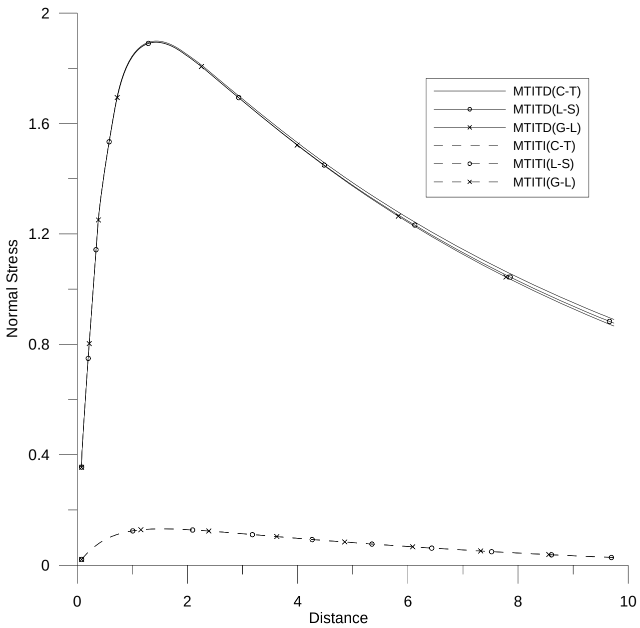

Fig.3. Variations of normal stress with distance.

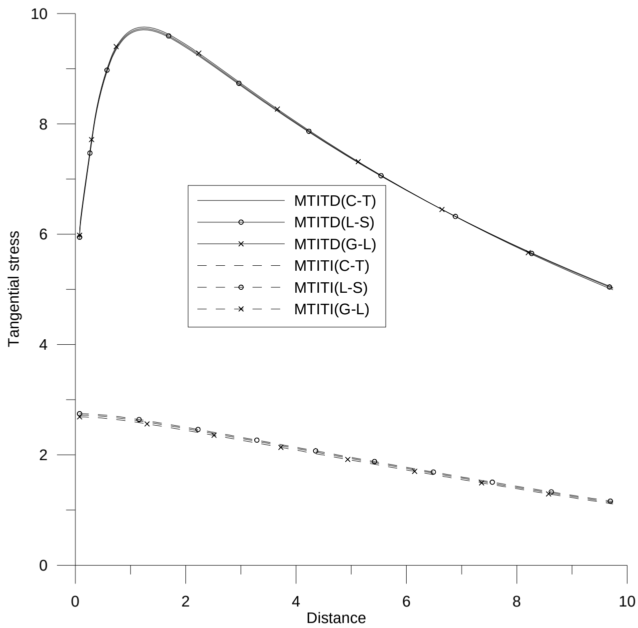

Fig.4. Variations of tangential stress with distance.

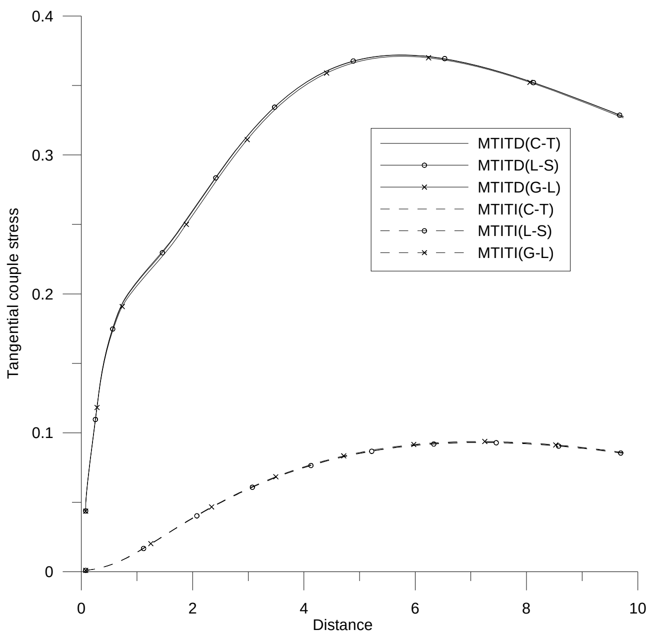

Fig.5. Variations of tangential couple stress with distance.

Figure 5 shows that the value of tangential couple stress goes on increasing with an increase in distance up to a value 6 of distance, but after that it decreases with a further increase in distance, when wave number is 0.25. However, as the value of wave number gets increased, the value of tangential couple stress initially oscillates and then decreases. Figures 6-9 depict the variation of amplitudes and displacement, microrotation and temperature distribution with distance. It is illustrated in these figures that the value of displacements and microrotation goes on decreasing as the distance from the surface increases. This variation pattern is physically admissible since the characteristics of Rayleigh waves are that amplitude of the wave decreases rapidly with depth. The rate of decrease depends on the wavelength. Figure 9 shows the value of temperature distribution which initially increases at a depth of 5 units from the surface and then goes on decreasing with a increase in distance from the free surface. The value of displacements and microrotation gets decreased with an increase in wave number while that of temperature distribution gets increased. The variation pattern for all the three theories of thermoelasticity varies in same pattern with a slight difference in the magnitude.

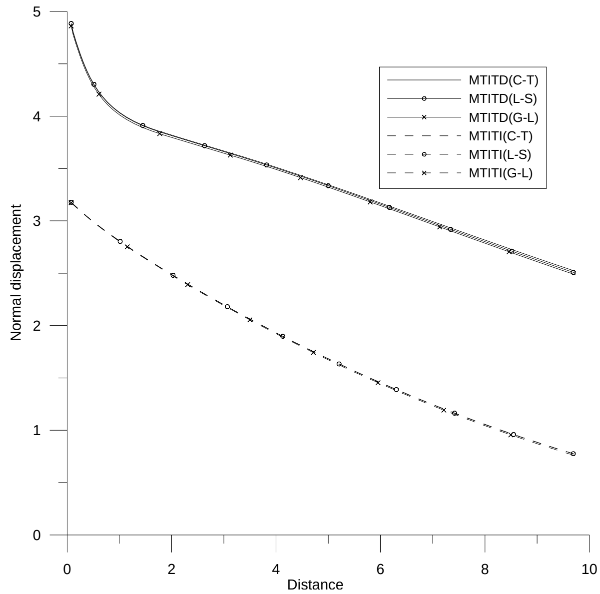

Fig.6. Variation of normal displacement with distance.

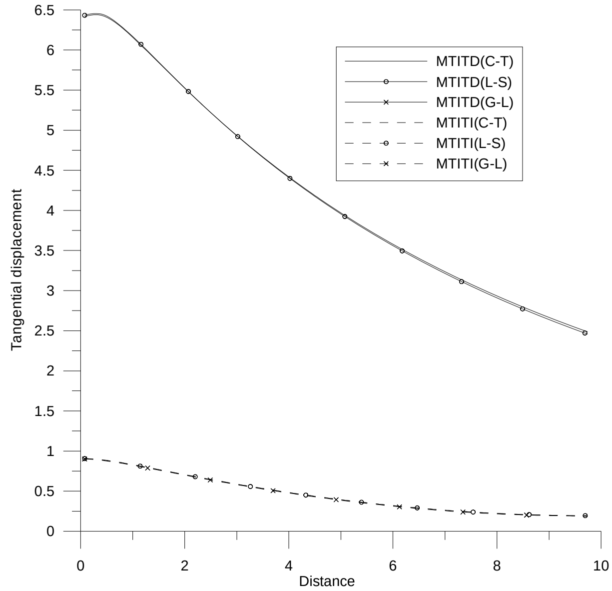

Fig.7. Variations of tangential displacement with distance.

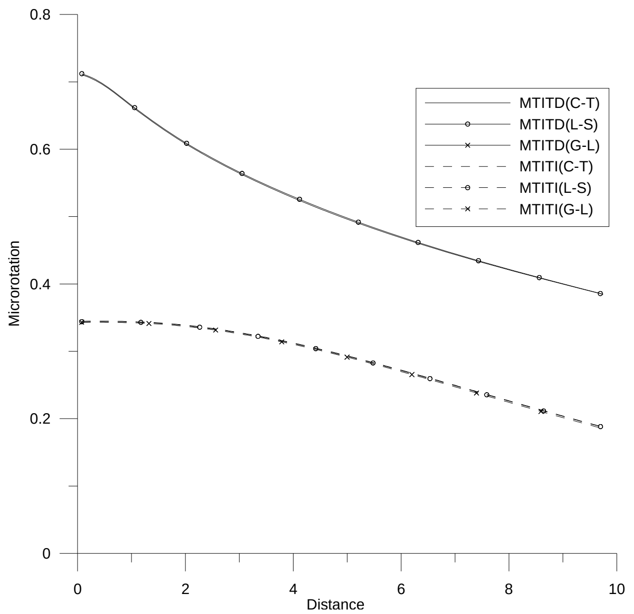

Fig.8. Variations of microrotation with distance.

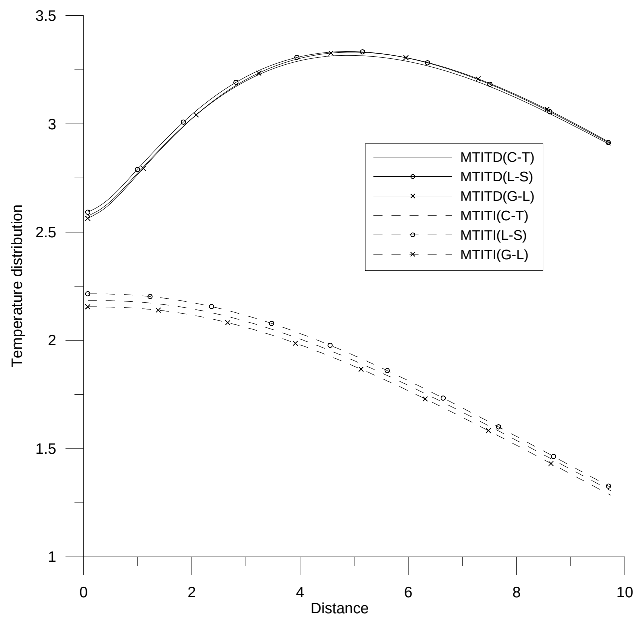

Fig.9. Variations of temperature distribution with distance.

Conclusions

The importance of thermal stresses in causing structural damages and changes in functioning of the structure is well recognized whenever thermal stress environments are involved. Propagation of waves in a transversely isotropic micropolar generalized thermoelastic half space under temperature dependent elastic properties have been discussed. The phase velocities and attenuation quality factor has been computed and plotted graphically for three different theories of elasticity. The expressions for amplitude of stresses, displacements, microrotation and temperature distribution have been derived and computed numerically. It is observed form all the figures that the variation pattern for all the three theories of thermoelasticty is similar with a very slight difference in the amplitude. The value of phase velocities of first 3 waves gets decreased due to temperature dependence elastic constants, while for the fourth wave a reverse behavior is observed. Similarly, the attenuating quality factor of all the waves also gets decreased due to temperature dependence elastic constants except for the case of the second wave. The numerically computed results are found to be in close agreement with the theoretical result.

Nomenclature

$A_{ijkl}, G_{ijkl}, B_{ijkl}$ – are characteristic constants of material following the symmetry properties given by [1]

C^* – specific heat at constant strain

j – microinertia

$K^*_{ij} = K_i \delta_{ij}$ – thermal conductivities

m_{ij} – components of couple stress tensor

T – temperature change

T_0 – uniform reference temperature

t_{ij} – components of stress tensor

u_i – components of displacement vector

$\beta_{ij} = \beta_i \delta_{ij}$ – is the thermal elastic coupling tensor

δ_{ij} – Kronecker delta

ε_{nmk} – permutation symbol

ρ – density

τ_0, τ_1 – thermal relaxation times

φ_i – components of microrotation vector

$\boldsymbol{\varphi}$ – microrotation vector

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