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ROLE OF SUDDEN APPLICATION OR WITHDRAWAL OF MAGNETIC FIELD ON MHD COUETTE FLOW

B.K. JHA and M.O. ONI*
Department of Mathematics, Ahmadu Bello University
Zaria, NIGERIA
E-mails: basant777@yahoo.co.uk; michaeloni29@yahoo.com

This article investigates the impact of a sudden application or sudden withdrawal of a magnetic field on an unsteady MHD Couette flow formation in a parallel plate channel. The governing momentum equation is derived and solved exactly in Laplace domain using the Laplace transform technique with the necessary initial and boundary conditions to capture the present physical situation for the cases; sudden application or sudden withdrawal of a magnetic field. Due to the complexity of the solution obtained, the Riemann-sum approximation technique is used to transform the Laplace domain to time domain. During the course of graphical and tabular representations, results show that the Hartmann number, time and nature of application of a magnetic field play an important role in the transition from hydrodynamic to magnetohydrodynamic flow and vice-versa. Also, fluid velocity steady-state solution is independent on whether the magnetic field is fixed relative to the moving plate or to the fluid for sudden withdrawal of magnetic field. In addition, the application of a sudden magnetic field leads to a delay in the attainment of steady-state solution.

Key words: MHD, Couette motion, Riemann-sum approximation, Hartmann number, parallel plates.

1. Introduction

Over the decades, there has been a continuous interest in the study of magnetohydrodynamics in the channel due to its latest application in purification of metals, administration of drugs for cancer patients and power generation in MHD (magnetohydrodynamics) generators.

Earlier studies dated back to the work of Hartmann [1], Hartmann and Lazarus [2]. Later, Rossow [3] studied the flow of electrically conducting fluids in the presence of a transverse magnetic field over a flat plate. He concluded that the skin-friction and rate of heat transfer are reduced when the transverse magnetic field is fixed relative to the plate but increases when fixed relative to the fluid. Singh and Kumar [4] examined the effect of the magnetic field in the absence of rotation on velocity and skin friction for impulsive and accelerated motion of the moving plates and deduced that fixing the magnetic field relative to the moving plate could increase the velocity field in both cases. Singh *et al.* [5] scrutinised the transient properties of the Couette flow of an electrically conducting fluid when one of the plates is set into the accelerated motion.

The study of fluid motion in a channel when one of the plates is moving was made by Couette [6]. It is of importance in ice skating, lubrication of bearings in automobiles, extrusion, power generation and pumps. Since then, several studies have been carried out to further investigate the significance of the movement of plates on flow formation and skin-friction [7-10].

On the other hand, few studies have been devoted to the investigation of an unsteady MHD flow in a channel when there is a sudden application or a sudden withdrawal of a magnetic field. The only research on this issue is the work of Kumaran *et al.* [11] where the transition of an MHD boundary layer flow past a stretching sheet is studied numerically. It was found that the steady state velocity and local skin-friction vary

* To whom correspondence should be addressed

with the magnetic field when there is a sudden application of the magnetic field whereas unchanged for a sudden removal of the magnetic field. However their work is limited in application since their numerical results cannot be used in situations when one of the plates is moving.

The aim of this article is to investigate the effect of a sudden application or sudden withdrawal of the magnetic field on an MHD Couette flow formation in a channel. The governing equations governing the problem are derived and solved in section 2 while results and conclusions are presented in sections 3 and 4, respectively. The solutions obtained in this article deserve great attention not only on account of their engineering applications in cooling devices and melting of ice, but also due to the fact that they can serve as an accuracy check for various conducted experimental results.

2. Mathematical formulation

The motion of a viscous, laminar, incompressible and electrically conducting fluid filling the gap between two parallel plates is considered. The fluid exists in the region $0 \le y' \le h,$ where the y' axis is the coordinate normal to the flow and h is the thickness of the channel. The fluid flow inside the channel is set up by movement of the lower plate which is located at $y' = 0$. Since the plates are of infinite length and the flow is fully developed, this implies that all physical parameters are functions of y' and t'. It is further assumed that no applied and polarisation voltage exists, in addition the magnetic Reynolds number is assumed to be negligible so that no magnetic field is induced. Under these assumptions, the steady hydrodynamic equations in the component form in the absence of pressure gradient is

$$\frac{d^2u'}{dy'^2} = 0 . \tag{2.1}$$

We imposed the following boundary conditions

$$u' = 1; \quad \text{at} \quad y' = 0 ,$$
$$u' = 0; \quad \text{at} \quad y' = 1 . \tag{2.2}$$

Now we have to find the velocity of the hydrodynamic flow represented by Eq.(2.1) subject to the boundary conditions (2.2). Using the following dimensionless quantities: $y = y'/h$, $u_{ss} = u'/U$, where u_{ss} is the dimensionless steady state velocity in the absence of the magnetic field. Then Eqs (2.1) and (2.2) in dimensionless form become

$$\frac{d^2u_{ss}}{dy^2} = 0 , \tag{2.3}$$

$$u_{ss} = 1; \quad \text{at} \quad y = 0 ,$$
$$u_{ss} = 0; \quad \text{at} \quad y = 1 . \tag{2.4}$$

The solution of Eq.(2.3) together with boundary conditions (2.4) is

$$u_{ss} = 1 - y . \tag{2.5}$$

The skin-friction at the channel walls is obtained as

$$\tau_0 = \tau_1 = -1 . \tag{2.6}$$

Also, the mass flow rate in dimensionless form is given as

$$Q = \int_0^1 u_{ss}(y)\,dy = \frac{1}{2} . \tag{2.7}$$

We now seek to obtain the steady state MHD solution by considering the equation below

$$\frac{d^2 u_s'}{dy'^2} - \frac{\sigma B_0^2}{\rho} u_s' = 0 . \tag{2.8}$$

Equation (2.8) is valid when the magnetic lines of force are fixed relative to the fluid. If the magnetic field is fixed relative to the moving plate, Eq.(2.8) can be replaced with

$$\frac{d^2 u_s'}{dy'^2} - \frac{\sigma B_0^2}{\rho}\left[u_s' - U\right] = 0 . \tag{2.9}$$

Equations (2.8) and (2.9) can be unified to obtain

$$\frac{d^2 u_s'}{dy'^2} - \frac{\sigma B_0^2}{\rho}\left[u_s' - KU\right] = 0 \tag{2.10}$$

where

$$K = \begin{cases} 0 & \text{when } B_0 \text{ is fixed relative to the fluid} \\ 1 & \text{when } B_0 \text{ is fixed relative to the moving plate} \end{cases} \tag{2.11}$$

and ρ, ν, σ and u_s are, respectively, the fluid density, kinematic viscosity, electrical conductivity steady-state dimensionless velocity profile.

Using the dimensionless quantities defined above and $= B_0 h / \sqrt{\sigma / \nu\rho}$, the solution of Eq.(2.10) using boundary conditions (2.4) is

$$u_s = \frac{(1-K)\sinh\left(M(1-y)\right)}{\sinh(M)} - K\left[1 - \frac{\sinh(yM)}{\sinh(M)}\right] . \tag{2.12}$$

Using Eq.(2.12), we can obtain

$$\tau_0 = \left.\frac{du_s}{dy}\right|_{y=0} = -KM\operatorname{cosech}(M) - M(1-K)\coth(M) , \tag{2.13}$$

$$\tau_1 = \left.\frac{du_s}{dy}\right|_{y=1} = -KM\coth(M) - M\left(1-K\right)\text{cosech}(M), \tag{2.14}$$

$$Q = \int_0^1 u_s(y)dy = \frac{\left(1-K\right)}{M}\coth(M) - \frac{\left(1-K\right)}{M}\text{cosech}\left(M\right) + \\ + \frac{K\left(M - \coth\left(M\right) + \text{cosech}\left(M\right)\right)}{M}. \tag{2.15}$$

In this paper, we are interested in two cases (*i*) suddenly imposed magnetic field and (*ii*) sudden withdrawal of the magnetic field from flow formation in a parallel-plates channel.

2.1. Flow due to suddenly imposed magnetic field (Case I)

At $t' > 0$, a transverse magnetic field of strength B_0 is applied throughout the fluid flow (see Fig.1a). The magnetic Reynolds number is assumed to be very small which corresponds to negligible induced magnetic field compared to the externally applied one as revealed in Pai [12], therefore, the uniform magnetic field $\boldsymbol{B}$ is a constant, $\boldsymbol{B} \equiv (0,0,B_0)$ and is considered as the total magnetic field acting on the fluid.

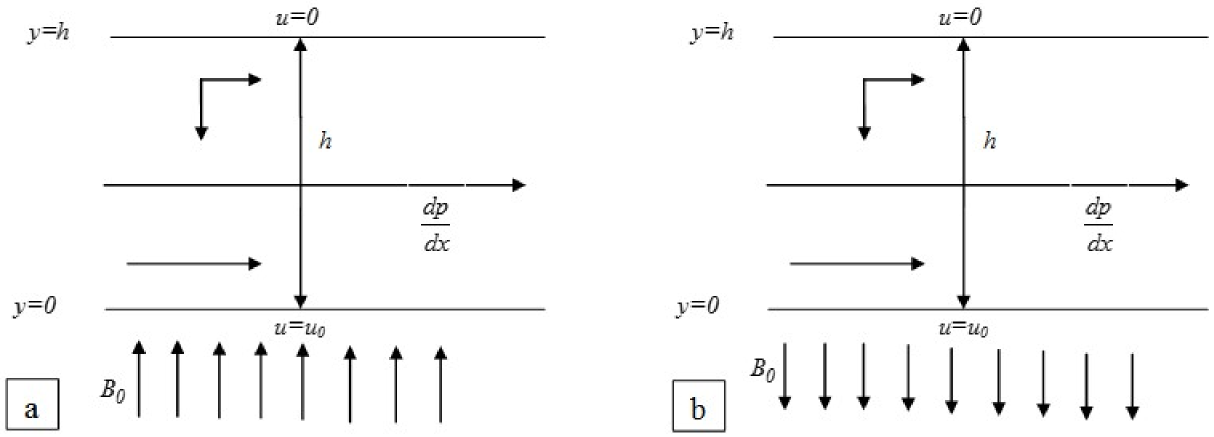

Fig.1. Schematic of the problem (a) Case I, (b) Case II.

The equation for the unsteady MHD Couette flow in dimensionless form is obtained as

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - H\left(t\right)M^2\left[u - K\right] \tag{2.16}$$

where $H\left(t\right)$ is a unit step function. In this case

$$H\left(t\right) = \begin{cases} 1 \text{ for } t > 0 \\ 0 \text{ for } t \le 0 \end{cases}. \tag{2.17}$$

The above Eq.(2.16) is subject to the following initial and boundary conditions

$$t \le 0, \qquad u = u_{ss}, \qquad 0 \le y \le 1,$$

$$t > 0 \begin{cases} u = 1; & \text{at} \quad y = 0 \\ u = 0; & \text{at} \quad y = 1 \end{cases}. \tag{2.18}$$

The solution of Eq.(2.16) can be obtained using the Laplace transform technique. Define the following transform variables

$$\bar{u}\left(y,r\right) = \int_0^\infty u(y,t)e^{-rt}dt\,, \quad \text{where } r \text{ is the Laplace parameter and } r > 0\,.$$

Taking the Laplace transform of Eq.(2.16), we obtain the following ordinary differential equation

$$\frac{d^2\bar{u}}{dy^2} - \left(M^2 + r\right)\bar{u} = -\frac{M^2K}{r} - \left(1 - y\right). \tag{2.19}$$

The solution of Eq.(2.19) subject to boundary condition (2.18) is obtained as

$$\begin{aligned} \bar{u} = &\left[\frac{1}{r} - \frac{1}{\left(M^2 + r\right)} - \frac{M^2K}{r\left(M^2 + r\right)}\right]\frac{\sinh\left(\beta\left(1 - y\right)\right)}{\sinh\left(\beta\right)} + \\ &+ \frac{M^2K}{r\left(M^2 + r\right)}\left[1 - \frac{\sinh(\beta y)}{\sinh(\beta)}\right] + \frac{\left(1 - y\right)}{\left(M^2 + r\right)} \end{aligned} \tag{2.20}$$

where $\qquad \beta = \sqrt{M^2 + r}\,.$

The skin-friction at the wall is obtained by taking the derivative of the velocity profile with respect to y as follows

$$\bar{\tau}_0 = \left.\frac{d\bar{u}}{dy}\right|_{y=0} = \left[\frac{1}{\left(M^2 + r\right)} - \frac{1}{r} + \frac{M^2K}{r\left(M^2 + r\right)}\right]\beta\coth(\beta) - \frac{M^2K\beta\operatorname{cosech}\left(\beta\right)}{r\left(M^2 + r\right)} - \frac{1}{\left(M^2 + r\right)}, \tag{2.21}$$

$$\bar{\tau}_1 = \left.\frac{d\bar{u}}{dy}\right|_{y=1} = \left[\frac{1}{\left(M^2 + r\right)} - \frac{1}{r} + \frac{M^2K}{r\left(M^2 + r\right)}\right]\beta\operatorname{cosech}(\beta) - \frac{M^2K\beta\coth\left(\beta\right)}{r\left(M^2 + r\right)} - \frac{1}{\left(M^2 + r\right)}. \tag{2.22}$$

Another important parameter of interest is the mass flow rate obtained from Eq.(2.20) as

$$\bar{Q}=\int_0^1 \bar{u}(y)dy=\left[\frac{1}{r}-\frac{1}{\left(M^2+r\right)}-\frac{M^2K}{r\left(M^2+r\right)}\right]\left(\frac{\coth(\beta)-\operatorname{cosech}(\beta)}{\beta}\right)+$$
$$+\frac{M^2K}{r\left(M^2+r\right)}\left(\frac{\beta-\coth(\beta)+\operatorname{cosech}(\beta)}{\beta}\right)+\frac{1}{2\left(M^2+r\right)}. \tag{2.23}$$

Equation (2.20)-(2.23) are to be inverted in order to determine the velocity, skin-friction and mass flow rate in time domain. Since these equations are difficult to invert in closed form, we use a numerical procedure used in [13-15] which is based on the Riemann-sum approximation which has been demonstrated to be a useful tool with high degree of accuracy [16]. In this method, any function in the Laplace domain can be inverted to the time domain as follows

$$u(y,t)=\frac{e^{\epsilon t}}{t}\left[\frac{1}{2}\bar{u}(y,\epsilon)+Re\sum_{n=1}^{N}\bar{u}\left(y,\epsilon+\frac{in\pi}{t}\right)(-1)^n\right] \tag{2.24}$$

where Re refers to the real part, $i=\sqrt{-1}$ is the imaginary number. N is the number of terms used in the Riemann-sum approximation and ϵ is the real part of the Bromwich contour that is used in inverting Laplace transforms. The Riemann-sum approximation for the Laplace inversion involves a single summation for the numerical process. Its accuracy depends on the value of ϵ and the truncation error dictated by N. According to Tzou [17], the value of ϵ must be selected so that the Bromwich contour encloses all the branch points. For faster convergence, the value of $\epsilon t=4.7$ gives the most satisfactory results.

2.2. Flow due to sudden withdrawal of magnetic field (Case II)

In this case, the flow of an electrically conducting fluid is considered initially. Suddenly, a uniform magnetic field is continually withdrawn from the flow to transform the fluid from magnetohydrodynamic to hydrodynamic (see Fig.1b). The equation for sudden withdrawal of the magnetic field in the dimensionless form is obtained from Eq.(2.16) when

$$H(t)=\begin{cases}0 \ \text{for}\, t>0\\ 1 \ \text{for}\, t\le 0,\end{cases} \tag{2.25}$$

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}\ . \tag{2.26}$$

The equation is subject to the following dimensionless initial and boundary conditions

$$t\le 0, \quad u=u_s, \quad 0\le y\le 1,$$
$$t>0\begin{cases}u=1; \ \text{at}\ y=0\\ u=0; \ \text{at}\ y=1\end{cases} \tag{2.27}$$

where u_s is defined in Eq.(2.10). Using the Laplace transform technique, the following ordinary differential equation is obtained

$$\frac{d^2\bar{u}}{dy^2}-r\bar{u}=-u_s\,. \tag{2.28}$$

The solution of Eq.(2.28) subject to boundary conditions (2.27) is

$$\bar{u}=\left[\frac{1}{r}-\frac{K}{r}+\frac{(1-K)}{\left(M^2-r\right)}\right]\frac{\sinh\left(\alpha(1-y)\right)}{\sinh(\alpha)}+\frac{K}{r}\left[1-\frac{\sinh(\alpha y)}{\sinh(\alpha)}\right]+$$
$$+\frac{K}{\left(M^2-r\right)}\left[\frac{\sinh(My)}{\sinh(M)}-\frac{\sinh(\alpha y)}{\sinh(\alpha)}\right]-\frac{(1-K)\sinh\left(M(1-y)\right)}{\left(M^2-r\right)\sinh(M)} \tag{2.29}$$

where $\alpha=\sqrt{r}$.

In a similar manner, the skin-friction at the channel walls is obtained as

$$\bar{\tau}_0=\left.\frac{d\bar{u}}{dy}\right|_{y=0}=\left[\frac{K}{r}-\frac{(1-K)}{\left(M^2-r\right)}-\frac{1}{r}\right]\alpha\coth(\alpha)-\frac{K\alpha\,\mathrm{cosech}(\alpha)}{r}+$$
$$+\frac{K}{\left(M^2-r\right)}\left[M\mathrm{cosech}(M)-\alpha\mathrm{cosech}(\alpha)\right]+\frac{(1-K)M\coth(M)}{\left(M^2-r\right)}, \tag{2.30}$$

$$\bar{\tau}_1=\left.\frac{d\bar{u}}{dy}\right|_{y=1}=\left[\frac{K}{r}-\frac{(1-K)}{\left(M^2-r\right)}-\frac{1}{r}\right]\alpha\mathrm{cosech}(\alpha)-\frac{K\alpha\coth(\alpha)}{r}+$$
$$+\frac{K}{\left(M^2-r\right)}\left[M\coth(M)-\alpha\coth(\alpha)\right]+\frac{(1-K)M\mathrm{cosech}(M)}{\left(M^2-r\right)}. \tag{2.31}$$

Similarly

$$\bar{Q}=\left[\frac{K}{r}-\frac{(1-K)}{\left(M^2-r\right)}-\frac{1}{r}\right]\left(\frac{\mathrm{cosech}(\alpha)-coth(\alpha)}{\alpha}\right)+\frac{K}{r}\left(\frac{\alpha-\coth(\alpha)+cosech(\alpha)}{\alpha}\right)+$$
$$+\frac{K}{\left(M^2-r\right)}\left[\frac{\coth(M)}{M}-\frac{\coth(\alpha)}{\alpha}-\frac{\mathrm{cos}ech(M)}{M}+\frac{\mathrm{cos}ech(\alpha)}{\alpha}\right]+$$
$$+\frac{(1-K)}{M\left(M^2-r\right)}\left[\mathrm{cos}ech(M)-\coth(M)\right]. \tag{2.32}$$

The time domain solution of Eqs (2.29)-(2.32) is similarly obtained by using the Riemann-sum approximation Laplace inversion of Eq.(2.24).

3. Results and discussion

In this section, graphical representations of the solutions obtained are presented. It is found for both cases that fluid transition from hydrodynamics to magnetohydrodynamics in the channel and vice-versa is controlled by the Hartmann number (M), time (t) and parameter indicating the nature of the magnetic field application (K). Throughout this article we have chosen $0 \le M \le 5$, $0 \le t \le 0.5$ and $0 \le K \le 1$ to describe the physical problem presented.

To verify the accuracy of the solutions obtained, it is expected that the steady state solution obtained from Eqs (2.5)-(2.15) should correspond with the unsteady solutions in Eqs (2.20)-(2.32) at large time.

Table 1. Numerical comparison of steady and unsteady state skin-friction and mass flow rate for suddenly application of magnetic field for different values of $\boldsymbol{M}$ and $\boldsymbol{K}$.

	Unsteady state at large time $(\boldsymbol{t} = 0.9)$						Steady state					
M	$-\tau_0$		$-\tau_1$		Q		$-\tau_0$		$-\tau_1$		Q	
	$K=0$	$K=1$	$K=0$	$K=1$	$K=0$	$K=1$	$K=0$	$K=1$	$K=0$	$K=1$	$K=0$	$K=1$
0.01	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000
0.5	1.0820	0.9595	0.9595	1.0820	0.4898	0.5102	1.0820	0.9595	0.9595	1.0820	0.4898	0.5102
1.0	1.3130	0.8509	0.8509	1.3130	0.4621	0.5379	1.3130	0.8509	0.8509	1.3130	0.4621	0.5379
1.5	1.6572	0.7045	0.7045	1.6572	0.4234	0.5766	1.6572	0.7045	0.7045	1.6572	0.4234	0.5766
2.0	2.0746	0.5514	0.5514	2.0746	0.3808	0.6192	2.0746	0.5514	0.5514	2.0746	0.3808	0.6192
5.0	5.005	0.0674	0.0674	5.005	0.1973	0.8027	5.005	0.0674	0.0674	5.005	0.1973	0.8027

Table 2. Numerical comparison of steady and unsteady state skin-friction and mass flow rate for sudden withdrawal of magnetic field at different values of $\boldsymbol{M}$ and $\boldsymbol{K}$.

	Unsteady state at large time $(\boldsymbol{t} = 0.9)$						Steady state					
M	$-\tau_0$		$-\tau_1$		Q		$-\tau_0$		$-\tau_1$		Q	
	$K=0$	$K=1$	$K=0$	$K=1$	$K=0$	$K=1$	$K=0$	$K=1$	$K=0$	$K=1$	$K=0$	$K=1$
0.01	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000
0.5	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000
1.0	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000
1.5	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000
2.0	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000
5.0	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000	1.0000	1.0000	1.0000	1.0000	0.5000	0.5000

Tables 1 and 2 compute numerical comparisons for a suddenly applied magnetic field and a sudden withdrawal of the magnetic field, respectively, and results show excellent agreement. Also, from these tables, for a small magnetic field parameter, skin-friction at the channel wall and mass flow rate are independent on either case considered (sudden application or sudden withdrawal of the magnetic field) as well as the nature of application of the magnetic field (*K*).

3.1. Flow due to suddenly imposed magnetic field (Case I).

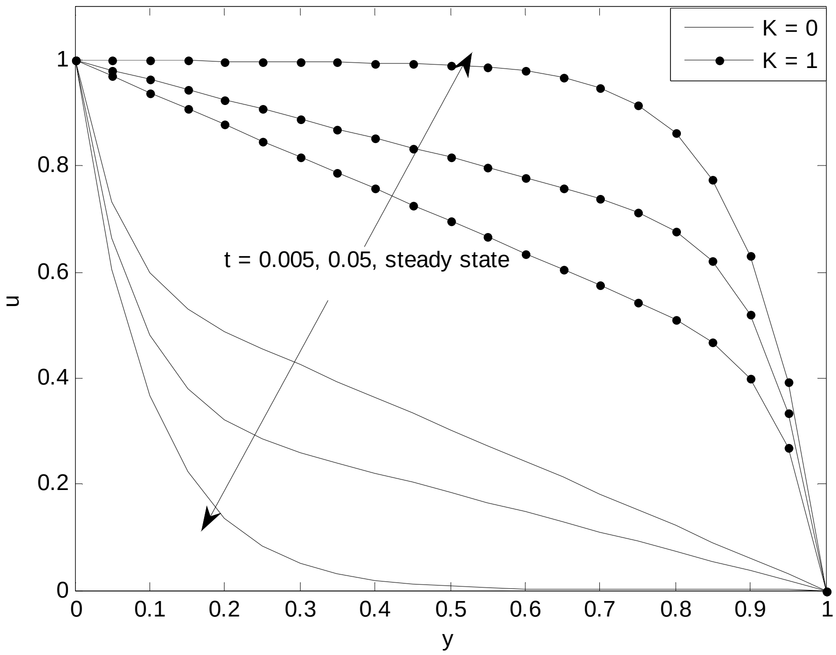

Fig.2. Velocity profile showing the effect of time for $K=0$ and $K=1$ with fixed value of $M=10.0$, (Case I).

Figure 2 depicts the effect of t and K on unsteady fluid velocity when a sudden transverse magnetic field is imposed relative to the fluid $(K=0)$ and relative to the moving plate $(K=1)$. It is found from this figure that fluid velocity increases with time, until a steady state is attained for $(K=1)$ while the reverse result is seen for $(K=0)$.

Figures 3 and 4 present the skin-friction at different time and magnetic field parameter at the wall $y=0$ and $y=1$, respectively. It is evident from these figures that a sudden imposition of the magnetic field increases the skin-friction at the wall $y=0$ when the magnetic field is applied relative to the moving plate while the opposite result is obtained at $y=1$. It is interesting to note that the skin-friction at $y=0$ relative to the moving plate $(K=1)$ is exactly the same as that of the skin-friction at $y=1$ relative to the fluid. In addition, the skin-friction at the moving wall increases with time for $K=1$ and decreases otherwise $(K=0)$. This increase can be attributed to the fact that the presence of the magnetic field and movement of the wall increases the drag force which in turn increases the skin-friction at this wall. The reverse trend is found for the skin-friction at the wall $y=1$.

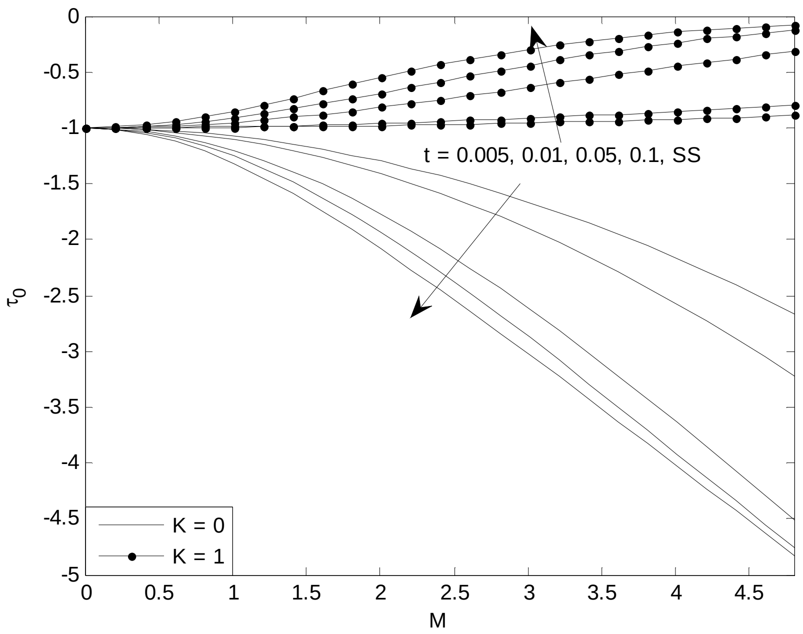

Fig.3. Skin-friction showing the effect of the magnetic field parameter and time for $K = 0$ and $K = 1$, at $(y = 0)$, (Case I).

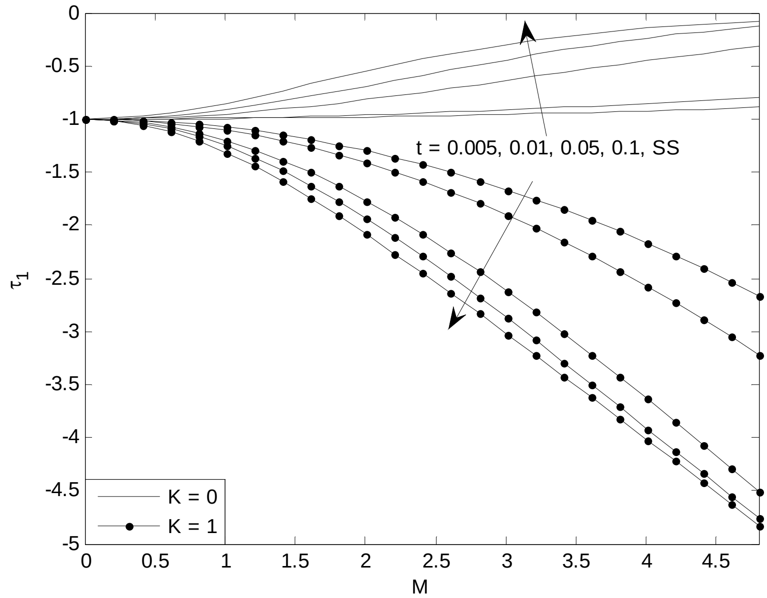

Fig.4. Skin-friction showing the effect of the magnetic field parameter and time for $K = 0$ and $K = 1$, at $(y = 1)$, (Case I).

In addition, it is easy to see from Figs 3 and 4 that the transient state skin-friction at both walls corresponds with that of steady state for a small magnetic field parameter. This correspondence can be viewed as an accuracy check on Eqs (2.13), (2.14), (2.21) and (2.22).

3.2. Flow due to sudden withdrawal of magnetic field (Case II)

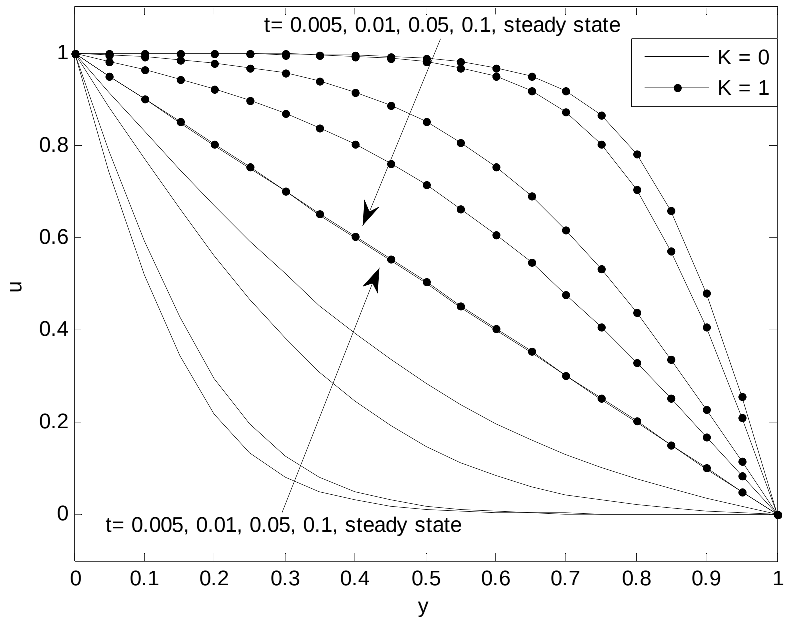

Fig.5. Velocity profile showing the effect of time for $K=0$ and $K=1$ with a fixed value of $M=10.0$, (Case II).

Figure 5 illustrates the impact of a sudden removal of the magnetic field on the unsteady velocity profile at different time and K. Results show that the removal of the magnetic field decreases fluid velocity as time increases until steady state is reached for the case when the magnetic field is relative to the moving place $(K=1)$ while the opposite trend is found when the magnetic field is fixed relative to the fluid $(K=0)$. In addition, at steady state, fluid velocity is exactly the same for either $(K=0)$ or $(K=1)$. This is attributed to the fact that removal of the magnetic field leads to an absence of the magnetic field and hence fluid velocity is independent of K.

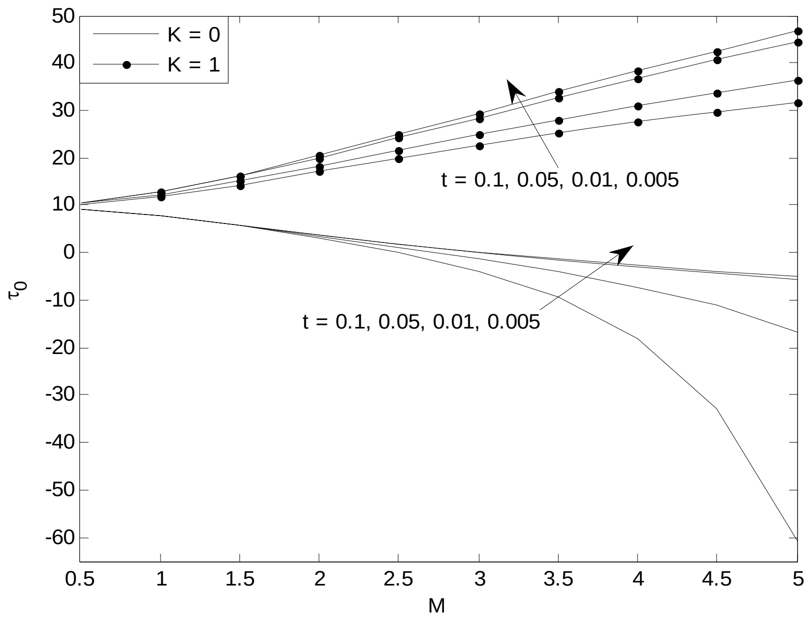

Fig.6. Skin-friction showing the effect of the magnetic field parameter and time for $K=0$ and $K=1$, at $(y=0)$, (Case II).

To see the effect of a sudden removal of the magnetic field on the skin-friction at the channel walls, Figs 6 and 7 exhibit the impact of the magnetic field and time of drag force at the walls. Figure 6 shows that the skin-friction decreases with an increase in time and increases with an increase in the magnetic field parameter for the case $K = 1$ while the reverse observation is found for $K = 0$. Generally, the reverse result is obtained for the skin-friction at the wall $y = 1$ (Fig.7).

Table 3 gives a numerical comparison of time required to attain velocity steady state for the cases when the magnetic field is fixed relative to the fluid $(K = 0)$ and relative to the moving plate $(K = 1)$ for a sudden application of the magnetic field and sudden withdrawal of the magnetic field. It is evident from this numerical computation that for both cases the time required to attain steady state velocity increases with an increase in the magnetic field parameter. In addition, the steady state time increases when the magnetic field is fixed relative to the moving plate compared to when it is fixed relative to fluid. Also, the attainment of steady state is faster when there is a sudden removal of the magnetic field than when the magnetic field is imposed suddenly.

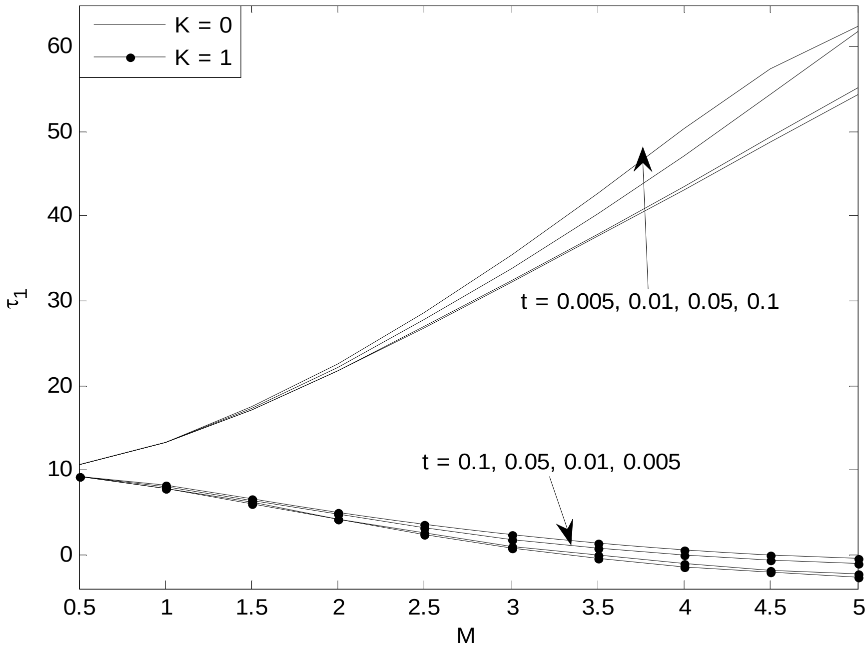

Fig.7. Skin-friction showing the effect of the magnetic field parameter and time for $K = 0$ and $K = 1$, at $(y = 1)$, (Case II).

Table 3. Steady state time of fluid velocity for different values of the magnetic field.

	Due to sudden imposition of magnetic field		**Due to sudden withdrawal of magnetic field**	
M	$K = 0$	$K = 1$	$K = 0$	$K = 1$
0.1	0.20	0.20	0.20	0.20
0.5	0.59	0.65	0.65	0.69
1.0	0.65	0.69	0.69	0.85
2.0	0.79	0.81	0.78	0.90
5.0	0.93	0.95	0.85	1.00
10.0	1.5	1.52	0.90	1.05
50.0	4.5	5.6	0.95	1.10
100.0	6.5	7.8	1.0	1.20

4. Conclusion

This article investigates the impact of sudden application or sudden withdrawal of the magnetic field on unsteady flow formation in a channel in which one of the plates is moving for the case when the magnetic field is applied fixed relative to the fluid or moving plate. The following conclusions can be drawn based on the solution and graphical representations obtained.

1. Fluid velocity decreases with time when the magnetic field is fixed relative to the fluid for the case of sudden imposition of the magnetic field while the reverse result is obtained for a sudden removal of the magnetic field.
2. A sudden removal of the magnetic field leads to an equilibrium in steady state velocity for the case when the magnetic field is fixed relative to the fluid or relative to the moving plate.
3. The skin-friction for a sudden application of the magnetic field is exactly the same at the moving wall when the magnetic field is fixed relative to the moving wall as that of the fixed wall when the magnetic field is fixed relative to the fluid.
4. A sudden application of the magnetic field leads to a delay in attainment of fluid velocity steady state.
5. The time required to transform from hydrodynamic to fully developed magnetohydrodynamic flow and vice-versa is dependent on the magnetic field strength.

Nomenclature

B_0 – magnetic field strength
h – spacing between the parallel plates
M – Hartmann number
P – dimensionless pressure
p – pressure
Q – mass flow rate
U – dimensionless axial velocity
u – axial velocity
u_0 – constant ref. velocity
$\bar{u}$ – mean velocity
$\vec{v}$ – vectorial velocity profile
x – dimensionless axial coordinate
x', y' – axial and transverse coordinates, respectively
y – dimensionless transverse coordinate
β – thermal expansion coefficient
k – fluid thermal conductivity
μ – fluid dynamic viscosity
ρ – fluid density
σ – electrical conductivity of the fluid
τ – skin-friction
ν – kinematic viscosity

Subscripts

s – steady-state

References

[1] Hartmann J. (1937): *Hg-Dynamics I. Theory of laminar flow of an electrically conductive liquid in a homogeneous magnetic field*. Kgl. Danske Videnskabernes Salskab, Mathematisk-Fysiske Meddelelser, vol.15, No.6.

[2] Hartmann J. and Lazarus F. (1937): *Hg-Dynamics II. Experimental investigation on the flow of mercury in a homogeneous magnetic field*. Kgl. Danske Videnskabernes Salskab, Mathematisk-Fysiske Meddelelser, vol.15, No.6.

[3] Rossow V.J. (1957): *Flow of electrically conducting fluids over a flat plate in the presence of a transverse magnetic field*. – Report 1358 National Advesory Committee for Aeronautics.

[4] Singh A.K. and Kumar N. (1983): *Unsteady hydromagnetic Couette flow*. – Wear, vol.89, pp.125-129.

[5] Singh A.K., Sacheti N.C. and Chandran P. (1994): *Transient effects on magnetohydrodynamics Couette flow with rotation: accelerated motion*. – International Journal of Engineering Science, vol.32, pp.133-139.

[6] Couette M.M. (1890): *Etudes sur le frottement des liquids.* – Ann Chim. Phys. 6, Ser., vol.21, pp.433-510.

[7] Jha B.K. and Apere C.A. (2010): *Unsteady MHD two-phase Couette flow of fluid-particle suspension in an annulus*. – J. Phy. Soc. Jpn, vol.79, No.12, pp.124403-124403-5.

[8] Mazumder B.S. (1991): *An exact solution of oscillatory Couette flow in a rotating system.* – ASME J. Appl. Mech., vol.58, No.4, pp.1104-1107.

[9] Jha B.K. and Odengle J.O. (2015): *Unsteady MHD Couette flow in composite channel partially filled with porous material: A semi-analytical Approach*. – Transp. Porous Med., vol.107, pp.219-234.

[10] Akonur A. and Lueptor R.M. (2003): *Three dimensional velocity field for wavy Taylor-Couette flow*. – Phys. Fluids, vol.15, pp.947-960.

[11] Kumaran V., Kumar A.V. and Pop I. (2010): *Transition of MHD boundary layer flow past a stretching sheet*. – Commun. Nonlinear Sci. Numer. Simulat., vol.15, pp.300-311.

[12] Pai S.I. (1962): *Magnetogasdynamics and Plasma Dynamics*. – Berlin: Springer.

[13] Jha B.K. and Apere C.A. (2010): *Unsteady MHD Couette flow in Annuli: the Riemann-sum approximation Approach*. – J. Phy. Soc. Jpn, vol.79, No.12, pp.124403-124403-5.

[14] Khadrawi A.F. and Al-Nimr M.A. (2007): *Unsteady natural convection fluid flow in a vertical microchannel under the effect of the Dual-phase-lag heat conduction model*. – Int. J. Thermophys., vol.28, pp.1387-1400.

[15] Jha B.K. and Oni M.O. (2018): *Transient Natural Convection flow between Vertical Concentric Cylinders Heated/Cooled Asymmetrically*. – Proc IMechE Part A: J Power and Energy, DOI: 10.1177/0957650918758743.

[16] Jha B.K. and Oni M.O. (2017): *An analytical solution for temperature field around a cylindrical surface subjected to a time dependent heat flux: An alternative approach.* – https://doi.org/10.1016/j.aej.2017.01.003

[17] Tzou D.Y. (1997): *Macro to Microscale Heat Transfer: The Lagging Behaviour*. – Washington: Taylor and Francis.

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