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WATER WAVE SCATTERING BY AN INFINITE STEP IN THE PRESENCE OF AN ICE-COVER

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The classical problem of water wave scattering by an infinite step in deep water with a free surface is extended here with an ice-cover modelled as a thin uniform elastic plate. The step exists between regions of finite and infinite depths and waves are incident either from the infinite or from the finite depth water region. Each problem is reduced to an integral equation involving the horizontal component of velocity across the cut above the step. The integral equation is solved numerically using the Galerkin approximation in terms of simple polynomial multiplied by an appropriate weight function whose form is dictated by the behaviour of the fluid velocity near the edge of the step. The reflection and transmission coefficients are obtained approximately and their numerical estimates are seen to satisfy the energy identity. These are also depicted graphically against thenon-dimensional frequency parameter for various ice-cover parameters in a number of figures. In the absence of ice-cover, the results for the free surface are recovered.

Key words: water wave scattering, ice-cover, infinite step, integral equation, Galerkin approximation, reflection and transmission coefficients.

1. Introduction

Study of water wave problems in which the water is covered by a thin elastic plate has gained considerable attentionin recent times due to the importance of finding the application in the construction of very large floating structures like oil storages bases, offshore pleasure cities, floating runways, etc. So, it is worthwhile to consider wave propagation problem where the bottom of the water consists of an infinite step which exists between regions of a combination of finite and infinite depth.

A mathematical model describing the interaction of ocean waves by shore fast sea-ice modelled as a thin elastic plate has been considered by Fox and Squire [1]. They used a matching technique based on minimizing a certain error integral evaluated at the point of discontinuity on the upper surface of water. Hermans [2] used a method based on solving an integral equation to investigate interaction of water waves with floating flexible strips modelled as a thin elastic plate. Gayen *et al*. [3] used Havelock's inversion theorem to reduce the boundary value problem describing interaction of water waves by a surface strip to two coupled singular integral equation of the Carleman type which were solved approximately for large strip width.

In our study, we investigate the infinite step problem in the presence of an ice-cover by reducing it to solving an integral equation for the horizontal component of velocity on the cut above the infinite step. The integral equation is solved numerically by using the Galerkin approximation in terms of simple polynomials

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multiplied by a weight function whose form is dictated by the behaviour of the fluid velocity near the edge of the step. The energy identity has been evaluated and matched with the numerical estimates for the reflection and transmission coefficients. These are also depicted graphically against a non-dimensional frequency parameter in a number of figures for different ice-cover parameters. When the ice-cover parameters are considered very small, they are seen to be very similar to the results obtained by Newman [4] for an infinite step with free surface. Both the cases of wave incidence from deep water region to the finite depth region and from finite depth region to deep water region have been considered.

It may be mentioned here that Newman [4] first considered this type of geometry of an infinite step below a *free surface* and studied water wave scattering by such a step. In our paper, we have replaced the free surface by a thin ice-cover modelled as a thin elastic plate and solved the corresponding water wave scattering problem by formulating the problem in terms of an integral equation which is solved numerically by the Galerkin approximation followed by numerical evaluation of the reflection and transmission coefficients. Due to a significant increase in the scientific activities in the cold regions during the last few decades, a section of the community of hydrodynamic researchers generalized many free surface problems to ice-cover problems wherein the ice-cover is modelled as a thin elastic plate. Thus it is felt that the problem considered here is of some interest for the researchers on water waves.

2. Mathematical formulation

A rectangular Cartesian co-ordinate system is chosen in which the y-axis is taken vertically downwards into the fluid region and the xz -plane coincides with the mean portion of the ice-sheet floating on water with an infinite step. The fluid region consists of two regions, one is of finite depth represented by $(0<y<h,-\infty<x<0)$ and the other is a deep water region represented by $(0<y<\infty,0<x<\infty)$ as shown in Fig.1.

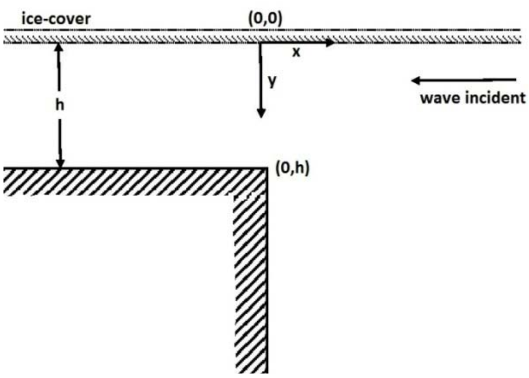

Fig.1. Diagram of the wave incidence.

Assuming linear theory and the motion in water to be irrotational, time harmonic with angular velocity ω, independent of the co-ordinate z and using linear theory if $\varphi(x,y,t)=Re\{\varphi(x,y)e^{-i\omega t}\}$ denotes the velocity potential describing the motion, then φ satisfies the Laplace equation

$$\nabla^2\varphi=0, \tag{2.1}$$

in the fluid region. The condition on the ice-cover is

$$K\varphi+\left(D\partial_x^4+1-\varepsilon K\right)\varphi_y=0 \qquad \text{on} \qquad y=0,-\infty<x<\infty. \tag{2.2}$$

The condition on the step

$$\frac{\partial \varphi}{\partial x} = 0, \qquad x = 0, \qquad h < y < \infty . \tag{2.3}$$

The bottom conditions are

$$\nabla \varphi \to 0 \quad \text{as} \quad y \to \infty, \qquad 0 < x < \infty \ \ \text{(for deep water region)}, \tag{2.4a}$$

$$\frac{d\varphi}{dy} = 0 \quad \text{on} \quad y = h, \qquad -\infty < x < 0 \quad \text{(for finite depth water region)}, \tag{2.4b}$$

the edge condition is

$$r^{1/3}\nabla\varphi \ \text{ is bounded as } r \to \infty \tag{2.5}$$

where $r = \left\{x^2 + (y-h)^2\right\}^{1/2}$ is the distance from the edge of the step, and finally the condition as $|x| \to \infty$ is given by

$$\varphi(x,y) \to \begin{cases} \left(e^{-i\mu_0 x} + R_1 e^{i\mu_0 x}\right)e^{-\mu_0 y} & \text{as} \quad x \to \infty \\ \dfrac{T_1 \cos h\lambda_0 (h-y) e^{-i\lambda_0 x}}{\cos h\lambda_0 h} & \text{as} \quad x \to -\infty \end{cases} \tag{2.6}$$

where μ_0 is the unique positive root of the dispersion relation

$$k\left(Dk^4 + 1 - \epsilon K\right) = K , \tag{2.7}$$

with $K = \dfrac{\omega^2}{g}$, g being the acceleration due to gravity and, $\epsilon = \dfrac{\rho_i d}{\rho}$, $D = \dfrac{Ed^3}{12\left(1-\nu^2\right)\rho g}$ is the flexural rigidity of the ice-sheet, E is the Young's modulus and v is the Poisson's ratio of the material of the elastic plate, ρ as the density of water, ρ_i as the density of the ice-sheet, d is the very small thickness of ice of which a still smaller part is immersed in water so that ϵK is always less than unity for all practical purposes. It may be noted that Eq.(2.7) has a real positive root λ_0 four complex roots $\pm\lambda_1, \pm\lambda_2$ (λ_1 has positive real and imaginary parts with $\lambda_2 = \overline{\lambda_1}$) and a set of countably infinite purely imaginary roots $\pm i\lambda_n^*, n = 1,2,3,\ldots\ldots\ldots \left(\lambda_n^* > 0\right)$ where $\left(n - \dfrac{1}{2}\right)\pi < \lambda_n^* h < \left(n + \dfrac{1}{2}\right)\pi$ and $\lambda_n^* h \to n\pi$ as $n \to \infty$ (c.f. Chung and Fox [5]) and λ_0 is the unique positive root of the dispersion relation

$$k\left(Dk^4 + 1 - \epsilon K\right)\tan hkh = K , \tag{2.8}$$

and R_1 and T_1 denote the reflection and transmission coefficients (unknown) for the case of incidence of waves from the deep water region to the finite depth region.

For the case of incidence from the finite depth region to the deep water region condition (2.6) is to be replaced by

$$\varphi(x,y)\to\begin{cases}\left(e^{-i\lambda_0 x}+R_2 e^{i\lambda_0 x}\right)\dfrac{\cos h\lambda_0(h-y)}{\cos h\lambda_0 h} & \text{as} \quad x\to-\infty\\ T_2 e^{-\mu_0 y+i\mu_0 x} & \text{as} \quad x\to\infty\end{cases} \tag{2.9}$$

where R_2 and T_2 denote, respectively, the reflection and transmission coefficients (unknown) in this case. Our task here is to obtain numerical estimates for R_j, T_j $(j=1,2)$ for different values of various ice-cover parameters.

3. Reduction to integral equation

For the case of incidence of waves from the deep water region, use of Havelock's [6] expansion for water wave potentials for deep and finite depth water produces

$$\begin{aligned}\varphi(x,y)&=e^{-\mu_0 y-i\mu_0 x}+R_1 e^{-\mu_0 y+i\mu_0 x}+A_1 e^{-\mu_1 y+i\mu_1 x}+A_2 e^{-\mu_2 y-i\mu_2 x}+\\&+\int_0^\infty f_1(y)M(k,y)e^{-kx}dk \qquad \text{for} \quad x>0, \quad 0<y<\infty,\end{aligned} \tag{3.1}$$

and

$$\begin{aligned}\varphi(x,y)&=T_1 I_0(y)e^{i\lambda_0 x}+B_1 I_1(y)e^{i\lambda_1 x}+B_2 I_2(y)e^{-i\lambda_2 x}+\\&+\sum_{n=1}^\infty B_n^* I_n^*(y)e^{i\lambda_n^* x} \qquad \text{for} \quad x<0, \quad -\infty<y<0,\end{aligned} \tag{3.2}$$

with

$$M(k,y)=k\left(Dk^4+1-\epsilon K\right)\cos ky-K\sin ky\,, \tag{3.3}$$

nd

$$I_n(y)=\frac{\cosh\lambda_n(h-y)}{\cosh\lambda_n h}, \qquad n=0,1,2\,, \tag{3.4}$$

$$I_n^*(y)=\frac{\cos\lambda_n^*(h-y)}{\cos\lambda_n^* h}, \qquad n=1,2,3,\ldots\ldots\ldots\,. \tag{3.5}$$

Here A_1, A_2, B_1, B_2 and B_n^* are unknown constants and $A(k)$ is an unknown function.

Let,

$$f_1(y)=\frac{d\varphi}{dx}(0,y), \tag{3.6}$$

then condition (3.6) provides

$$f_1(y)=0, \qquad h<y<\infty\,, \tag{3.7}$$

while the edge condition (2.5) gives

$$f_1(y) = O\left((h-y)^{-1/3}\right) \quad \text{as} \quad y \to h-0. \tag{3.8}$$

Thus, $f_1(y)$ is unknown for $0<y<h$. An integral equation for $f_1(y)$ $(0<y<h)$ is now derived following a procedure similar to Manam *et al.* [7] for the deep water region and also for the finite depth region.

As in Manam *et al.* [7], the unknown coefficients in Eq.(3.1) are given by

$$1-R_1 = \frac{i}{\mu_0 a_0}\int_0^\infty f_1(u)e^{-\mu_0 u}du, \tag{3.9}$$

$$A_1 = \frac{i}{\mu_1 a_1}\int_0^\infty f_1(u)e^{-\mu_1 u}du\,, \tag{3.10}$$

$$A_2 = -\frac{i}{\mu_2 a_2}\int_0^\infty f_1(u)e^{-\mu_2 u}du\,, \tag{3.11}$$

and

$$A(k) = -\frac{2}{\pi}\frac{1}{k\left(k^2\left(Dk^4+1-\epsilon K\right)^2+K^2\right)}\int_0^\infty f_1(y)M(k,y)du\,, \tag{3.12}$$

with

$$a_n = \frac{5D\mu_n^4-1+\epsilon K}{2K}, \quad n=0,1,2\,. \tag{3.13}$$

Also, the unknown coefficients Eq.(3.2) are given by

$$T_1 = V(\lambda_0)\int_0^h f_1(u)I_0(u)du\,, \tag{3.14}$$

$$B_1 = V(\lambda_1)\int_0^h f_1(u)I_1(u)du\,, \tag{3.15}$$

$$B_2 = V(\lambda_2)\int_0^h f_1(u)I_2(u)du \tag{3.16}$$

and

$$B_n^* = -V(i\lambda_n^*)\int_0^h f_1(u)I_n^*(u)du\,, \tag{3.17}$$

with

$$V(k)=\frac{4i(Dk^4+1-\epsilon K)\cosh^2 kh}{\left(5Dk^4+1-\epsilon K\right)2kh+(5Dk^4+1-\epsilon K)\sinh 2kh}. \tag{3.18}$$

Now the continuity of pressure above the step $(x=0, 0<y<h)$ gives

$$\varphi(+0,y)=\varphi(-0,y), \qquad 0<y<h. \tag{3.19}$$

This provides the integral equation for $f_1(y)$ as given by

$$\int_0^h f_1(u)L_1(y,u)du=-(1+R_1)e^{-\mu_0 y}, \quad 0<y<h \tag{3.20}$$

where

$$L_1(y,u)=\frac{i}{\mu_1 a_1}e^{-\mu_1(u+y)}-\frac{i}{\mu_2 a_2}e^{-\mu_2(u+y)}-\frac{2}{\pi}\int_0^\infty \frac{M(k,u)M(k,y)}{k\left(k^2\left(Dk^4+1-\epsilon K\right)^2+K^2\right)}dk+$$

$$-V(\lambda_0)I_0(y)I_0(u)-V(\lambda_1)I_1(y)I_1(u)+V(\lambda_1)I_2(y)I_2(u)+\sum_{n=1}^{\infty}V\left(i\lambda_n^*\right)I_n^*(y)I_n^*(u),$$

$$0<y,u<h. \tag{3.21}$$

If we write

$$g_1(u)=\frac{f_1(u)}{1+R_1}, \tag{3.22}$$

then Eq.(3.20) is reduced to

$$\int_0^h g_1(u)L_1(y,u)du=-e^{-\mu_0 y}, \qquad 0<y<h. \tag{3.23}$$

For the case of incidence from the finite depth water region to the deep water region the integral equation for $f_2(y)(=\frac{d\varphi}{dx}(-0,y), 0<y<h)$ is given by

$$\int_0^h f_2(u)L_2(y,u)du=-(1+R_2)I_0(u), \quad 0<y<h \tag{3.24}$$

where

$$L_2(y,u)=V(\lambda_1)I_1(y)I_1(u)-V(\lambda_1)I_2(y)I_2(u)-\sum_{n=1}^{\infty}V\left(i\lambda_n^*\right)I_n^*(y)I_n^*(u)+$$

$$-\frac{i}{\mu_0 a_0}e^{-\mu_0(u+y)}-\frac{i}{\mu_1 a_1}e^{-\mu_1(u+y)}+\frac{i}{\mu_2 a_2}e^{-\mu_2(u+y)}+$$

$$+\frac{2}{\pi}\int_0^\infty \frac{M(k,u)M(k,y)}{k\left(k^2\left(Dk^4+1-\epsilon K\right)^2+K^2\right)}dk \qquad 0<y,\ u<h. \tag{3.25}$$

If we write

$$g_2(u) = \frac{f_2(u)}{1+R_2}, \tag{3.26}$$

then Eq.(3.24) is reduced to

$$\int_0^h g_2(u) L_2(y,u)\,du = -I_0(u), \quad 0<y<h. \tag{3.27}$$

It may be noted that both the integral Eqs (3.23) and (3.27) are weakly singular integral equations.

4. Numerical solutions of the integral equations

Numerical solutions of the integral Eqs (3.23) and (3.27) are now obtained by using the Galerkin approximation in terms of simple polynomials multiplied by an appropriate weight function whose form is dictated by the edge condition (2.5). Thus we expand $g_1(y)$ and $g_2(y)$ as

$$g_1(y) = \left(\frac{h}{h-y}\right)^{1/3} \sum_{n=0}^{N} c_n \left(\frac{y}{h}\right)^n \tag{4.1}$$

and

$$g_2(y) = \left(\frac{h}{h-y}\right)^{1/3} \sum_{n=0}^{N} d_n \left(\frac{y}{h}\right)^n \tag{4.2}$$

where N is an integer to be chosen.

Substituting the expression of $f_1(y)$ in terms of $g_1(y)$ given by Eq.(4.1) in Eqs (3.9) and (3.14) we obtain R_1 and T_1 as given by

$$R_1 = \frac{1 - \dfrac{i}{\mu_0 a_0}\sum_{n=0}^{N} c_n \int_0^h \left(\dfrac{h}{h-y}\right)^{1/3}\left(\dfrac{y}{h}\right)^n e^{-\mu_0 y}\,dy}{1 + \dfrac{i}{\mu_0 a_0}\sum_{n=0}^{N} c_n \int_0^h \left(\dfrac{h}{h-y}\right)^{1/3}\left(\dfrac{y}{h}\right)^n e^{-\mu_0 y}\,dy}, \tag{4.3}$$

and

$$T_1 = (1+R_1)V(\lambda_0)\sum_{n=0}^{N} c_n \int_0^h \left(\frac{h}{h-y}\right)^{1/3}\left(\frac{y}{h}\right)^n I_0(y)\,dy. \tag{4.4}$$

Similarly, using Eq.(4.2) we obtain R_2 and T_2 as given by

$$R_2 = \frac{1 - V(\lambda_0)\sum_{n=0}^{N} d_n \int_0^h \left(\dfrac{h}{h-y}\right)^{1/3}\left(\dfrac{y}{h}\right)^n I_0(y)\,dy}{1 + V(\lambda_0)\sum_{n=0}^{N} d_n \int_0^h \left(\dfrac{h}{h-y}\right)^{1/3}\left(\dfrac{y}{h}\right)^n I_0(y)\,dy}, \tag{4.5}$$

and

$$T_2 = (1+R_2)\frac{i}{\mu_0 a_0}\sum_{n=0}^{N} c_n \int_0^h \left(\frac{h}{h-y}\right)^{\frac{1}{3}} \left(\frac{y}{h}\right)^n e^{-\mu_0 y} dy \,. \tag{4.6}$$

To find the unknown constants $c_n\ (n = 0, 1, 2,\ldots., N)$ we put $y = y_i\ \ (i = 0, 1, 2,\ldots., N)(0 < \mathrm{y}_i < h)$, in relation (3.23) to obtain the linear system

$$\sum_{n=0}^{N} c_n A_{in} = p_i, \quad i = 0,1,2,\ldots\ldots\ldots.N \tag{4.7}$$

where

$$A_{in} = \int_0^h \left(\frac{h}{h-u}\right)^{\frac{1}{3}} \left(\frac{u}{h}\right)^n L_1(y_i,u)du, \quad i = 0,1,2,\ldots\ldots\ldots.N \tag{4.8}$$

and

$$p_i = e^{-\mu_0 y_i}, \quad i = 0,1,2,\ldots\ldots\ldots.N\,. \tag{4.9}$$

The collocation points y_i are to be chosen suitably. Here we have chosen

$$y_i = \frac{ih}{n}, \qquad i = 0,1,2,\ldots\ldots\ldots.N\,. \tag{4.10}$$

The linear system (4.7) is solved by any standard method to obtain the constants $c_n\ (n = 0, 1, 2,\ldots., N)$ and thus the approximate solution of the integral Eq.(3.23) is obtained.

Similarly, to find the unknown constants $d_n\ (n = 0, 1, 2,\ldots., N)$ we put $y = y_i$ $(i = 0, 1, 2,\ldots., N)(0 < y_i < h)$, in relation (3.27) to obtain the linear system

$$\sum_{n=0}^{N} d_n B_{in} = q_i, \quad i = 0,1,2,\ldots\ldots\ldots.N \tag{4.11}$$

where

$$B_{in} = \int_0^h \left(\frac{h}{h-u}\right)^{\frac{1}{3}} \left(\frac{u}{h}\right)^n L_2(y_i,u)du, \quad i = 0,1,2,\ldots\ldots\ldots.N \tag{4.12}$$

and

$$p_i = I_0(y_i), \quad i = 0,1,2,\ldots\ldots\ldots.N\,. \tag{4.13}$$

The collocation points y_i are chosen similarly as given by Eq.(4.10).

Now, the linear system (4.11) is solved by any standard method to obtain the constants $d_n\ (n = 0, 1, 2,\ldots., N)$ and thus the approximate solution of the integral Eq.(3.27) is obtained.

Using Eqs (4.3) and (4.4), numerical estimates for R_1 and T_1 for different values of D and ε choosing $N = 0,1,2,\ldots..$ against $(Kh)^{1/2}$, it is observed that fairly accurate numerical estimates are obtained for $N = 2$. Also, these numerical estimates satisfy the energy identity given by

$$|R_1|^2 + \gamma |T_1|^2 = 1 \tag{4.14}$$

where

$$\gamma = \frac{2\lambda_0 h + \sinh 2\lambda_0 h}{2\cosh^2 \lambda_0 h}\left(\frac{\lambda_0^{\ 4} + 1 - \varepsilon K}{\mu_0^{\ 4} + 1 - \varepsilon K}\right). \tag{4.15}$$

For the case of incidence of wave from the finite depth water region, numerical estimates for R_2 and T_2 are obtained from Eqs (4.5) and (4.6), respectively, after solving the linear system (4.11). These estimates satisfy the energy identity derived

$$|R_2|^2 + \delta |T_2|^2 = 1 \tag{4.16}$$

where

$$\delta = \frac{2\cosh^2 \lambda_0 h}{2\lambda_0 h + \sinh 2\lambda_0 h}\left(\frac{\mu_0^{\ 4} + 1 - \varepsilon K}{\lambda_0^{\ 4} + 1 - \varepsilon K}\right). \tag{4.17}$$

The energy identities given by Eqs (4.14) and (4.17) can be derived as in Das *et al*. [8].

5. Numerical results

The values of $|R_1|$, $|T_1|$, $|R_2|$ and $|T_2|$ are computed numerically for the non-dimensional parameters $D^{'} = \dfrac{D}{h^4}$ and $\varepsilon^{'} = \dfrac{\varepsilon}{h}$ against the non-dimensional frequency parameter $(Kh)^{1/2}$ in different figures. For the purpose of computing the reflection and transmission coefficients, we have chosen $N = 2$.

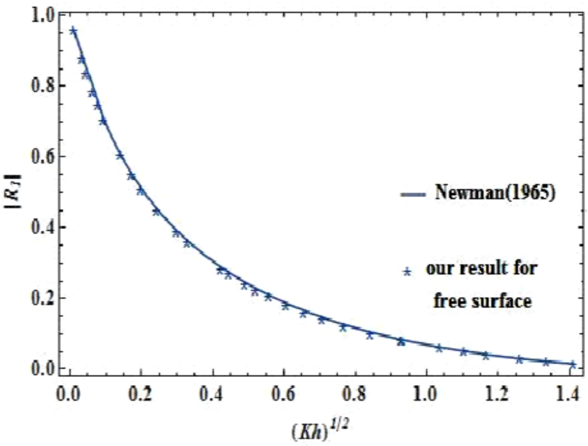

Fig.2. Graph for $|R_1|$ vs$(Kh)^{1/2}$.

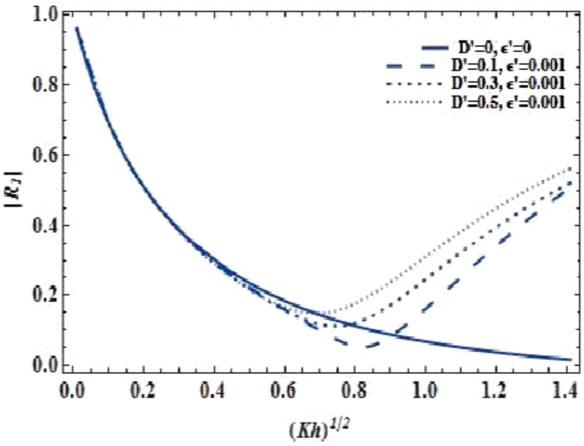

Fig.3. Graph for $|R_1|$ vs$(Kh)^{1/2}$.

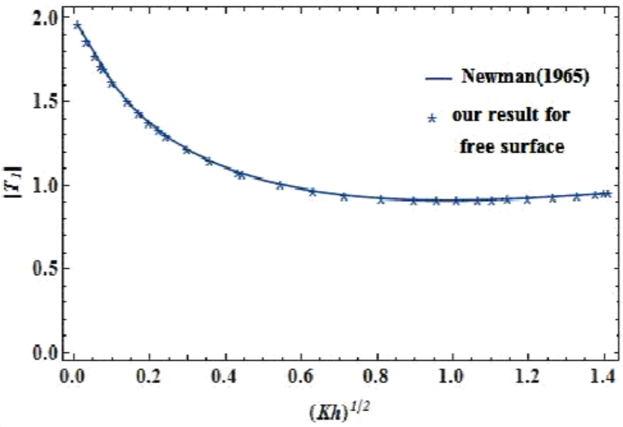

Fig.4. Graph for $|T_1|$ vs$(Kh)^{1/2}$.

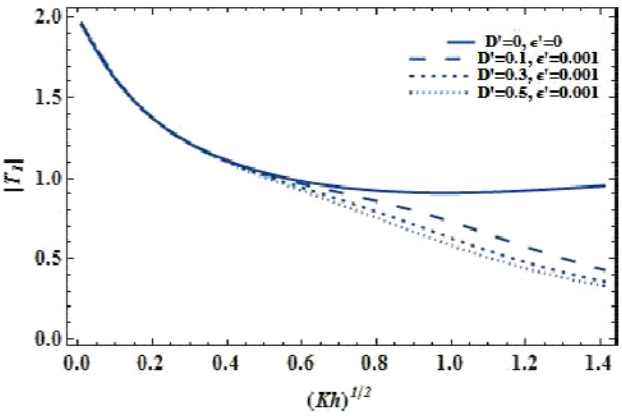

Fig.5. Graph for $|T_1|$ vs$(Kh)^{1/2}$.

In Fig.2, it is observed that, the computed values of $|R_1|$ for small values of $D^{'}$ and $\varepsilon^{'}$ (e.g. $D^{'}=0.0001,\ \varepsilon^{'}=0.001$) and Newman's [4] free surface results (i.e., $D^{'}=0,\ \varepsilon^{'}=0$) almost coincide providing a check on the correctness of the numerical method employed here. In Fig. 3, the numerical results of $|R_1|$ are presented for different values of $D^{'}$ and $\varepsilon^{'}$ (e. g. $D^{'}, \varepsilon^{'} = 0,0$; $0.1,0.001$; $0.3,0.001$; $0.5,0.001$) against the wave number $(Kh)^{1/2}$. Again, in Fig.4, it is observed that, the numerical estimates for $|T_1|$ for very small values of $D^{'}$ and $\varepsilon^{'}$ (e.g. $D^{'}=0.0001,\ \varepsilon^{'}=0.001$) almost coincide with Newman's [4] free surface results for $D^{'}=0$ and $\varepsilon^{'}=0$ in most cases. In Fig.5, the numerical estimates for $|T_1|$ are presented for the same set of values of $D^{'}$ and $\varepsilon^{'}$ used in Fig.3. In addition, our present numerical values of $|R_1|$ and $|T_1|$ also satisfy the energy identity given by Eq.(4.15), which demonstrates another check on the correctness of the numerical values used here. Figures 3 and 5 demonstrate the effect of the presence of the ice-cover on the reflection and transmission coefficients.

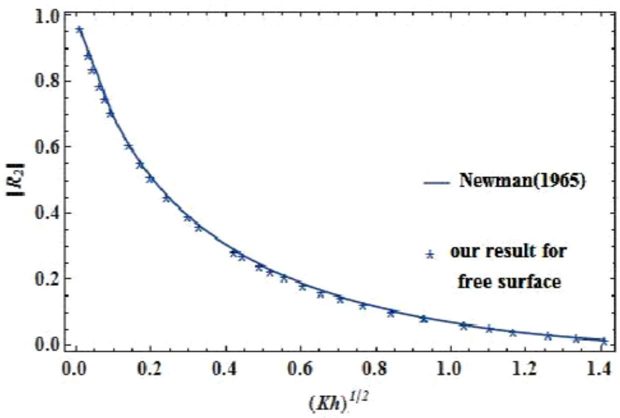

Fig.6. Graph for $|R_2|$ vs$(Kh)^{1/2}$.

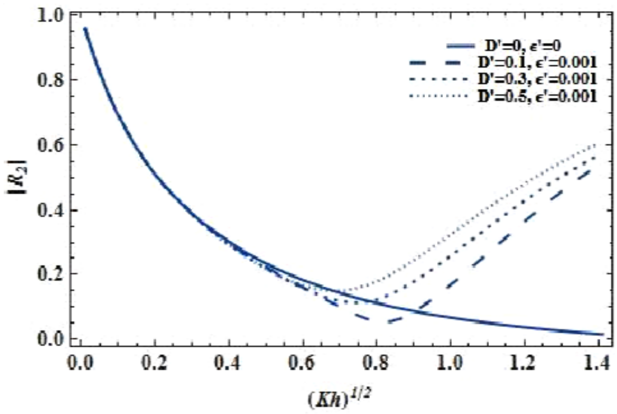

Fig.7. Graph for $|R_2|$ vs$(Kh)^{1/2}$.

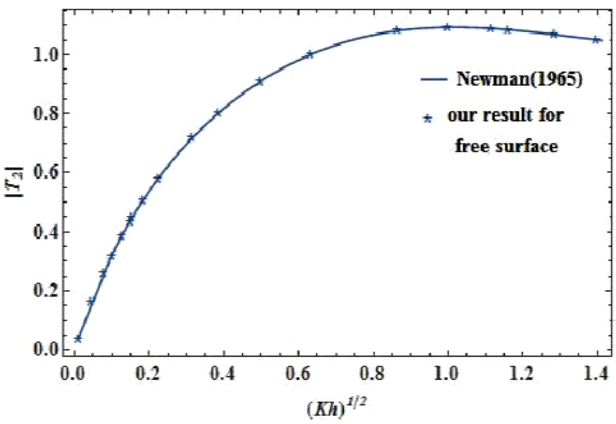

Fig.8. Graph for $|T_2|$ vs$(Kh)^{1/2}$.

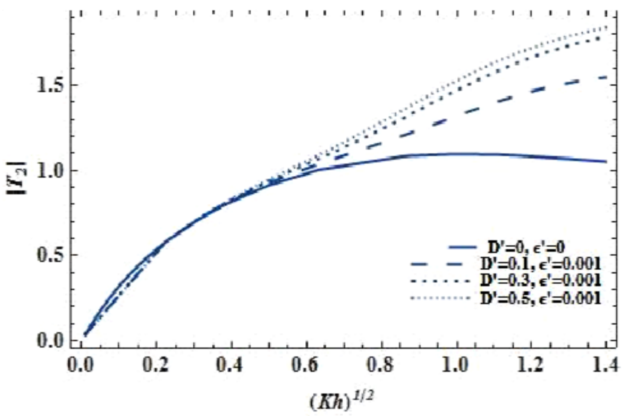

Fig.9. Graph for$|T_2|$ vs$(Kh)^{1/2}$.

In Figs 6 and 7, the numerical estimates for $|R_2|$ and in Figs 8 and 9, the numerical estimates for $|T_2|$ are presented, i.e. when incidence is from the deep water region to the finite depth region. It is observed that the numerical estimates for $|R_2|$ are almost same as $|R_1|$, which was previously obtained by Newman [4]. Also, for very small values of D' and ε' (e.g.$D' = 0.0001$, $\varepsilon' = 0.001$), both the values of $|R_2|$ and $|T_2|$ almost coincide~~s~~ with Newman's [4] results for the free surface (as in Figs 6 and 8). In Figs 7 and 9, the values of $|R_2|$ and $|T_2|$ are depicted against $(Kh)^{1/2}$ for the same set of values of D' and ε' used in $|R_1|$ and $|T_1|$, respectively. The numerical estimates for $|R_2|$ and $|T_2|$ also satisfy the energy identity (4.17) to ensure the correctness of the results obtained here. Figures 6 and 8 also demonstrate a significant effect of the presence of ice-cover on the reflection and transmission coefficients. However, the method employed in the mathematical analysis is simple compared to the same used by Newman [4].

6. Conclusion

The classical two-dimensional problem of water wave scattering by a step of infinite depth beneath a free surface is extended here to the same beneath an ice-cover modelled as a thin elastic plate considering two cases, i.e. when the waves are incident from the deep water region to the finite depth water region and vice versa. For each case a weakly singular integral equation is formed for the horizontal component of velocity across the cut above the step and this is solved numerically by the Galerkin technique wherein the unknown function is expanded in terms of simple polynomial multiplied by an appropriate weight function whose form is dictated by the edge condition at the corner of the infinite step. Very accurate numerical estimates for the reflection and transmission coefficients are obtained and are depicted in a number of figures. The effect of the presence of ice cover is seen to be insignificant for the small wave number but becomes significant for moderately large wave numbers.

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Nomenclature

E – Young's modulus
g – acceleration due to gravity
h – constant depth of water
K – wave number
ν – Poisson ratio
ρ – density of water
φ – velocity potential
$\epsilon\rho$ – surface density of the ice-cover

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