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# RADIATION EFFECTS ON MHD BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER A NONLINEAR STRETCHING SURFACE WITH VARIABLE WALL TEMPERATURE IN THE PRESENCE OF NON-UNIFORM HEAT SOURCE/SINK

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In this paper, an investigation is made to analyze the effects of radiation on an MHD boundary layer flow and heat transfer over a nonlinear stretching surface with variable wall temperature and non-uniform heat source/sink. A suitable similarity transformation is used to transform the governing nonlinear partial differential equations into a system of nonlinear ordinary differential equations by using the Nachtsheim Swigert shooting iteration technique together with the fourth order Runge Kutta method. The effects of various physical parameters over a dimensionless velocity and dimensionless temperature are presented graphically. The numerical results for the skin friction co-efficient and non- dimensional rate of heat transfer are presented and discussed for several sets of values of the parameters. Comparisons of numerical results are made with the earlier published results under limiting cases.

**Key words:** MHD, radiation, non-uniform heat source/sink, variable wall temperature.

## 1. Introduction

The study of a boundary layer flow over a stretching sheet has gained considerable attention in the contemporary world of fast technological applications such as extrusion processes, wire and fiber coating, polymer processing, manufacturing process of artificial films, artificial fibers, dilute polymer solution and design of various heat exchangers. The analysis of momentum and thermal transports within the fluid on a continuously stretching surface is important for gaining some fundamental understanding of such processes. Sakiadis [1, 2] initiated the study of the boundary layer flow over a continuous stretching surface. Later, Crane [3] obtained closed-form similarity solutions for a steady two-dimensional incompressible boundary layer flow caused by a stretching sheet with linear velocity. Further, many investigations over a stretching

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sheet were carried out under various physical situations. Banks [4] obtained a power law velocity variation over the flow field of a stretching wall for different values of the velocity exponent. Kumaran and Ramanaiah [5] considered the quadratic stretching on the viscous fluid flow over a stretching surface and obtained the closed form solutions.

The study of the laminar boundary layer flow and heat transfer over a nonlinearly stretching sheet has gained great interest of several authors due to its enormous applications in industries. Soundalgekar and Murty [6] investigated the power law temperature variation with constant surface. Carragher and Crane [7] discussed the heat transfer in a two dimensional flow past a stretching sheet when the temperature difference between the surface and the ambient fluid is proportional to a power of distance from the fixed point. Grubka and Bobba [8] examined the temperature field in the flow over a stretching surface. But they restricted their research to analyze the heat transfer and flow characteristics in the absence of a magnetic field. The effect of non-uniform surface temperature over nonlinearly stretching surface was studied by Noor Afzal [9].

The power law velocity and temperature distribution at the surface was discussed by Ali [10]. Vajravelu [11] investigated the flow and heat transfer characteristic over a nonlinearly stretching sheet. Cortell [12] focused on heat transfer over a nonlinearly stretching sheet. The flow and heat transfer over a nonlinear stretching sheet was studied by Akyildiz and Siginer [13].

The study of a magneto-hydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. In metallurgical processes, the rate of cooling and stretching of the strips can be controlled by drawing the strip in an electrically conducting fluid subject to a magnetic field, so that a final product of desired characteristics can be achieved. Due to these applications, Chakrabarti and Gupta [14] obtained an analytical solution for the linear stretching problem with hydromagnetic effect. Andersson [15] carried out an exact solution of the Navier–Stokes equations for the magneto hydrodynamic flow. Chiam [16] presented an analytical and numerical solution of an MHD flow and heat transfer over a non- linear stretching surface with power law velocity by using the Runge–Kutta shooting algorithm with Newton iteration. Anjali Devi and Thiyagarajan [17] discussed the steady nonlinear MHD flow and heat transfer over a stretching surface with variable temperature. Anjali Devi and David Maxim Gururaj [18] studied the effects of variable viscosity on the nonlinear MHD flow and heat transfer over a stretching surface with power law velocity.

In several practical applications, there exist significant temperature differences between the surface and the ambient fluid. This necessitates the consideration of temperature-dependent heat sources or sinks which may exert a strong influence on the heat transfer characteristics. The study on heat generation or absorption effect is important in view of several physical problems of such fluids undergoing exothermic or endothermic chemical reactions. Although exact modeling of internal heat generation or absorption is quite difficult, some simple mathematical models can express its average behavior for most physical situations. Vajravelu and Rollins [19] investigated heat transfer characteristics in an electrically conducting fluid over a stretching sheet with internal heat source/sink. Elbashbeshy and Bazid [20] examined the heat transfer over a stretching surface with internal heat generation. Abo-Eldahab and El Aziz [21] considered the blowing/suction effect on hydromagnetic heat transfer by mixed convection from an inclined continuously stretching surface with internal heat generation/absorption, but the work considered both the space and temperature dependent heat source/sink, in a viscous flow. Heat transfer in a viscoelastic boundary layer flow over a stretching sheet with non-uniform heat source/sink was studied by Abel *et al*. [22]. Nandeppanavar *et al*. [23] investigated the heat transfer of a viscoelastic fluid flow due to nonlinear stretching sheet with internal heat source.

All the above investigations were restricted to MHD flows and heat transfer problems. However, of late, the radiation effects on the MHD flow and heat transfer problem has become more important in the engineering field. Many processes in engineering occur at high temperature and the full understanding of the effects of radiation on the rate of heat transfer is necessary in the design of equipment. The radiative flow of an electrically conducting fluid with high temperature in the presence of a magnetic field are encountered in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry, nuclear engineering applications and in other industrial areas. Viskanta and Grosh [24] analysed a boundary layer flow in thermal radiation for absorbing and emitting media by using the Rosseland approximation.

Raptis and Massales [25] discussed the MHD flow over a flat plate in the presence of radiation. Raptis *et al*. [26] investigated the first numerical solution of the flow field with the influence of both the magnetic field and radiation. Bataller [27] investigated the effects of thermal radiation on the Blasius flow. Recently, the hydromagnetic boundary layer flow over a stretching surface with thermal radiation has been examined by Siti *et al*. [28]. Yahaya and Daniel [29] analysed the theoretical influence of buoyancy and thermal radiation on the MHD flow over a stretching porous sheet. Mahantesh *et al*. [30] obtained an optimal homotopy asymptotic method for a fluid flow and heat transfer over a nonlinear stretching sheet.

So far no attempt has been made to analyse the radiation effects on a viscous boundary layer forced convective flow over a nonlinear stretching surface with non-uniform heat source/sink and variable wall temperature in the presence of a magnetic field and hence the present work is focused on this. The equations of continuity, momentum and energy, which govern the flow field, are solved numerically using the Nachtsheim Swigert shooting iteration scheme together with the fourth order Runge Kutta integration method. In this analysis, estimation of the skin friction co-efficient and the non-dimensional rate of heat transfer which are considered significant from the industrial applications perspective are also made. In the absence of the variable magnetic field and radiation, the results obtained are in good agreement with these of Ali [10] and Cortell [12].

## 2. Mathematical formulation

A steady, two-dimensional laminar boundary layer flow of a viscous, incompressible, electrically conducting and radiating fluid over a nonlinear stretching surface with variable temperature in the presence of a variable magnetic field and non-uniform heat source/sink has been considered. Two equal and opposite forces are introduced along the $x$-axis so that the surface is stretched keeping the origin as fixed. The $x$-axis is taken along the stretching surface $y=0$ in a direction of motion and the $y$-axis is perpendicular to the surface in the outward direction. The surface is considered to be nonlinearly stretching along the $x$-axis with the distance $x^n$. The variable magnetic field of strength $\boldsymbol{B}(x)$ is applied in the normal direction to the surface. The flow configuration and the coordinate system are shown in Fig.1.

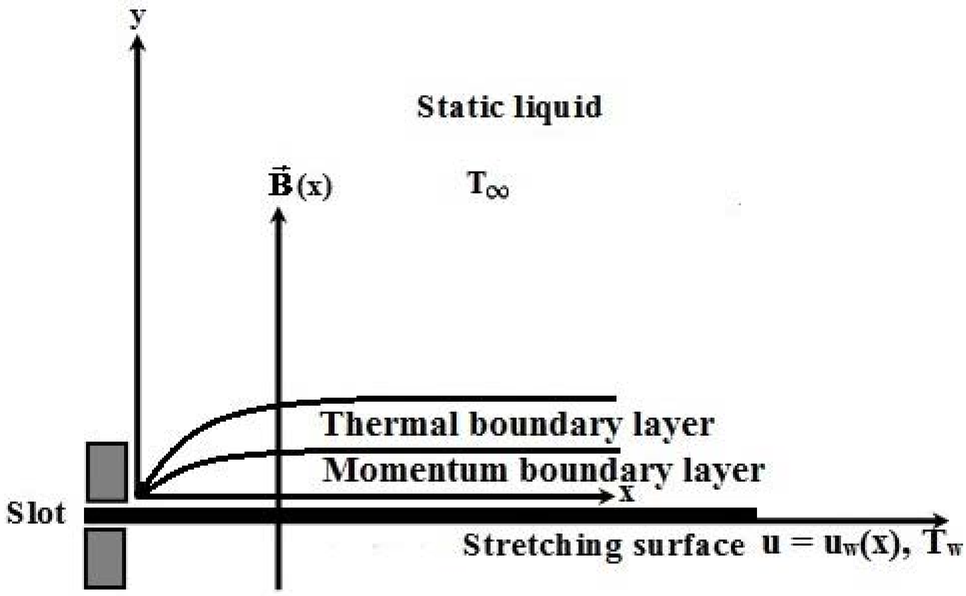


Fig.1. Schematic diagram of the problem.

The magnetic Reynolds number is assumed to be small so that the induced magnetic field is assumed to be negligible in comparison to that of the applied magnetic field. Since the induced magnetic field is assumed to be negligible and $\boldsymbol{B}(x)$ is independent of time, curl $\boldsymbol{E} = 0$ also div $\boldsymbol{E} = 0$ in the absence of surface charge density. Hence $\boldsymbol{E} = 0$ and it is also assumed that the electric field due to the polarization of charge is negligible. The pressure gradient, body forces, viscous dissipation and Joule heating effects are assumed to be negligible. The temperature of the plate surface is held uniform at $T_w$ which is higher than the ambient temperature $T_\infty$. The fluid is considered to be a gray, absorbing, emitting radiation but non scattering medium. The Rosseland approximation is used in the energy equation to describe the radiative heat flux. The radiative heat flux in the $x$-direction is considered to be negligible when compared to that in the $y$-direction and the boundary layer approximations are made.

Under these assumptions, the governing boundary layer equations for conservation of mass, momentum and thermal energy in the presence of the magnetic field, non-uniform heat source/sink and radiation are given below.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma[B(x)]^2}{\rho}u\,, \tag{2.2}$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{q'''}{\rho C_p}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{2.3}$$

where $u$ and $v$ are the velocity components of the fluid in the $x$ and $y$ direction respectively. $\sigma$ is the electrical conductivity of the fluid, $B(x)=B_0 x^{\frac{n-1}{2}}$ is the applied variable magnetic field, $\rho$ is the fluid density of the fluid, $\alpha=\frac{k}{\rho C_p}$ is the thermal diffusivity, $B_0$ is a constant, $\nu=\frac{\mu}{\rho}$ is the kinematic viscosity, $\mu$ is the viscosity of the fluid, $T$ is the temperature of the fluid, $k$ is the thermal conductivity of the fluid, $C_p$ is the specific heat at constant pressure, $q_r$ is the radiative heat flux and $q'''$ is non-uniform heat source/sink defined as [21]

$$q'''=\frac{k\,u_w(x)}{x\,\nu}\left(A^*\left(T_w-T_\infty\right)e^{-\eta}+B^*\left(T-T_\infty\right)\right) \tag{2.4}$$

where $u_w(x)=b\,x^n$ is the stretching velocity, $b$ is the constant, $n$ is a parameter related to the stretching surface, $A^*$ and $B^*$ are parameters of space and temperature dependent internal heat source/sink. $T_w$ is the temperature of the surface and $T_\infty$ is the constant temperature far away from the surface. For an optically thick fluid, the radiative heat flux $q_r$ [26], can be approximated by the Rosseland approximation as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{2.5}$$

where $\sigma^*$ and $k^*$ are the Stephan-Boltzman constant and mean absorption coefficient respectively.

We assume that the temperature difference between the fluid within the boundary layer region is very small so that $T^4$ can be expressed as a linear function of temperature $T$. Thus, expanding $T^4$ in Taylor series about $T_\infty$ and neglecting the second and higher order terms, we obtain

$$T^4 \cong 4T^3_\infty T - 3T_\infty^{\,4}, \tag{2.6}$$

using Eqs (2.5) and (2.6), Eq.(2.3) can be reduced as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^{\,3}}{3k^*\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{q'''}{\rho C_p}. \tag{2.7}$$

The associated boundary conditions are

$$\begin{aligned} &u = u_w(x), \quad v = 0, \quad T = T_w(x) = Ax^\lambda \quad \text{at} \quad y = 0, \\ &u = 0, \quad T \to T_\infty \quad \text{as} \quad y \to \infty \end{aligned} \tag{2.8}$$

where λ is the temperature parameter.

## 3. Similarity analysis

The velocity components $u$ and $v$ are defined as

$$u = \frac{\partial\psi}{\partial y}, \qquad v = -\frac{\partial\psi}{\partial x}. \tag{3.1}$$

Introducing the similarity variable η and the dimensionless variable θ as

$$\eta = y\sqrt{\frac{b(n+1)}{2\nu}x^{\frac{n-1}{2}}}, \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \qquad \text{where} \qquad T_w - T_\infty = Ax^\lambda, \tag{3.2}$$

the velocity components are obtained as

$$u = \frac{\partial\psi}{\partial y} = bx^n f'(\eta), \qquad v = -\frac{\partial\psi}{\partial x} = -\sqrt{\frac{b\nu(n+1)}{2}}x^{\frac{n-1}{2}}\left(f(\eta) + \frac{n-1}{n+1}\eta f'(\eta)\right). \tag{3.3}$$

Equations (2.2) and (2.3) can be written as

$$f'''(\eta) + f(\eta)f''(\eta) - \frac{2n}{n+1}f'^2(\eta) - M^2 f'(\eta) = 0, \tag{3.4}$$

$$\left(1 + \frac{4}{3R_d}\right)\theta''(\eta) + \Pr f(\eta)\theta'(\eta) + \frac{2}{n+1}\left(B^* - \Pr\lambda f'(\eta)\right)\theta(\eta) + \frac{2}{n+1}A^* e^{-\eta} = 0, \tag{3.5}$$

with the appropriate boundary conditions

$$f'(\eta)=1, \quad f(\eta)=0, \quad \theta(\eta)=1 \quad \text{at} \quad \eta=0,$$
$$f'(\eta)\to 0, \quad \theta'(\eta)\to 0 \quad \text{as} \quad \eta\to\infty \tag{3.6}$$

where $M^2=\dfrac{2\sigma B_0^{\,2}}{\rho b(n+1)}$ is the magnetic interaction parameter, $R_d=\dfrac{kk^*}{4\sigma^* T_\infty^{\,3}}$ is the radiation parameter and $\Pr=\dfrac{\nu}{\alpha}$ is the Prandtl number.

The important physical quantities of interest in this problem are the skin friction coefficient and the local Nusselt number which are defined as

$$\mathrm{Re}_x^{\,1/2}C_f=\frac{2\tau_w}{\rho u_w^{\,2}}=\sqrt{2(n+1)}\,f''(0), \tag{3.7}$$

$$\frac{\mathrm{Nu}_x}{\mathrm{Re}_x^{\,1/2}}=\frac{xq_w}{k(T_w-T_\infty)}=-\sqrt{\frac{n+1}{2}}\left(1+\frac{4}{3R_d}\right)\theta'(0) \tag{3.8}$$

where $\mathrm{Re}_x=\dfrac{u_w x}{\upsilon}$ is the local Reynolds number, the surface shear stress $\tau_w$ and the surface heat flux $q_w$ are given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \qquad q_w=\left(-k\frac{\partial T}{\partial y}+q_r\right)_{y=0}. \tag{3.9}$$

## 4. Numerical solution of the problem

The set of nonlinear differential Eqs (3.4) and (3.5) along with the boundary conditions (3.6) constitute a nonlinear boundary value problem. The analytical solution of these equations is very difficult to obtain, and hence the solution is obtained numerically by reducing the nonlinear boundary value problem to an initial value problem using the Nachtsheim-Swigert shooting iteration scheme. The reduced initial value problem is solved utilizing the fourth order Runge-Kutta method.

The boundary value problem consisting of the nonlinear differential equations are solved with initial guesses for $f''(0)$ and $\theta'(0)$. The initial guesses are made for $f''(0)$ and $\theta'(0)$ using the Nachtsheim Swigert shooting iteration technique. The numerical solutions are obtained using the fourth order Runge-Kutta method for different values of the physical parameters over the flow field and dimensionless temperature distribution. Numerical values of the dimensionless skin friction coefficient and the local Nusselt number are obtained and tabulated.

## 5. Results and discussion

Radiation effects on the MHD boundary layer flow and heat transfer over a nonlinear stretching surface with variable wall temperature in the presence of non-uniform heat source/sink have been investigated. Numerical solutions of the problem are obtained using the Nachtsheim Swigert shooting iteration technique together with the fourth order Runge Kutta integration method for various values of the physical parameters such as the magnetic interaction parameter $M^2$, power-law index $n$, radiation parameter $R_d$, Prandtl number Pr, temperature parameter $\lambda$, space dependent heat source/sink parameter $A^*$ and temperature dependent heat source/sink parameter $B^*$. The dimensionless velocity and dimensionless temperature distributions are demonstrated graphically.

In the absence of the magnetic field, a comparative study has been made for different values of $n$ with the results of Cortell [12]. It is obvious that the numerical values of $-f''(0)$ are in excellent agreement with the results of Cortell [12] which are presented in Tab.1. In the case when $1/R_d=0$, $M^2=0$, $A^*=0$, $B^*=0$, the results of temperature gradient are excellent with these of Ali [10] which are depicted in Tab.2. Table 3 depicts the non-dimensional rate of heat transfer for various physical parameters. From this table, it can be seen that the effect of the magnetic interaction parameter, radiation parameter $R_d$, space dependent heat source/sink parameter $A^*$ and the temperature dependent heat source/sink parameter $B^*$ are all similar so as to reduce the non-dimensional rate of heat transfer while the influence of power law index parameter $n$, Prandtl number Pr and the wall temperature parameter $\lambda$ is to enhance the non-dimensional rate of heat transfer. Figure 2 displays a comparison graph of dimensionless temperature for different values of Pr. In the absence of non-uniform heat source/sink, Magnetic field and the radiation effects, the results are in good agreement with that of Ali [10] when $\lambda=0$.

Table 1. Comparison of the values of velocity gradient at the wall $-f''(0)$ for various values of $n$.

| $n$ | $-f''(0)$ | |
|---|---|---|
| | Cortell [12] | Author's result |
| 0.5 | 0.889477 | 0.889843 |
| 0.75 | 0.953786 | 0.954524 |
| 1.0 | 1.000000 | 1.000046 |
| 1.5 | 1.061587 | 1.061610 |
| 3.0 | 1.148588 | 1.148593 |
| 7.0 | 1.216847 | 1.216851 |
| 10.0 | 1.234875 | 1.234875 |
| 20.0 | 1.257418 | 1.257424 |
| 100.0 | 1.276768 | 1.276774 |

Table 2. Comparison of the values of temperature gradient $-\theta'(0)$ for various values of Pr.

| $n$ | $\lambda$ | $-\theta'(0)$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | Ali [10] | | | Author's result | | |
| | | Pr = 0.72 | Pr = 1.0 | Pr = 3.0 | Pr = 0.72 | Pr = 1.0 | Pr = 3.0 |
| 3.0 | 0.0 | 0.4469 | 0.5633 | 1.1373 | 0.44789 | 0.56466 | 1.14227 |

Table 3. Variation of non-dimensional rate of heat transfer for various values of $Pr$, $M^2$, $\lambda$, $n$, $R_d$, $A^*$, $B^*$

| $M^2$ | Pr | $\lambda$ | $n$ | $R_d$ | $A^*$ | $B^*$ | $-\theta'(0)\left(1+\frac{4}{3R_d}\right)$ | $-\theta'(0)\sqrt{\frac{n+1}{2}}\left(1+\frac{4}{3R_d}\right)$ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 7.0 | 6.0 | 3.0 | 2.0 | 0.1 | 0.1 | 10.55199 | 14.92263 |
| 1.0 | | | | | | | 10.38386 | 14.68485 |
| 1.5 | | | | | | | 10.31304 | 14.58470 |
| 2.0 | | | | | | | 10.24774 | 14.49235 |
| 1.5 | 1.0 | 6.0 | 3.0 | 2.0 | 0.1 | 0.1 | 4.265219 | 6.031872 |
| | 1.5 | | | | | | 4.692994 | 6.636822 |
| | 2.3 | | | | | | 5.527927 | 7.817595 |
| | 7.0 | | | | | | 10.31304 | 14.58470 |
| 1.5 | 7.0 | 2.0 | 3.0 | 2.0 | 0.1 | 0.1 | 5.914202 | 8.363860 |
| | | 4.0 | | | | | 8.379818 | 11.85074 |
| | | 6.0 | | | | | 10.31304 | 14.58470 |
| | | 8.0 | | | | | 11.95542 | 16.90736 |
| 1.5 | 7.0 | 6.0 | 1.0 | 2.0 | 0.1 | 0.1 | 13.96460 | 13.96460 |
| | | | 2.0 | | | | 11.49728 | 14.08118 |
| | | | 3.0 | | | | 10.31304 | 14.58470 |
| | | | 4.0 | | | | 9.767345 | 15.44315 |
| 1.5 | 7.0 | 6.0 | 3.0 | 2.0 | 0.1 | 0.1 | 11.13249 | 15.74356 |
| | | | | 3.0 | | | 10.06845 | 14.23880 |
| | | | | 5.0 | | | 10.28157 | 14.54020 |
| | | | | $10^9$ | | | 8.117460 | 11.47971 |
| 1.5 | 7.0 | 6.0 | 3.0 | 2.0 | -0.2 | 0.1 | 10.39749 | 14.70413 |
| | | | | | -0.1 | | 10.36934 | 14.66432 |
| | | | | | 0.1 | | 10.31304 | 14.58470 |
| | | | | | 0.2 | | 10.28489 | 14.54489 |
| 1.5 | 7.0 | 6.0 | 3.0 | 2.0 | 0.1 | -0.2 | 10.35681 | 14.64660 |
| | | | | | | -0.1 | 10.34224 | 14.62600 |
| | | | | | | 0.1 | 10.31304 | 14.58470 |
| | | | | | | 0.2 | 10.29839 | 14.56398 |

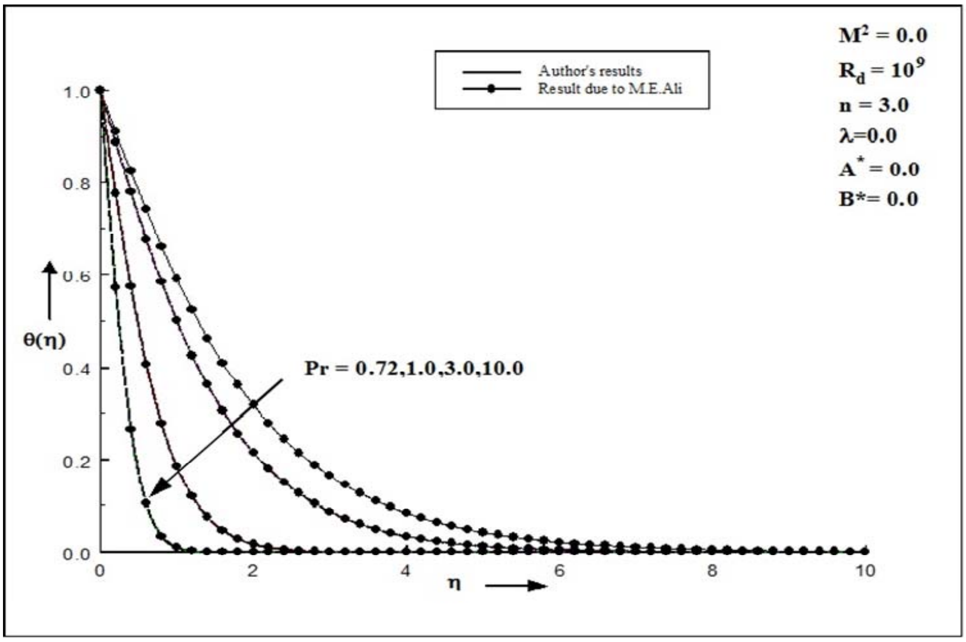


Fig.2. Temperature profiles for various values of Pr.

The influence of the magnetic field over the dimensionless velocity is demonstrated in Fig.3. Increasing values of the magnetic interaction parameter is found to retard the velocity at all points of the flow field. It is because that the application of the transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. The dimensionless velocity distribution for different values of the power law index parameter *n* is shown in Fig.4. For increasing values of the power law index *n* the velocity gets decelerated and further it is noted that the effect of *n* over the velocity is less significant.

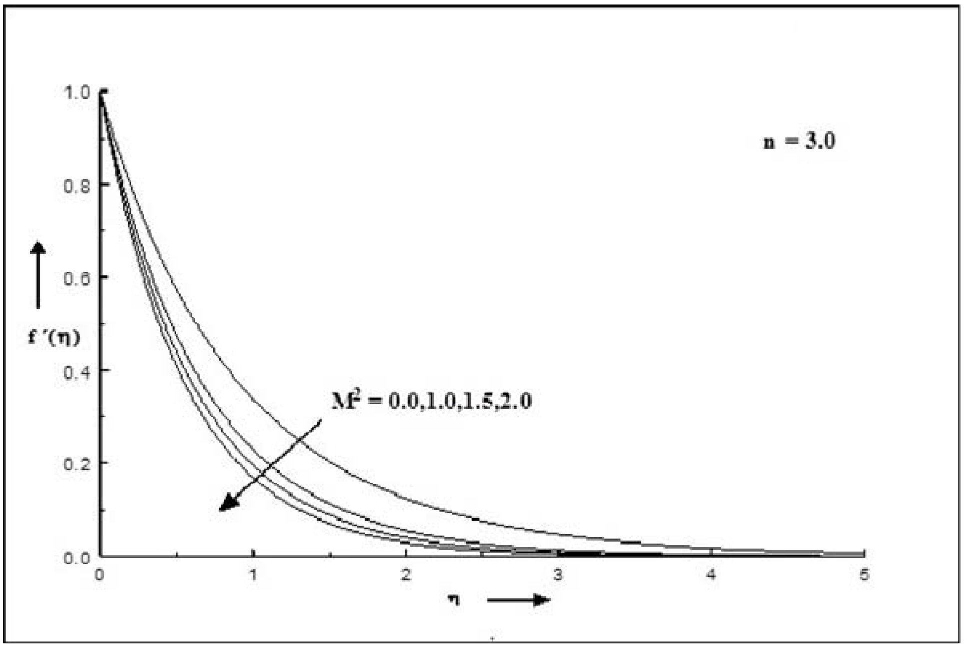


Fig.3. Dimensionless velocity profiles for different values of $M^2$.

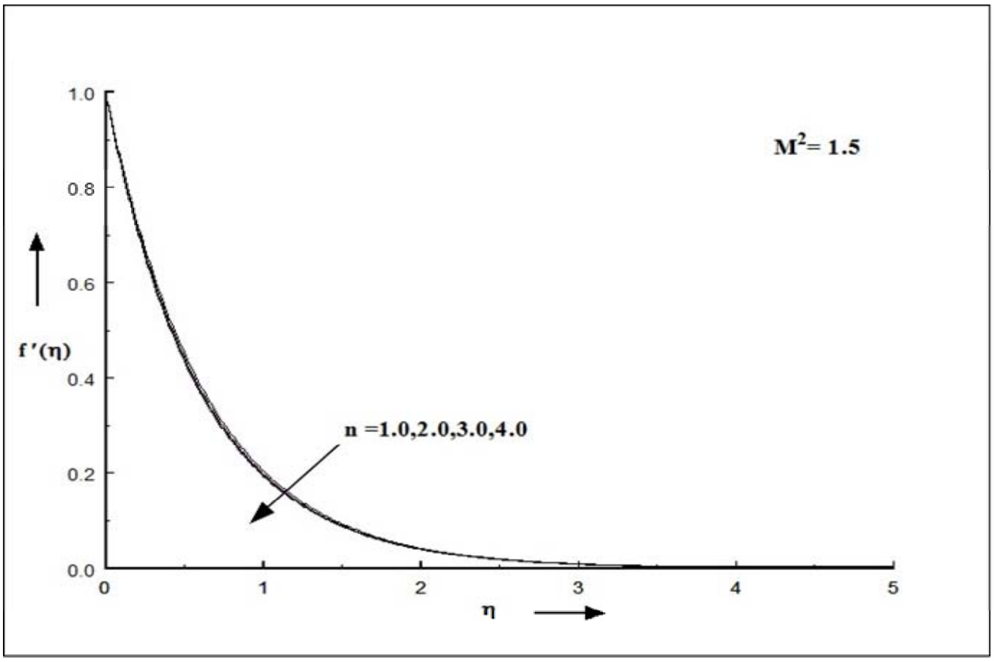


Fig.4. Dimensionless velocity profiles for different values of *n*.

The influence of the magnetic interaction parameter on the temperature distribution is disclosed through Fig.5. It reveals that the induced Lorentz force suppressed the flow motion. This causes the enhancement in the temperature and simultaneously the thermal boundary layer also gets thicker.

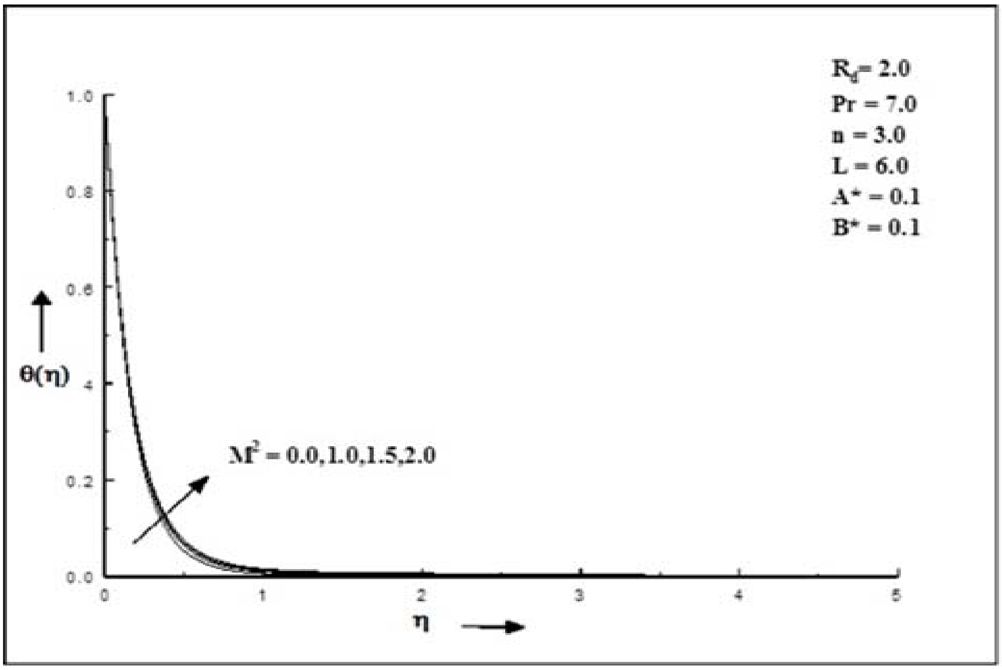


Fig.5. Temperature distribution for different values of $M^2$.

The temperature distribution for various values of the power law index $n$ is illustrated in Fig.6. It is noted that temperature distribution increases with the increase of the power-law index $n$. The effect of radiation $R_d$ on the temperature distribution in the boundary layer region is in Fig.7. When the value of the radiation parameter is amplified, the temperature declines, consequently the thermal boundary layer thickness becomes smaller. Since $R_d$ decreases for $k$ and $T_\infty$, the Rosselend mean absorption co-efficient $k^*$ also decreases, as $k^*$ decreases $\frac{\partial q_r}{\partial y}$ increases which exhibits the fact that the rate of heat transfer increases.

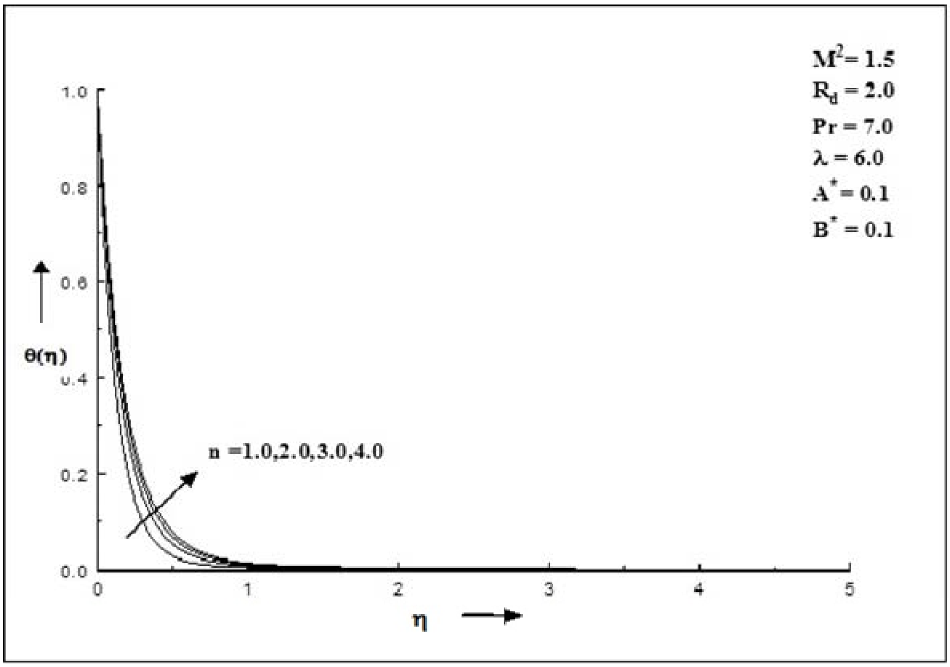


Fig.6. Effect of $n$ over the temperature distribution.

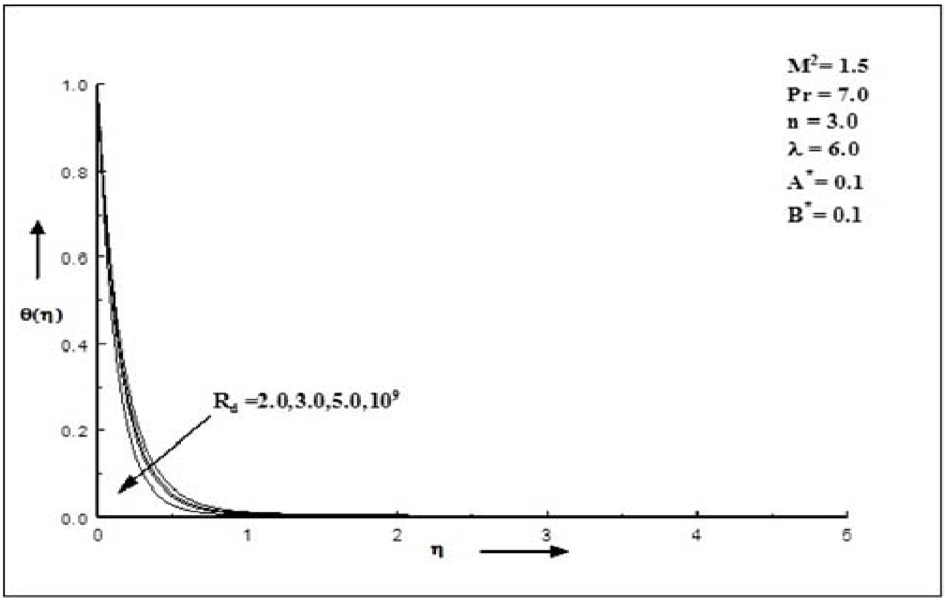


Fig.7. Temperature distribution for different values of $R_d$.

Figure 8 depicts the effect of the Prandtl number Pr over the temperature distribution. It can be noted that the dimensionless temperature distribution decreases continuously within the boundary layer for increasing values of Pr. Thus, the higher Prandtl number reduces thermal boundary layer thickness.

Figure 9 shows the behavior of the temperature distribution for different values of the temperature parameter $\lambda$. It is noted that the increasing effect of $\lambda$ is to suppress the thickness of the thermal boundary layer. The influence of the space-dependent heat source/sink parameter $A^*$ over temperature distribution is shown in Fig.10. Physically, the presence of $A^* > 0$ has a tendency to increase the fluid temperature. Hence the thermal boundary layer thickness increases. In case of the space- dependent heat sink $A^* < 0$, the thermal boundary absorbs energy.

The effect of temperature dependent heat source/sink parameter $B^*$ on the temperature distribution is shown in Fig.11. The energy is created in the boundary layer for increasing values of the temperature dependent heat source parameter $B^*$ $(B^* > 0)$. Hence the thermal boundary layer thickness rises. While in the presence of heat sink $B^* < 0$, energy is being absorb for increasing values of $B^*$. Hence, the thermal boundary layer thickness as well as the temperature decreases. However, this effect is not prominent.

Figure 12 displays the variation of the skin friction coefficient against the magnetic interaction parameter $M^2$ for different values of the power-law index $n$. It is seen that the skin friction coefficient decreases with an increase of the power-law index $n$ and it increases for increasing values of the magnetic interaction parameter $M^2$.

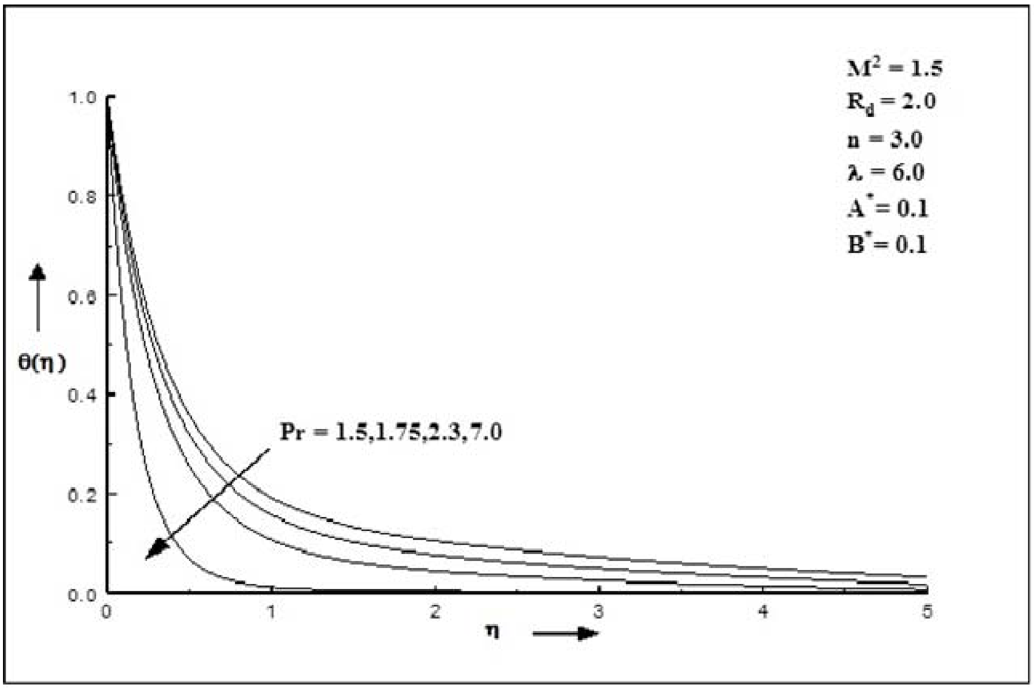


Fig.8. Temperature distribution for different values of Pr.

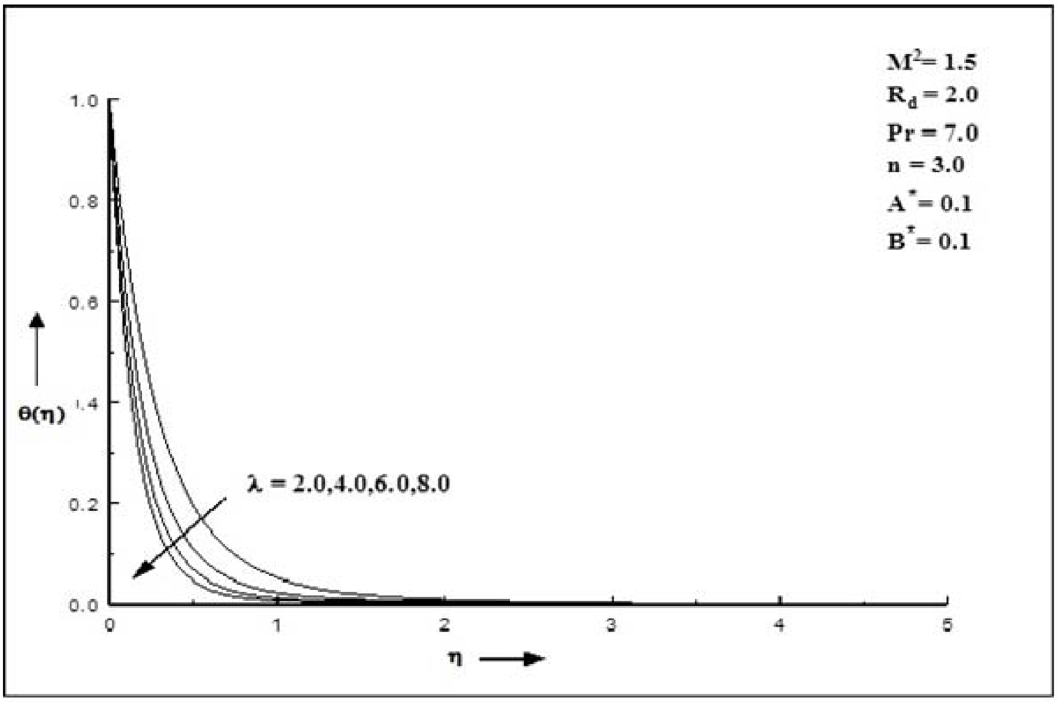


Fig.9. Temperature distribution for different values of λ.

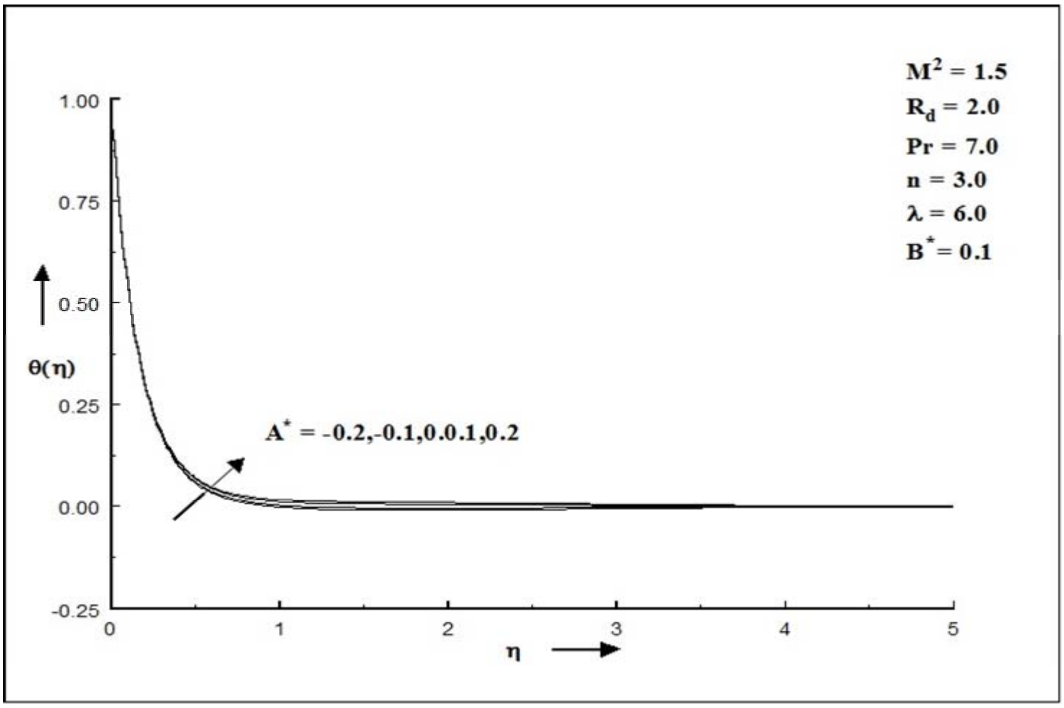


Fig.10. Effect of $A^*$ over temperature distribution.

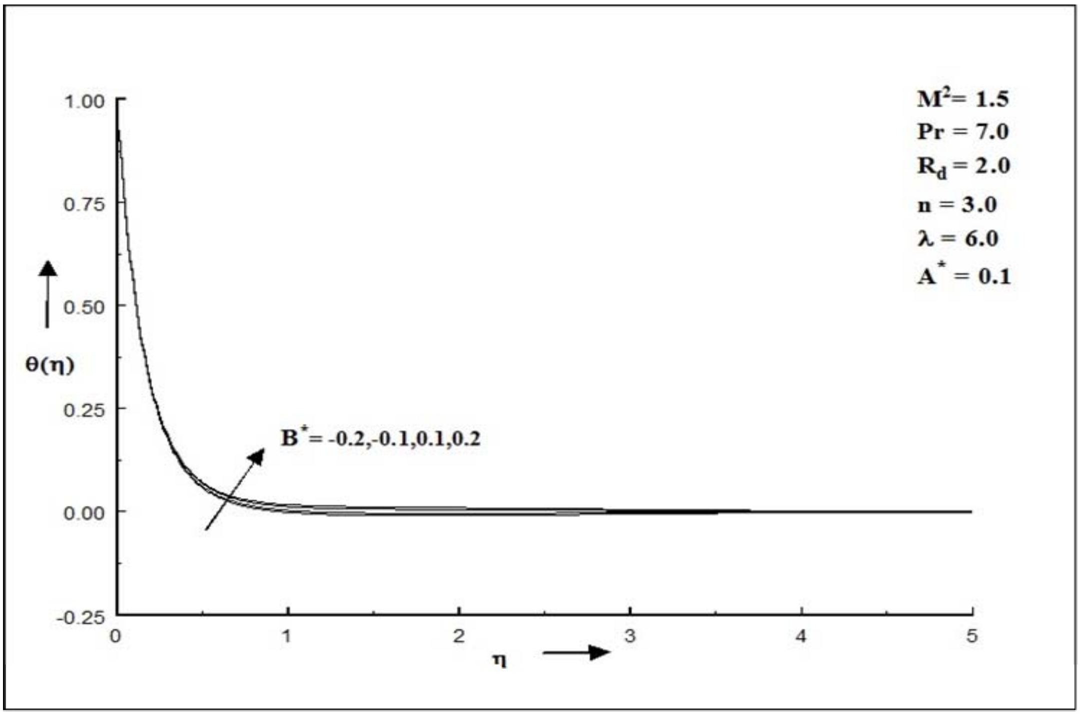


Fig.11. Temperature distribution for different values of $B^*$.

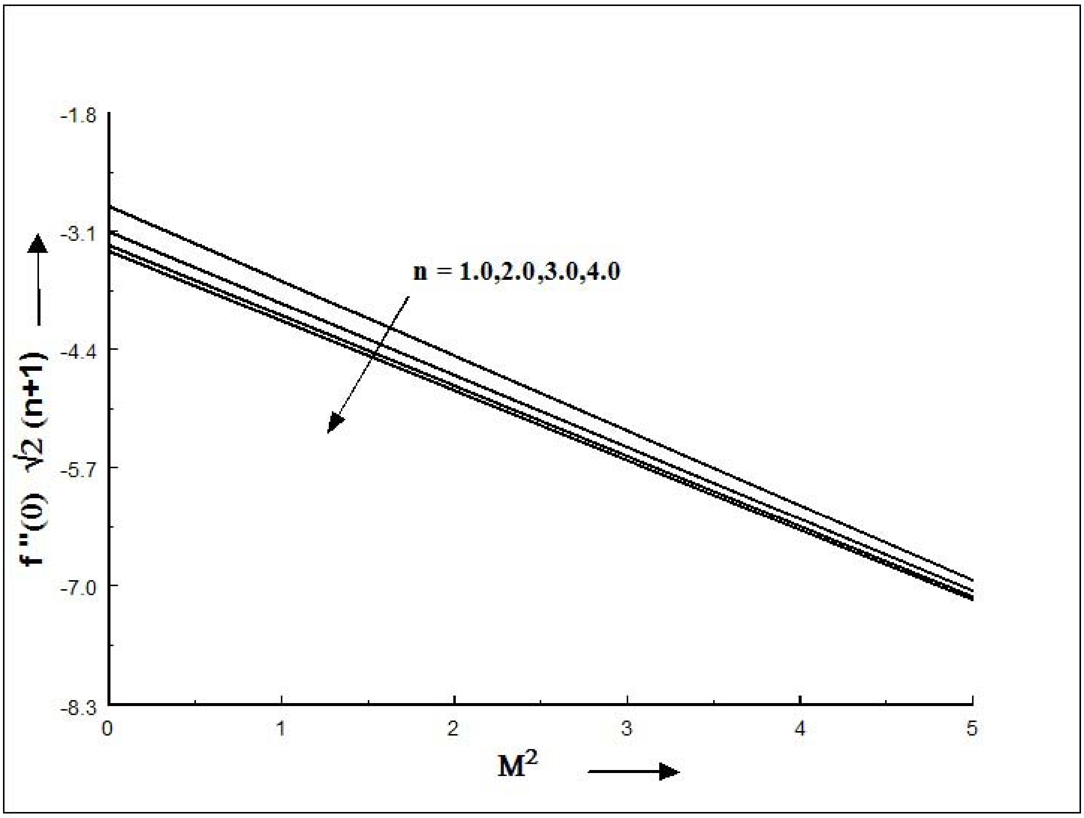


Fig.12. Skin friction coefficient for different *n.*

## 6. Conclusion

The present study gives the numerical solution for the MHD boundary layer flow with heat transfer over a nonlinear stretching sheet subjected to variable wall temperature in the presence of radiation and non-uniform heat source/sink. The governing equations were transformed to a set of ordinary differential equations and are solved numerically using the shooting technique such as the Nachtsheim-Swigert iteration method along with the fourth order Runge Kutta integration method. The effects of various non-dimensional parameters on dimensionless velocity and temperature distribution are discussed and presented through graphs. Also, the effect of physical parameters on the friction factor and Nusselt number are analysed. Comparisons with previous published results are presented in the absence of the magnetic field and radiation. The main findings of this investigation are as follows:

- The effect of the transverse magnetic field $M^2$ on a viscous fluid flow is to suppress the velocity of the fluid which in turn causes the enhancement of the temperature field.
- Increasing values of the power law index $n$ decreases the dimensionless velocity and local skin friction coefficient whereas it increases the dimensionless temperature as well as the non-dimensional rate of heat transfer.
- The effect of the radiation parameter $R_d$ is to reduce the temperature distribution and non-dimensional rate of heat transfer for its increasing values.
- The effect of the Prandtl number Pr and temperature parameter $\lambda$ decrease the thermal boundary layer thickness and enhance the non-dimensional rate of heat transfer at the wall.
- The space dependent heat source parameter $A^*$ ($A^* > 0$) and temperature dependent heat source parameter $B^*$ ($B^*>0$) enhance the thermal boundary layer thickness as well as the temperature while the space dependent heat sink ($A^*<0$) and temperature dependent heat sink ($B^*<0$) curtails both the temperature and thermal boundary layer thickness. It is also noted that the dimensionless rate of heat transfer decreases with the heat source parameter and enhances with increasing values of the heat sink parameter.

## Nomenclature

$A^*$ – space dependent heat source/sink parameter
$B^*$ – temperature dependent heat source/sink parameter
$B_0$ – magnetic field strength
$b$ – stretching rate
$C_f$ – skin friction coefficient
$C_p$ – specific heat at constant pressure
$f$ – dimensionless stream function
$k$ – thermal conductivity of the fluid
$k^*$ – Rosseland mean absorption coefficient
$M^2$ – magnetic interaction parameter
$\mathrm{Nu}_x$ – local Nusselt number
$n$ – power-law index
Pr – Prandtl number
$R_d$ – radiation parameter
$\mathrm{Re}_x$ – local Reynolds number
$T$ – temperature of the fluid
$T_w$ – temperature at the wall
$T_\infty$ – temperature of the free stream fluid
$u$ – velocity in the $x$ direction
$u_w$ – velocity of the stretching surface
$q_r$ – radiative heat flux

$q_w$ – heat flux at the surface
$q'''$ – non-uniform heat source/sink
$v$ – velocity in the $y$ direction
$x$ – horizontal coordinate
$y$ – vertical coordinate
$\alpha$ – thermal diffusivity of the fluid
$\eta$ – similarity variable
$\theta$ – dimensionless temperature
$\lambda$ – temperature parameter
$\mu$ – viscosity of the fluid
$\nu$ – kinematic viscosity of the fluid
$\rho$ – density of the fluid
$\sigma$ – electrical conductivity of the fluid
$\sigma^*$ – Stefan Boltzmann constant
$\tau_w$ – shear stress at the wall
$\chi$ – stream function

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