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AN OSCILLATORY RADIATING HYDROMAGNETIC INTERNAL HEAT GENERATING FLUID FLOW THROUGH A VERTCAL POROUS CHANNEL WITH SLIP AND TEMPERATURE JUMP

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The present study concerns the natural convective heat generating/absorbing, radiative magnetohydrodynamic, oscillatory fluid flow through a vertical porous channel with slip and temperature jump. The effect of Joule dissipation is taken into consideration while it is assumed that the flow is fully developed. The differential transforms method(DTM) is employed to solve the system of non-linear ordinary differential equations that is obtained from the non-linear partial differential equations governing the flow. Semi analytical solutions of the steady and unsteady part of the flow in the slip flow regime through a vertical porous channel are obtained. The effects of various flow parameters on the velocity and temperature profiles as well as Nusselt and skin friction are presented graphically and discussed. An excellent agreement between the results of this article and those available in the literature validated the presented approach.

Key words: temperature jump, velocity slip, hydromagnetic, oscillatory and porous channel.

1. Introduction

Investigations of a natural convective magnetohydrodynamic (MHD) flow through a vertical channel are of importance due to its application in various manufacturing processes and devices. These include transpiration, cooling of reentry vehicles, petroleum industries, power generators, accelerators, electrostatic precipitation, MHD pumps and rocket boosters. An appreciable number of studies has also been reported in the literature. Ranna *et al.* [1] examined the MHD unsteady natural convection water's memory flow with constant suction and heat sink. Earlier on, Das *et al.* [2] studied the transient free convection flow past an infinite vertical plate with periodic temperature variation.

Many research works on the analysis of heat transfer and fluid flow at micro scale level have been carried out due to the wide application in micro - electro - mechanical systems(MEMS) involving temperature jump and velocity slip [3]. A lot of attention has been paid to this flow due to its application in heat exchangers, physiological flows, drying process and electronic cooling. An interesting example is the work of Mehmood and Ali [4] who examined the influence of velocity slip on an unsteady MHD oscillatory flow of a viscous fluid in a planar channel. It was shown that no slip velocity condition may not be suitable for hydrophilic flows over hydrophobic boundaries at both the micro-scale and nano-scale levels. It is also well known that when Knudsen number (Kn) is zero the no slip condition holds while if Kn is less than *0.001* the continum flow assumption holds. However, when Kn lies between the range [*0.001*, *0.1*] the flow is termed slip and in this regime the classical energy equation as well as the Navier Stokes equation hold (see [3], [5] and [6]). More works on slip flows with various flow configurations can be found in [8, 9, 10 and

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11]. On the other hand, interesting works on both velocity slip and temperature jump are carried out in [24-27].

For a high fluid temperature, radiation occurs and plays a significant role. In particular for plates of the channel having high temperatures the influence of radiation cannot be ignored. As a matter of fact, in such a case, the influence of radiation and natural convection must be taken into account. Several authors have modeled thermal radiating MHD flows with applications in astrophysical fluid dynamics. For example, Abo-elahab [7] studied the effects of temperature-dependent fluid properties on a free convective flow along a semi-infinite vertical plate in the presence of radiation using the CogleyVincentine-Giles equilibrium model. Gbadeyan and Dada [16] examined the influence of radiation and heat transfer on an unsteady MHD non Newtonian flow with slip in a porous medium. They observed that the temperature increases with a decrease in either the Prandtl number or radiation parameter and also noticed that the velocity profile decreases as the radiation parameter or Grashof number decrease. Hayat *et al*. [17] investigated the effect of radiation and the magnetic field on the mixed convection stagnation point flow over a vertical stretching sheet in a porous medium.

The importance of a natural convective flow in the case of internal heat generation /absorption fluid through a porous vertical channel has been well discussed in [5], [12] and [22]. In particular, it is pointed out in [12] that the study of such flow has recently been well recognized as a result of the fact that an appreciable increase in temperature difference may cause the volumentric heat generation/absorption to have a great effect on the heat transfer and hence on the flow. Variouse researchers have carried out interesting studies involving internal heat generating fluid. For instance, Ostrach[13] examined the combined natural and forced convection flow and heat transfer of fluids with and without a heat source in a channel with linearly varying wall temperature. The author also discussed a laminar natural convective flow and heat transfer of fluids with and without a heat source in a channel with constant wall temperature in [18]. An experimental study of temperature distribution in a laminar tube flow of a fluid with internal heat generation was carried out in Inman [14]. It is further remarked in [22] that internal heat generation/absorption plays a significant role in various physical phenomena e.g., application in the field of nuclear energy [19], fire and combustion modeling [20] and as convection in earth's mantles [21].

Adesanya [5] investigated the unsteady natural convective flow of an internally heat generating/absorbing fluid through a porous vertical channel under the effect of slip and temperature jump boundary conditions. However, the analysis of the oscillatory flow problem did not take into account the influence of both the magnetic field strength and radiation. It is observed that an increase in the slip parameter leads to a decrease in the shear stress at the suction wall while it enhances the flow velocity. Earlier on, the effect of slip and jump boundary conditions on an MHD oscillatory flow of a radiating fluid through a vertical porous channel, neglecting heat source/sink, was studied in [15] .

The present article is mainly motivated by the work [5], [15] and the considerable amount of studies mode on the natural convection with internal heat generation/absorption. Hence, this article aims at investigating the combined influence of radiation, MHD and heat source/sink on an oscillatory fluid flow through a vertical porous channel with slip velocity and temperature jump boundary conditions. In other words, the current work extends the studies of both [5] and [15].

To achieve this aim, the governing nonlinear partial differential equations were solved by first splitting the velocity and temperature into the steady and unsteady parts after which the semi analytical technique known as the differential transform method (DTM) is then used to solve the resulting set of ordinary differential equations. The rest of the paper is organized as follows. The mathematical formulation is presented in the next section. Section 3 deals with the concept of the differential transform method and its application in solving the present nonlinear problem. The real part of the results are presented and discussed in section 4. In section 5, the concluding remarks were given.

2. Mathematical analysis

Consider a fully developed laminar flow of a free convective incompressible viscous electrically conducting and heat generating/absorbing fluid in a vertical porous channel. The left channel wall is heated with slip and temperature jump. The channel walls are taken vertically and parallel to the *x*-axis at $y = \pm h$ (see Fig.2). On one side of the plates $y = +h$, the fluid is injected into the channel with constant velocity v_0 and it is sucked off from the other plate $y = -h$ at the same velocity. It is assumed that there exist interfacial interactions between the fluid molecules and atoms of the surface of the wall of the channel. Hence, the molecules of the fluid may be absorbed on to the surface which is then reflected after some time lag. Such a time lag results in a microscopic velocity slip and temperature jump [5]. A uniform magnetic field of strength B0 is applied perpendicular to the channel. The corresponding magnetic Reynold number is assumed to be very small hence the induced magnetic field is neglected [28]. The radiation effect is also taken in to account. The radiative heat flux in the energy equation is assumed to follow Rosland approximation. The buoyancy effect sets in as a result of the temperature gradient between the plates and the fluid [23]. Under the usual Boussinesq's approximation the basic equations governing the flow of the viscous incompressible fluids and heat transfer in a vertical periodic porous channel are [5, 15, and 23]

$$\frac{\partial u'}{\partial t'} - v_0 \frac{\partial u'}{\partial y'} = v \frac{\partial^2 y}{\partial y'^2} + g\beta\left(T - T_0\right) - \frac{\sigma B_0^2 u'}{\rho}, \tag{2.1}$$

$$\frac{\partial T}{\partial t'} - v_0 \frac{\partial T}{\partial y'} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial y'^2} + \frac{Q}{\rho C_p}\left(T_0 - T\right) - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y'} + \frac{\sigma B_0^2}{\rho C_p} u'^2 . \tag{2.2}$$

Together with appropriate initial conditions

$$u'\left(t', y'\right) = 0, \qquad T\left(t', y'\right) = 0 \qquad \text{at} \quad t' = 0, \tag{2.3}$$

according to [5], for rarefied flow with temperature jump, the appropriate boundary conditions can be written as

$$u'\left(t', y'\right) = \frac{2 - \varepsilon}{\varepsilon} \gamma \frac{\partial u'}{\partial y'}, \qquad y' = -h \quad \text{at} \quad t' > 0, \tag{2.4}$$

$$T\left(t', y'\right) = T_1 + T_2 \cos\left(\omega t\right) + \frac{2 - \sigma_T}{\sigma_T} \frac{2\psi}{\psi + 1} \frac{\gamma}{P_r} \frac{dT}{dy'}, \qquad y' = -h \quad \text{at} \quad t' > 0 . \tag{2.5}$$

The non-moving wall and isothermal condition gives

$$u'\left(t', y'\right) = 0, \qquad T\left(t', y'\right) = T_1 + T_2 \cos\omega t, \quad y' = +h \quad \text{at} \quad t' > 0 . \tag{2.6}$$

The radiation heat flux q_r in the energy equation is $q_r = \frac{-4\gamma^*}{3\alpha^*} \frac{\partial T^4}{\partial y'}$ where γ^* and α^* are Stephan boltzman constant and mean absorption constant. Assuming that the temperature difference within the fluid is sufficiently small, T^4 may be expressed as a linear function of temperature T. This can be achieved by expanding T^4 in a Taylor series about T_1 and omitting higher order terms we arrived at $T^4 = \left(4T_1\right)^3 T - 3T_1^4$. The other physical quantities used in this work are defined in the nomenclature. To solve Eqs (2.1) - (2.6) we follow Ajibade [23] and Adesanya [5] thereby splitting the velocity and temperature into a steady and periodic part, respectively, as follows

$$u'\left(t',y'\right)=\frac{g\beta h^2}{v}\left(\left(T_1-T_0\right)A\left(y\right)+T_2B\left(y\right)e^{i\omega t}\right), \tag{2.7}$$

$$T\left(t',y'\right)=T_0+\left(\left(T_1-T_0\right)F\left(y\right)+T_2G(y)e^{i\omega t}\right) \tag{2.8}$$

Where $A\left(y\right),F(y)$ stands for the steady parts and $B\left(y\right),G(y)$ for the periodic parts of the velocity and temperature, respectively. The dimensionless quantities used are

$$\begin{gathered} y=\frac{y'}{h},\quad \mathrm{S_t}=\frac{h^2\omega}{v},\quad \mathrm{P_r}=\frac{\mu C_p}{K},\quad S=\frac{hv_0}{v},\quad \delta=\frac{Q_0h^2}{K},\quad K_n=\frac{\lambda}{h}\left(\frac{2-\sigma_t}{\sigma_t}\right)\frac{2\phi}{\phi+1}, \\ \gamma=\frac{\left(2-\xi\right)\lambda}{\xi h},\quad H^2=\frac{\sigma B_0^2h^2}{\mu},\quad N=\frac{4\gamma^*T_1^3}{\alpha^*K},\quad \mathrm{E_c}=\frac{g^2\beta^2h^4}{Kv}\rho\left(T_1-T_0\right) \end{gathered} \tag{2.9}$$

Substituting Eqs (2.7)-(2.9) into Eqs (2.1)-(2.6) and equating orders of $e^{i\omega t}$ resulted in the following dimensionless non-linear ordinary differential equations.

$$A''\left(y\right)+SA'-H^2A\left(y\right)+F\left(y\right)=0, \tag{2.10}$$

$$\left(1+\frac{4}{3}N\right)F''\left(y\right)+SP_rF'\left(y\right)-\delta F\left(y\right)+H^2E_c\left(A\left(y\right)\right)^2, \tag{2.11}$$

$$B''\left(y\right)+SB'-\left(iS_t+H^2\right)B\left(y\right)+G\left(y\right)=0, \tag{2.12}$$

$$\left(1+\frac{4}{3}N\right)G''\left(y\right)+SP_rG'\left(y\right)-\left(iS_tP_r+\delta\right)G\left(y\right)+2H^2E_cA\left(y\right)B\left(y\right)=0, \tag{2.13}$$

subject to the following boundary conditions

$$\begin{gathered} A\left(-1\right)=\gamma A'\left(-1\right),\qquad A\left(1\right)=0, \\ F\left(-1\right)=1+\frac{K_n}{P_r}F\left(-1\right)',\qquad F\left(1\right)=1, \\ B\left(-1\right)=\gamma B'\left(-1\right),\qquad B\left(1\right)=0, \\ G\left(-1\right)=1+\frac{K_n}{P_r}G\left(-1\right)',\qquad F\left(1\right)=1. \end{gathered} \tag{2.14}$$

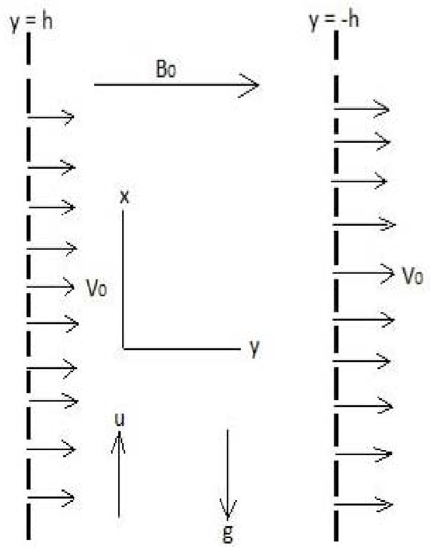

Fig.1. Physical geometry of the flow.

3. Analysis of Differential Transform Method (DTM)

The DTM is an iterative procedure to obtain analytic Taylor series solutions of differential equations. The basic definitions and the application procedure of this method are introduced as follows. Consider a function $F(y)$ which is analytic in a domain *D* and let $y = y_0$ represent any point in *D*. The function $F(y)$ is then represented by a power series whose center is located at y_0. The differential transform of the function $F(y)$ is given by [29]

$$F(k) = \frac{1}{k!}\left(\frac{d^k F(y)}{dy^k}\right)_{y=y_0} \tag{3.1}$$

where $F(x)$ is the original function and $F(k)$ is the transformed function. The inverse transformation is defined as

$$F(y) = \sum_{k=0}^{\infty} (y - y_0)^k F(k) \ . \tag{3.2}$$

For practical purposes, Eq.(3.2) can be written in a finite series as

$$F(y) = \sum_{k=0}^{n} (y - y_0)^k F(k) \tag{3.3}$$

where *n* is the truncating point depending on the convergence of the solution. Equation (3.3) implies that $F(y) = \sum_{k=n+1}^{\infty} (y - y_0)^k F(k) i$.

Table 1. Differential Transform Theorems [29].

Original Function	Transform Function
$F(y)=G(y)\pm H(y)$	$F(k)=G(k)\pm H(k)$
$F(y)=\lambda G(y)$	$F(k)=\lambda G(k)$
$F(y)=G(y)H(y)$	$F(k)=\sum_{i=0}^{k}G(k-i)H(i)$
$F(y)=\dfrac{d^nG(y)}{dy^n}$	$F(k)=\dfrac{(n+1)!}{n!}G(k+n)$
$F(y)=y^n$	$F(k)=\delta(k-n)=\begin{cases}0\text{ if } k\neq n\\ 1\text{ if } k=n\end{cases}$

The differential transforms of Eqs (2.10) - (2.13) are taken by using the theorem introduced in Tab.1 so that the following recurrence relations are obtained.

$$A(k+2)=\frac{H^2a(k)-F(k)-(k+1)A(k+1)S}{(k+1)(k+2)}, \tag{3.4}$$

$$F(k+2)=\frac{\delta F(k)-SP_r(k+1)F(k+1)-H^2E_c\sum_i^k A(i)A(k-i)}{\left(1+\frac{4}{3}N\right)(k+1)(k+2)}, \tag{3.5}$$

$$B(k+2)=\frac{\left(H^2+iS_t\right)B(k)-(k+1)B(k+1)S-G(k)}{(k+1)(k+2)}, \tag{3.6}$$

$$G(k+2)=\frac{(P_riS_t+\delta)G(k)-SP_r(k+1)G(k+1)-2H^2E_c\sum_i^k A(i)B(k-i)}{\left(1+\frac{4}{3}N\right)(k+1)(k+2)}. \tag{3.7}$$

The functions $A(k),F(k),B(k)$ and $G(k)$ are differential transforms of $A(y),F(y),B(y)$ and $G(y)$ respectively.

Assuming that $A(0),A(1),F(0),F(1),B(0),B(1),G(0)$ and $G(1)$ are constant $a_0,a_1,f_0,$ f_1,b_0,b_1,g_0 and g_1 are also constant, using Eq.(3.3) and recurrence relations in Eqs (3.4) - (3.7) the following series solutions are obtained

$$A(y)=a_0+a_1y+\left(\frac{H^2a_0-Sa_1-f_0}{2}\right)y^2+\left(\frac{H^2a_1-f_1-S\left(H^2a_0-Sa_1-f_0\right)}{2}\right)y^3+\ldots \tag{3.8}$$

$$F(y) = f_0 + f_1 y + \left(-\frac{H^2 E_c a_0^2}{2+\frac{8N}{3}} - \frac{SP_r f_1}{2+\frac{8N}{3}} + \frac{\delta f_0}{2+\frac{8N}{3}} \right) y^2 +$$
$$+ \left(\frac{\delta f_1}{6+8N} + \frac{2SP_r H^2 E_c a_0^2}{(6+8N)\left(2+\frac{8N}{3}\right)} + \frac{2S^2 P_r^2 f_1}{(6+8N)\left(2+\frac{8N}{3}\right)} - \frac{2SP_r \delta f_0}{(6+8N)\left(2+\frac{8N}{3}\right)} - \frac{2H^2 E_c a_0 a_1}{6+8N} \right) y^3 + \ldots, \tag{3.9}$$

$$B(y) = b_0 + b_1 y + \left(\frac{H^2 b_0 + iS_t b_0 - Sb_1 - g_0}{2} \right) y^2 +$$
$$+ \left(\frac{H^2 b_1 + iS_t b_1 - SH^2 b_0 - iS_t b_0 + S^2 b_1 + Sg_0 - g_1}{6} \right) y^3 + \ldots, \tag{3.10}$$

$$G(y) = g_0 + g_1 y + \left(\frac{ig_0 S_t P_r}{2+\frac{8N}{3}} + \frac{g_0 \delta}{2+\frac{8N}{3}} - \frac{SP_r g_1}{2+\frac{8N}{3}} - \frac{2H^2 E_c a_0 b_0}{2+\frac{8N}{3}} \right) y^2 +$$
$$+ \left(\frac{ig_1 S_t P_r}{6+8N} + \frac{g_1 \delta}{(6+8N)} - \frac{2iSP_r^2 g_0 S_t}{(6+8N)\left(2+\frac{8N}{3}\right)} - \frac{2SP_r \delta g_0}{(6+8N)\left(2+\frac{8N}{3}\right)} + \right. \tag{3.11}$$
$$\left. + \frac{2S^2 P_r^2 g_1}{(6+8N)\left(2+\frac{8N}{3}\right)} + \frac{4SP_r H^2 E_c a_0 b_0}{(6+8N)\left(2+\frac{8N}{3}\right)} - \frac{2E_c H^2 a_0 b_1}{(6+8N)} - \frac{2E_c H^2 a_1 b_0}{(6+8N)} \right) y^3 + \ldots.$$

Using the boundary conditions in Eq.(2.14) we get the values of constants $a_0, a_1, f_0, f_1, b_0, b_1, g_0$ and g_1 . The rate of heat transfer (Nu) and shear stress (τ) at the walls can be determined from $\frac{dG(y)}{dy}$ and $\frac{dB(y)}{dy}$, respectively .

4. Results and discussion

This section deals with effects of parameter variations for different values of parameters like the Navier slip, Knudsen number, Prandtl number, Hartmann's number, heat generation/absorption, suction/injection and radiation on the fluid flow. Tables 2 and 3 show the convergence of constants a_0, a_1, f_0 and f_1 and solution $B(y)$ and $G(y)$ respectively. In Tabs 4 and 5, the present results are compared with previous results for $K_n = \gamma = N = H = E_c = 0$ to show the accuracy of the solution. Note that $\delta < 0$ is the internal heat generation while $\delta > 0$ is the heat absorption.

Table 2. Convergence of constants a_0, a_1, f_0 and f_1 for $E_c = 0.1$, $\gamma = 0.1$, $K_n = 0.005$, $N = 0.5$, $H=1$, $\delta = 1$, $P_r = 1$, $S = 1$ and $S_t = 1$.

n	a_0	a_1	f_0	f_1
1	0	0	1	0
2	0.26440735	-0.022991943	0.77023001	0.011402972
3	0.25040664	-0.080869086	0.77908481	0.040737917
4	0.28153848	-0.080049599	0.76604413	0.040063760
5	0.27826993	-0.091260472	0.76675292	0.042983417
6	0.28127001	-0.091431183	0.76627708	0.042948596
7	0.28104318	-0.092323922	0.76629746	0.043035726
8	0.28120811	-0.092347653	0.76630020	0.043034947
9	0.28119978	-0.092385692	0.76629895	0.043029898
10	0.28120576	-0.092387074	0.76630162	0.043029945
11	0.28120551	-0.092388101	0.76630136	0.043028851
12	0.28120571	-0.092388162	0.76630168	0.043028858
14	0.28120570	-0.092388182	0.76630167	0.043028758
15	0.28120570	-0.092388182	0.76630167	0.043028752
16	0.28120570	-0.092388182	0.76630167	0.043028752

Table 3. Convergence of solutions $B(y)$ and $G(y)$ for : $E_c = 0.1$, $\gamma = 0.1$, $K_n = 0.005$, $N = 0.5$, $H = 1$, $\delta = 1$, $P_r = 1$, $S = 1$ and $S_t = 1$.

n	B_n	$\sum_m^n B_m$	G_n	$\sum_m^n G_m$
0	0.23545753	0.23545753	0.73079109	0.73079109
1	-6.7832506×10^{-3}	0.22867428	5.2069159×10^{-3}	0.73599800
2	-1.5151091×10^{-3}	0.22715917	2.5467977×10^{-3}	0.73854480
3	2.0876157×10^{-5}	0.22718005	-4.7913881×10^{-5}	0.73849689
4	-4.8925267×10^{-6}	0.22717515	1.2881914×10^{-6}	0.73849818
5	1.5529150×10^{-7}	0.22717531	-2.5287538×10^{-8}	0.73849815
6	-4.5032961×10^{-9}	0.22717530	3.5323135×10^{-11}	0.73849815
7	1.0542221×10^{-10}	0.22717530	1.6036173×10^{-12}	0.73849815

Table 4. Comparison of present DTM solution of $B(y)$ with previous work: $E_c = 0$, $\gamma = 0$, $K_n = 0$, $N = 0$, $H = 0$, $\delta = 1$, $P_r = 1$, $S = 1$ and $S_t = 1$.

y	Adesanya [5]	Jha and Ajibade [22]	present result
-1	-4.44089×10^{-16}	-2.22050×10^{-16}	3.450894×10^{-10}
-1.75	0.141466	0.141466	0.1414657
-0.5	0.213966	0.213966	0.2139661
-0.25	0.243139	0.243139	0.2431395
0	0.242830	0.242830	0.2428303
0.25	0.218953	0.218953	0.2189527
0.5	0.172309	0.172309	0.1723091
0.75	0.100621	0.100621	0.1006207
1	-2.22045×10^{-16}	-6.66134×10^{-16}	1.7294792×10^{-10}

Table 4. Comparison present DTM solution of $G(y)$ with previous works for: $E_c = 0$, $\gamma = 0$, $K_n = 0$, $N = 0$, $H = 0$, $\delta = 1$, $P_r = 1$, $S = 1$ and $S_t = 1$.

y	Adesanya [5]	Jha and Ajibade [22]	present result
-1	*1*	*1*	*0.9999999*
-1.75	*0.77398*	*0.773980*	*0.7739799*
-0.5	*0.648087*	*0.648087*	*0.6480865*
-0.25	*0.596908*	*0.596908*	*0.5969082*
0	*0.602555*	*0.602555*	*0.6025550*
0.25	*0.652739*	*0.652739*	*0.6527391*
0.5	*0.739168*	*0.739168*	*0.7391683*
0.75	*0.856242*	*0.856242*	*0.8562422*
1	*1*	*1*	*0.9999999*

The figures above show the response of fluid temperature and velocity to the variation of parameters. In Fig.2a, the effect of the Hartmann number on the temperature profile is depicted. From the figure, an increase in the magnetic field strength is seen to slightly increase the temperature profile. This is due to ohmic heating of the fluid. Figure 2b shows the temperature profile for different values of the temperature jump parameter. It is shown that the temperature profile is decreasing as the temperature jump parameter is increasing. The reason for this is that as the Kn is increasing there is an increase in the molecular distance of the fluid from the channel which leads to a decrease in the heat flux in the channel. Figure 3a depicts the effect of the radiation parameter on the temperature profile of the fluid. We realized that as the parameter is increasing the temperature profile is also increasing. This is due to breaking of the bond that holds the fluid particle by the heat produced from thermal radiation. In Fig.3b, the fluid is symmetrical at the channel half width due to the absence of injection/suction parameter. As the parameter is increasing, the temperature profile increases along the injection wall but decreases towards the suction wall.

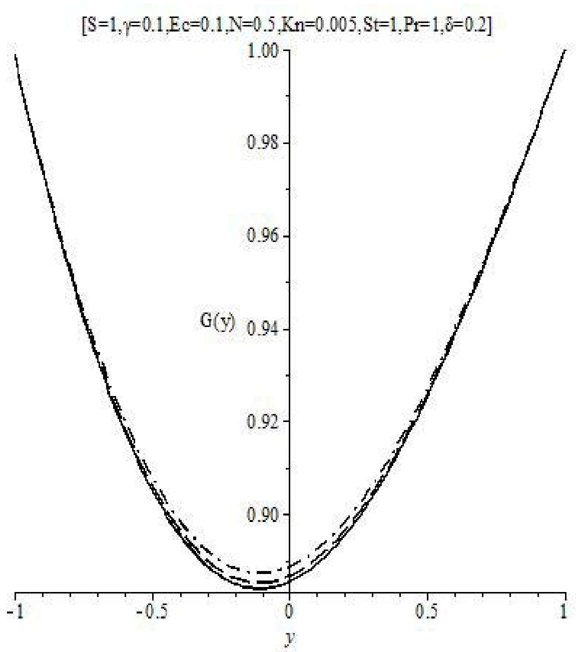

Fig.2a. Temperature profile with a change in the Hartman number.

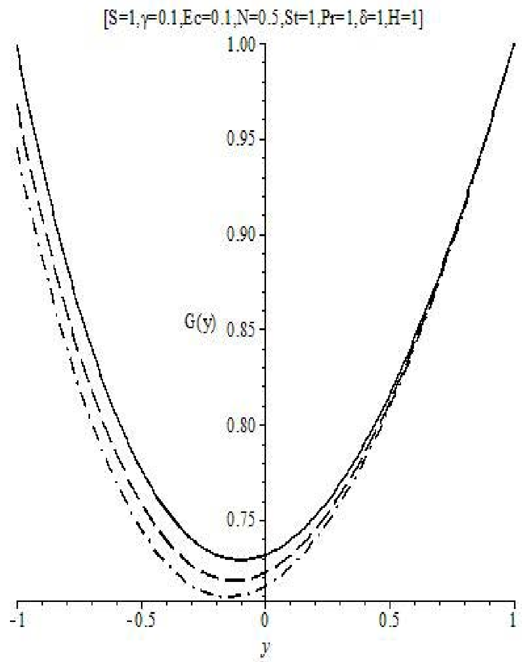

Fig.2b. Temperature profile with a change in the temperature jump.

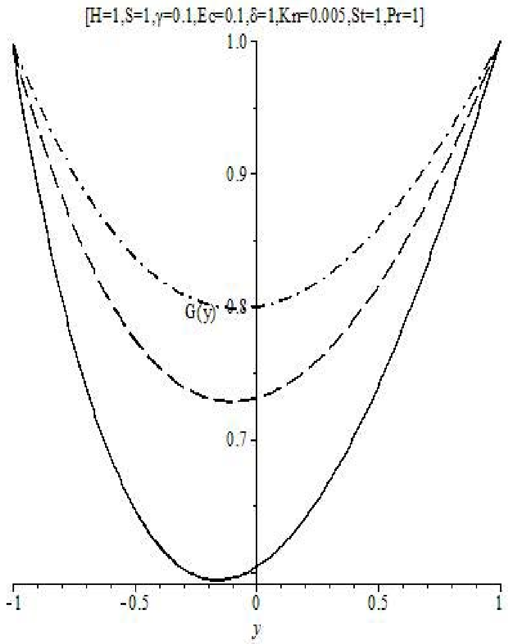

Fig.3a. Temperature profile with a change in the radiation parameter.

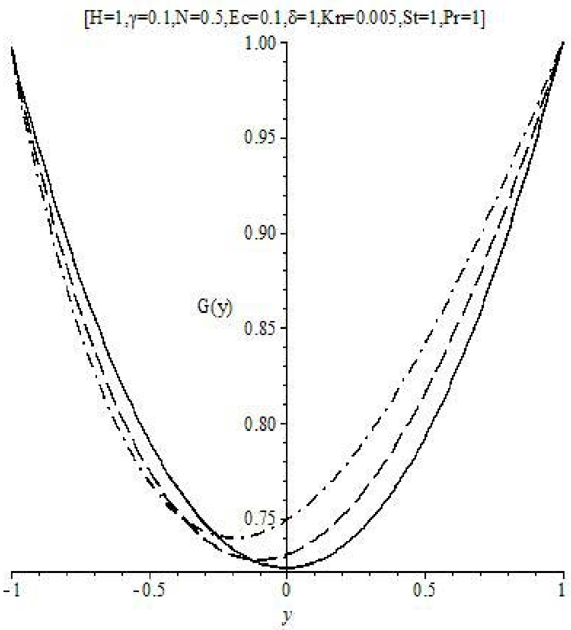

Fig.3b. Temperature profile with a change in the suction/injection parameter.

Figure 4a depicts the effect of heat absorption on the temperature profile. It is seen from the result that an increase in heat absorption leads to a decrease in the temperature of the fluid. The reverse is the case in Figure 4b where an increase in heat generation enhances the temperature profile. The reason for this is that the temperature within the channel increases as heat generation is increasing. The effect of the Strouhal number is presented in Fig.5a. From the results, it follows that an increase in the Strouhal number reduces the temperature profile of the fluid. This is due to a decrease in the intensity of heating the boundary plates. In Fig.5b, as heat absorption parameter is increasing we observed a decrease in velocity of the fluid. Figure 6a shows that an increase in heat generation enhances the fluid velocity. This is due to increase in temperature of the fluid which increases the molecular interaction of the fluid particle there by strengthening the convection current. Figure 6b shows the effect of the Hartmann number on the velocity profile. From the result, it follows that an increase in the magnetic field strength is seen to lower the fluid velocity. The reason for this is the retarding effect of lorentz forces present in the magnetic field. Figure 7a shows the effect of velocity slip parameter on the velocity profile. It is observed that an increase in the velocity slip enhances the flow. This is due to an increase in the gas molecules interaction. In Fig.7b, the effect of the oscillation parameter on the velocity profile lowers the fluid velocity due to a decrease in the intensity of heating the boundary plate as heating frequency is increased. In Fig.8a, an increase in suction/injection decreases the flow velocity along the injection wall but increases towards the suction wall. Also, it is observed in Fig.8b that an increase in the radiation parameter increases the velocity profile. This is because when the heat produced by thermal radiation is increased, it breaks the bond that holds fluid particles, thereby enhances the fluid flow. Figure 9a shows the effect of the velocity slip parameter on the skin friction. In the result, it is observed that an increase in the Navier slip parameter leads to the weakness of the wall skin friction. Additionally, in Fig.9b we can see that an increase in the Hartmann number weakens the skin friction at the wall. From Fig.10a it follows that an increase in radiation leads to an increase in the skin friction while an increase in the radiation parameter (Fig.10b) results in a decrease in the rate of heat transfer. Finally, an increase in temperature jump (Fig.11) leads to a decrease in the rate of heat transfer.

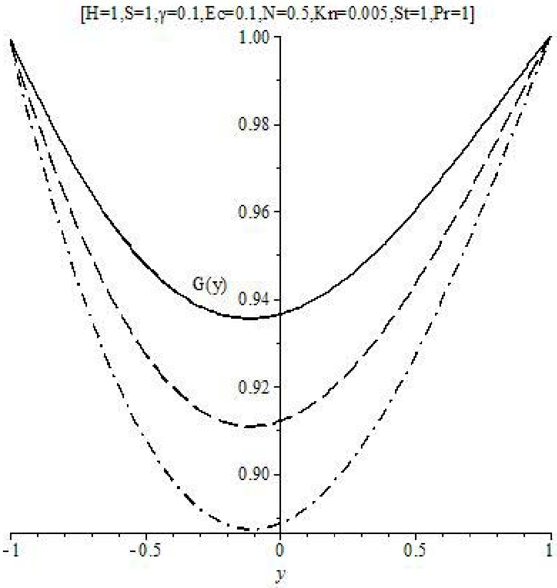

Fig.4a. Temperature profile with a change in the heat absorption parameter.

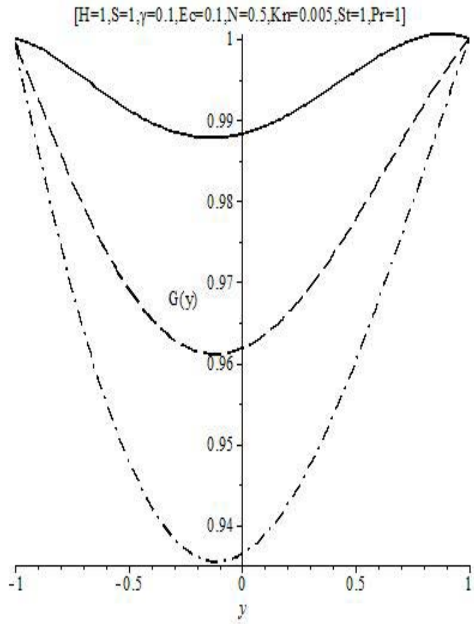

Fig.4b. Temperature profile with a change in the heat generation parameter.

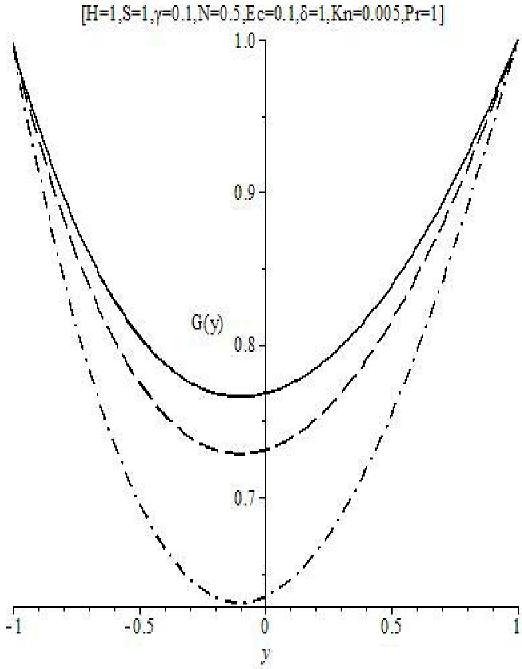

Fig.5a. Temperature profile with a change in the Strouhal number.

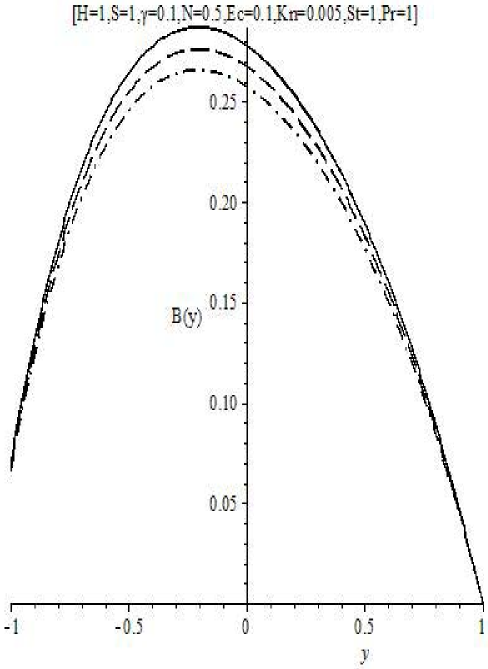

Fig.5b. Velocity profile with a change in the heat absorption parameter.

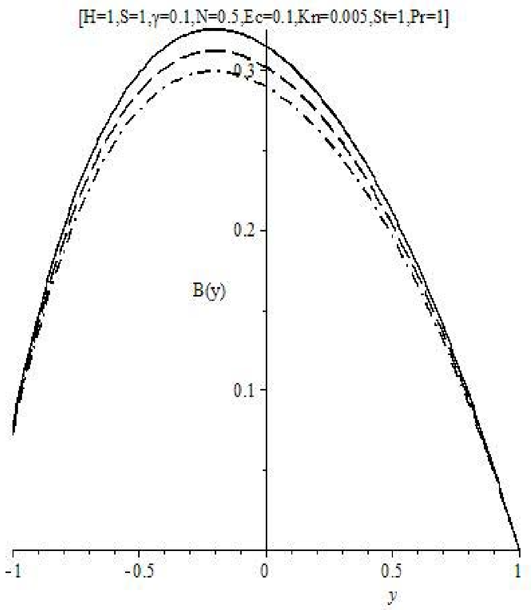

Fig.6a. Velocity profile with a change in the heat absorption parameter.

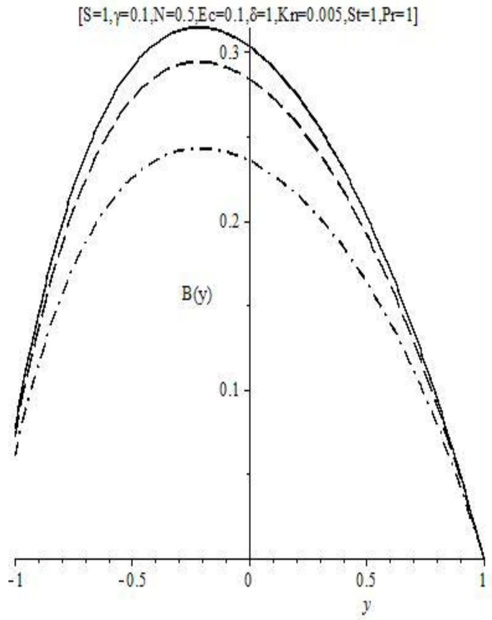

Fig.6b. Velocity profile with a change in the Hartmann number.

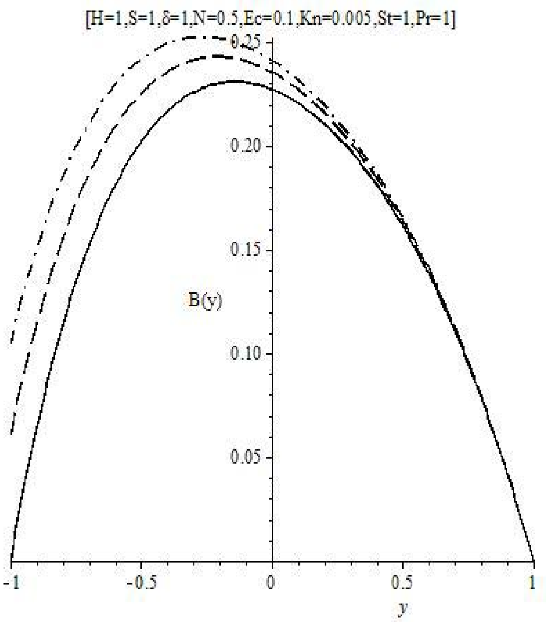

Fig.7a. Velocity profile with a change in the velocity slip parameter.

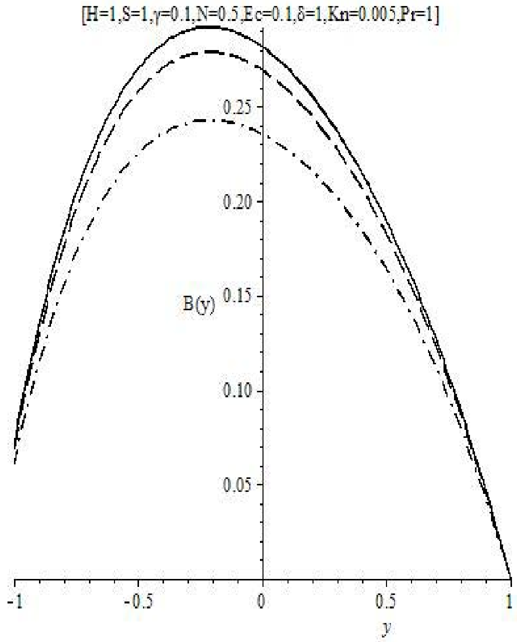

Fig.7b. Velocity profile with a change in the Strouhal number.

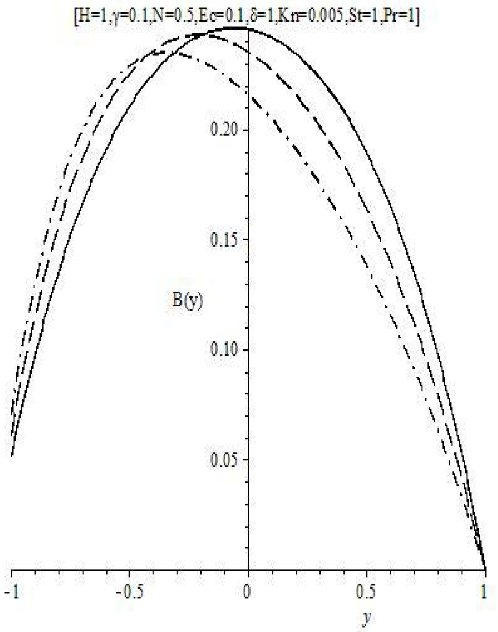

Fig.8a. Velocity profile with a change in the suction/injection parameter.

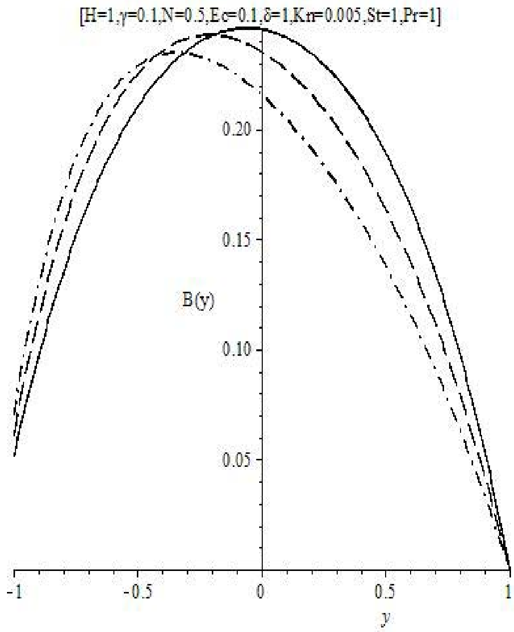

Fig.8b. Velocity profile with a change in the radiation parameter.

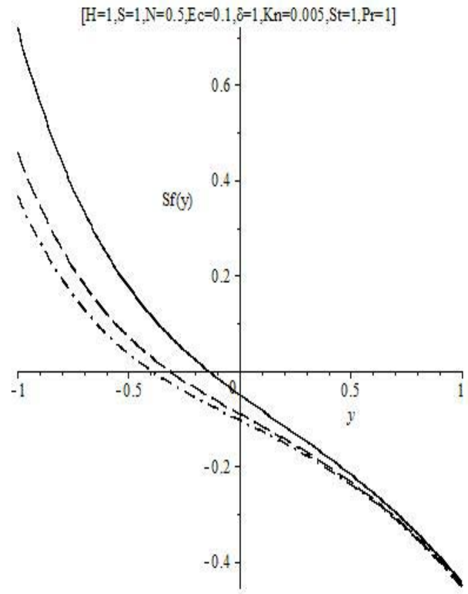

Fig.9a. Wall shear stress with a change in the velocity slip parameter.

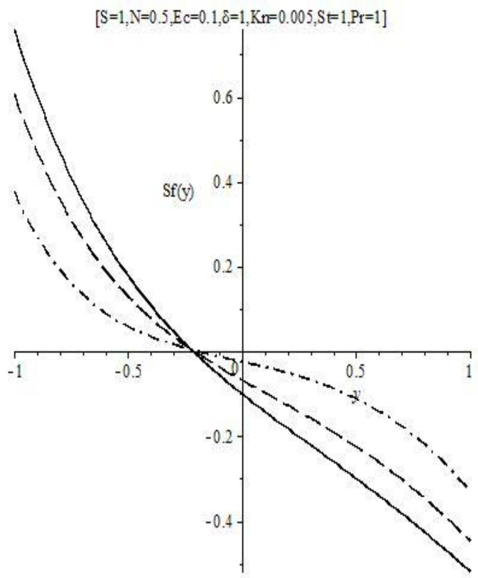

Fig.9b. Wall shear stress with a change in the Hartmann number.

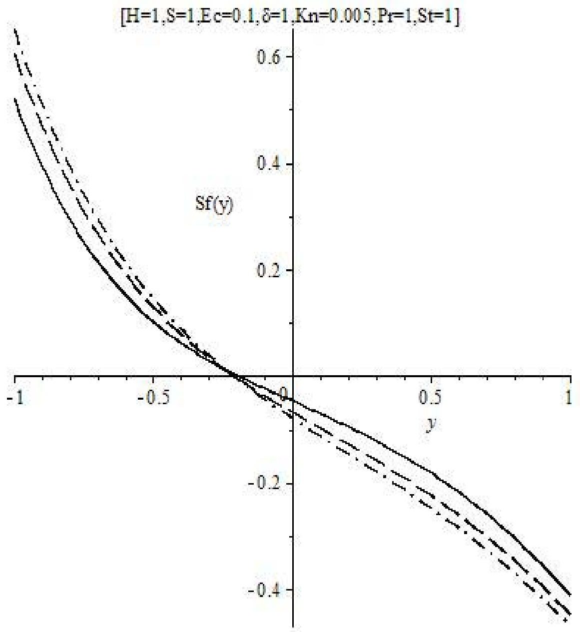

Fig.10a. Wall shear stress with a change in the radiation parameter.

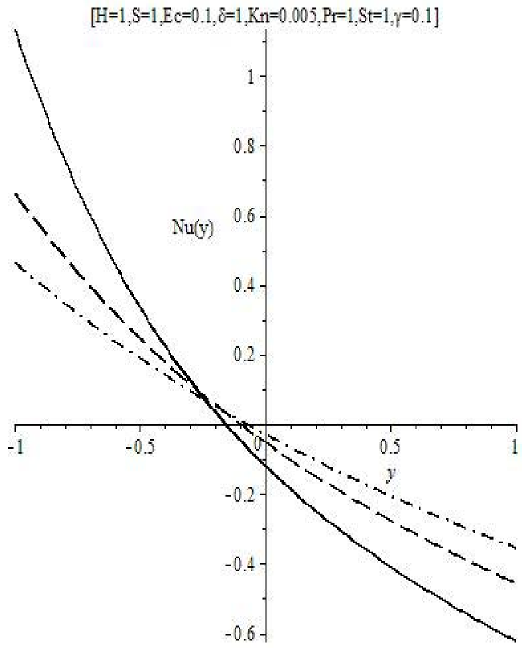

Fig.10b. Rate of heat transfer with a change in the radiation parameter.

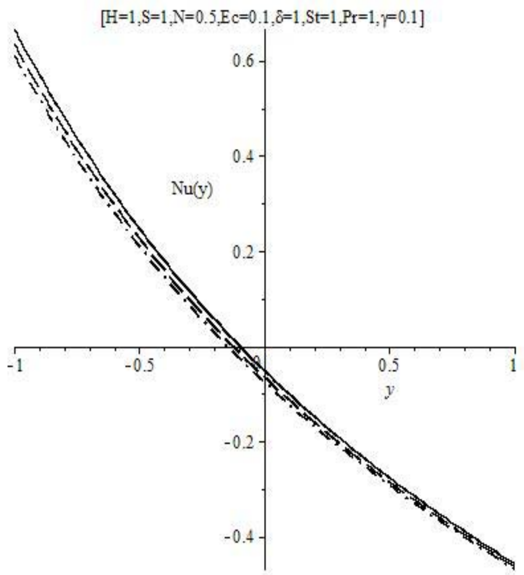

Fig.11. Rate of heat transfer with a change in the temperature jump parameter.

5. Conclusion

This paper investigates a radiative magnetohydrodynamic oscillatory natural convective flow through a vertical porous channel with slip, temperature jump and heat source. The velocity and temperature profiles are obtained using the differential transform method (DTM). The effects of different parameters are studied. The velocity profile increases with increasing velocity slip, heat generation, suction/injection and radiation parameter while it reduces with increase in heat absorption, the Hartmann number and Strouhal number. An increase in radiation, suction/injection, the Hartmann number and heat sink enhances temperature profile while increasing the Strouhal number, temperature jump and heat source parameter reduces the temperature profile. Finally, increasing velocity slip, temperature jump and the Hartmann number reduces the wall skin friction and heat transfer rate but increasing the radiation parameter increases the wall skin friction and decreases the heat transfer rate.

Nomenclature

B_0 – magnetic field strength
C_p – specific heat capacity at constant pressure
E_c – viscous heating parameter
g – gravitational acceleration
H – Hartmann number
h – half channel width
K – thermal conductivity
Kn – Knudsen number
Pr – Prandtl number
S – suction/injection parameter
St – Strouhal number
T – fluid temperature
T_0, T_1 and T_2 – referenced fluid temperature
t' – time
u' – velocity
Q – term due to internal heat generation
x – vertical coordinate
y – horizontal coordinate
β – volumetric expansion
γ – Navier's slip parameter
δ – heat generation parameter
λ – molecular mean free path
ν – kinematic viscosity
ξ – tangential momentum accommodation coefficient
ρ – fluid density
σ – electrical conductivity
σ_T – thermal accommodation coefficient
ϕ – specific heat ratio
ω – frequency

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