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Int. J. of Applied Mechanics and Engineering, 2017, vol.22, No.3, pp.749-757
DOI: 10.1515/ijame-2017-0047

Brief note

STUDY ON THE SIZE EFFECT OF AUXETIC CELLULAR MATERIALS

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The objective of this paper is to investigate the effects of scale of an auxetic cellular material sample on the evaluation of elastic properties. Size and boundary effects are studied in detail. This is achieved by conducting computer simulations of the auxetic structure under the typical loading exerted by the compression and simple shearing test performed by means of ABAQUS FEA. The material microstructure is discretized by the plane network of Timoshenko beam elements. The results of the studies give insight to the scale effects. Structures with designed properties can be potentially used for engineering applications.

Key words: negative Poissons ratio, auxetic structures, size effect.

1. Introduction

First experiments conducted on microstructured materials showed a dependence of measured mechanical constants on the dimension of samples. Experiments show that when the specimen size and the representative volume element size are of the same order of magnitude the representative volume element response varies considerably throughout the specimen. This leads to differences in the macroscopic properties of specimens of a different size. This effect can be also observed in cellular materials. The individual response to a load significantly differs from one cell to another when the cell size is of the order of the smallest structural length of specimen. In such a case so called higher-order effects occur and the classical theory of continuum is insufficient to describe mechanical behaviour. Such a phenomenon called the size effect designates the influence of the relation of cell size to the sample size on the mechanical properties. The classical continuum theory does not incorporate a length scale and cannot describe size effect. To consider size effects it is necessary to take into account the cellular morphology by discrete model of microstructure. This allows a precise description of the microstructural deformation mechanisms and predict how the macroscopic response is related to the structural parameters. Such a discrete model is computationally very expensive. Numerical studies based on FEM models are presented by Onck [1], Chen and Fleck [2], Diebels and Steeb [3].

Another way is to use a generalized continuum theory. Nonclassical elasticity includes the effects of microstructural deformations by inserting new independent degrees of freedom into the continuum. These kinds of theories can be referred to as higher order theories. The most important feature is that such theories incorporate the material length scale. For a cellular material it is the cell size in relation to the sample dimension. Experimental results confirm the applicability of the linear micromorphic Eringen theory and microstructure Mindlin theory for cellular materials. A comparison of the results obtained by application of the nonclassical continnum theory and study by FEM model is given in works by Onck [1].

A description of the size effects in materials with negative Poisson's ratios and influence on their elastic properties can be found in works by Donescu *et al.* [4]. The behavior of linear elastic auxetic materials with size effects can be interpreted in the light of Cosserat elasticity. The majority of the studies have concentrated on a description within the frames of micropolar or microstretch theory. The names micropolar theory and Cosserat theory are used interchangeably by many authors in the literature. The main advantage of applying the generalized continuum theory is that numerical calculations are not expensive.

The first experiments on cellular solids with the aim to study the dependence of the macroscopic material properties on the specimen size date back to the 1980's. They were performed on circular cylindrical compact bones by Lakes [5] and showed that torsional rigidity decreases with increasing sample dimensions. Lakes conducted a test on polymeric foams and proved that it can be described by the theory of micropolar elasticity. Young's modulus and the strength of open cell carbon foam was measured during experiment by Brezny and Green [6]. As a result it was observed that the modulus and the strength of this material decreased with decreasing specimen size. The weakening effects in both bending and torsion were also observed for closed cell polymethacrylimide foam and open cell copper foam. Size effects in foams under uniaxial compression were also experimentally investigated by Bastawros *et al*. [7]. He measured Young's modulus and the compressive strength of closed cell aluminium foam, by changing the area under compression while keeping the length of the samples in the compression direction constant. Andrews *et al*. [8] conducted uniaxial compression tests on square prisms of closed cell Alporas and open cell Duocel foams, where the samples had identical geometry but different absolute size. Experiments showed that Young's modulus and the compressive strength of the samples decrease significantly with decreasing specimen size.

Shear experiments performed by Andrews *et al*. [8], Chen *et al*. [9] indicated an increased shear strength with decreasing sample thickness. In these experiments the shear load was applied through face sheets that are perfectly bonded to the sample. As a result the surface cells that were perfectly bonded to the top and the bottom face sheets were much more constrained compared to those located in the bulk. This raised a gradient deformation in the layers adjacent to the face sheets. The volume fraction of these boundary layers increased with decreasing thickness and it leads to a higher shear strength. Kesler and Gibson [10] conducted three point bending experiments on sandwich panels with an Alporas foam core. The conclusion of these studies was that these size effects are actually edge effects. The edge effects are related to an incomplete cell layer located at the surface of the specimens, which is included in the total specimen volume. Surface damage introduced by cutting or machining of specimens enhances these edge effects. Anderson and Lakes [11] concluded that the edge effects and the micropolar effects are usually both present and it is possible to observe weakening or strengthening behaviour depending on which one is more dominant.

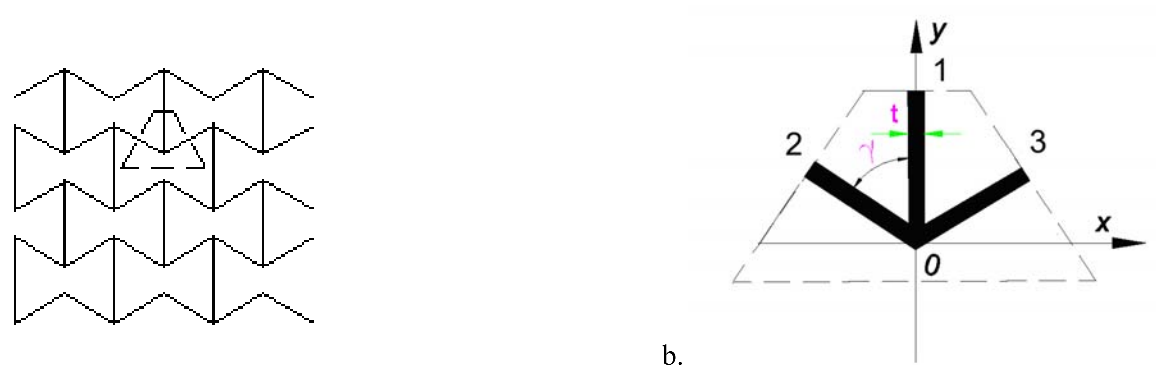
The aim of this paper is to explore the microstructural mechanisms that are responsible for the size-dependent elastic behaviour of cellular auxetics by using a discrete microstructural model.

2. Auxetic cellulars

Auxetic materials are related to negative Poisson's ratio. First auxetic cellular structures were first realised in the form of 2D silicone rubber or aluminum honeycombs in 1980. Auxetics exhibit lower stiffness, greater resistance to identations, show better strain redistribution under external load than ordinary materials. Negative Poisson's ratio can significantly influence stress distribution in contact problem, reduce stress concentration, give differences in Saint Venant effect and produce double curvature effect in 3D bending problems. This property can be useful in some engineering applications. Examples are matresses made of auxetic foams and auxetic skeletons of ergonomic seats. (Jasińska *et. al.* [12]). The overall effective properties are determined by with the use of transition between two scales of observation: micro and macro. It corresponds to the effective model construction (Janus-Michalska [13]). The effective properties are then applied to determine the response of structural elements on a macro scale and emerge naturally as a consequence of micro-macro relations. Deformation takes place on micro or macro level. This means that we can consider auxetic matrials and also auxetic structures.

2.1. Auxetic microstucture

Two dimensional material exhibiting reentrant hexagonal microstructure as shown in Fig.1a are studied.



a. b.

Fig.1. Auxetic material a. auxetic microstructure, b. representative unit cell.

A material with a repetitive microstructure is represented by an idealized regular repeating pattern of unit cells. A skeleton of a cell is modeled as an elastic beam structure with stiff joints. Each structure may be represented by a unit cell in part filled by skeleton with half struts $\boldsymbol{b}_i^0$ of the length $\left|\boldsymbol{b}_i^0\right| = L_{0-i}/2$ measured from the vertex 0 (node) to point i as shown in Fig.1b.

2.2. Strain and stress measures for equivalent continuum

Equivalent continuum properties (stiffness matrix and material constants) are determined by the FEM calculation based on the idea of micromechanical framework (Janus-Michalska [13]). The unit cell is treated as a model on the basis of which effective relations between strains and stresses are established. These strains and stresses are defined as volumetric averages of the micro field variables, which are defined as given below

$$\boldsymbol{\varepsilon} = \left\langle \boldsymbol{\varepsilon}^s \right\rangle_V = \frac{1}{V}\sum_{A_i} \mathrm{sym}\left(\boldsymbol{n}_i \otimes \boldsymbol{u}_i\right)\, dS\,, \qquad \boldsymbol{\sigma} = \left\langle \boldsymbol{\sigma}^s \right\rangle_V = \frac{1}{V}\sum_{A_i}\left(\boldsymbol{t}_i \otimes \boldsymbol{n}_i\right) dS$$

where: $\langle\ \rangle_V$ stands for the volumetric average in skeleton s taken over V, $\boldsymbol{n}_i$ is the outer unit normal to the boundary A_i and $\boldsymbol{u}_i$ and $\boldsymbol{t}_i$ are respectively the midpoint displacement on the surface A_i and surface traction defined as follows $t_i = F_i/A_i$.

A representative volumetric element with its mechanical model is used to determine the effective properties on a macro-scale together with the assumption of uniform strain and stress state. These properties are related to infinite sample dimensions.

2.3. Compliance tensor

The Hooke law for a material with a planar microstructure can be written in Voigt notation as follows

$$\boldsymbol{\varepsilon} = \boldsymbol{C}\boldsymbol{\sigma}\,,$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\mathrm{E}_x} & -\dfrac{\nu_{xy}}{\mathrm{E}_y} & \dfrac{\eta_{xy/x}}{G_{xy}} \\ -\dfrac{\nu_{yx}}{\mathrm{E}_x} & \dfrac{1}{\mathrm{E}_y} & \dfrac{\eta_{xy/y}}{G_{xy}} \\ \dfrac{\eta_{x/xy}}{\mathrm{E}_1} & \dfrac{\eta_{y/xy}}{\mathrm{E}_2} & \dfrac{1}{G_{xy}} \end{bmatrix} \cdot \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

where the compliance matrix $\mathbf{C}$ is expressed by engineering constants:

E_x, E_y - Young moduli in *x* and *y* direction

G_{xy} - Kirchoff modulus in *xy* plane

$\eta_{xy/x}$, $\eta_{xy/y}$ - coefficients describing the influence of strains in the *xy* plane on normal stress in *x* and *y* direction

$\eta_{x/xy}$, $\eta_{y/xy}$ - coefficients describing the influence of stress in *x* and *y* direction on strain in the *xy* plane.

Due to symmetry of the compliance matrix

$$C_{ij} = C_{ji} .$$

3. Loading cases

Two homogeneous deformations are considered: simple shearing and deformation relevant to uniaxial compressive load. The loadings are chosen in such a way that they correspond to typical experiments. Small strain anisotropic elasticity is considered.

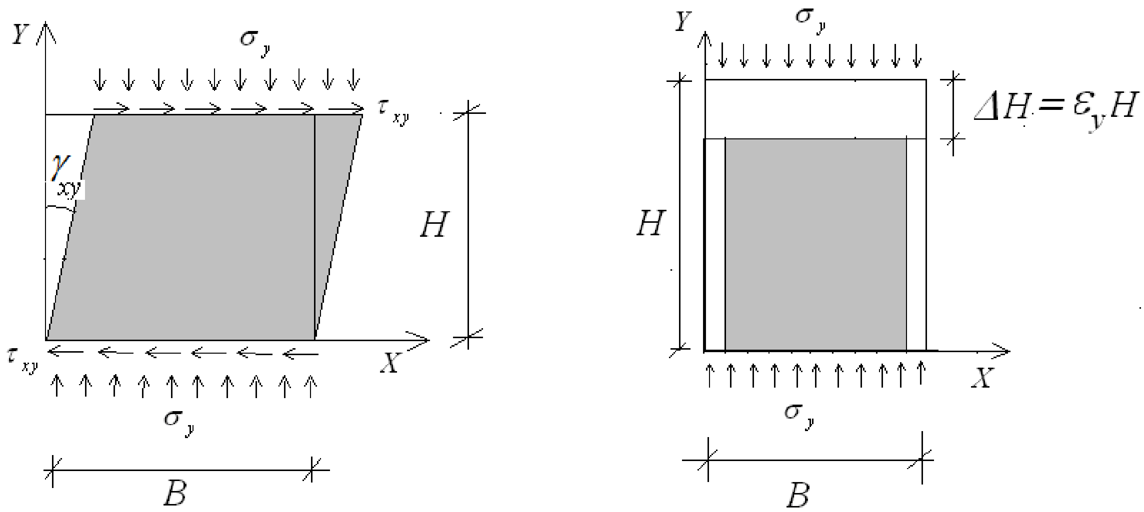


Fig.2. Homogeneous deformations in tests a. simple shearing, b. compression.

4. Numerical analysis - scope of the study

4.1. Size effects

The aim of this study is to explore the microstructural origin of the size effects in the mechanical behaviour of cellular auxetics. The effect of specimen size on the elastic response of regular auxetic cellular structures is investigated through the discrete simulations of two-dimensional beam networks in material sample. Simple shear and uniaxial compression tests are performed on samples with different microstructures to calculate the change in the macroscopic mechanical properties corresponding to a change in size. The specimen size is changed gradually to detect the size effects. For materials in which the structure size is sufficiently large, non-classical effects can be substantial. In such cases, the individual response to a load differs significantly from one cell to another and non-affine deformation of cells is observed. The assumption of the classical continuum theory that the mechanical properties of a material are uniformly distributed throughout its volume fails. The term size effect designates the effect of the macroscopic (sample) size, relative to the cell size, on the mechanical behavior.

In order to verify size effect series of numerical simulations are performed by means of ABAQUS FEA. The material microstructure is discretized by the plane network of Timoshenko beam elements.

Numerical tests are carried out for two microstructured materials with different combinations of geometric parameters. Specifications of these parameters for microstructures are given in Tab.1. Geometric proportions of two microstructures with repetitive structural segments are illustrated in Fig.3. Addition of these segments to specimens guarantees the same boundary conditions in each specimen. Calculations for chosen materials are also performed for two different microstructural orientations as shown in Fig.4. (for *i* total segments and i +1/2 structure with half segment on one boundary).

Table 1. Specification of microstructures.

type	Geometric parameters of skeleton [*mm*]	Skeleton material parameters
1)	$L_{01}=10.0$ $L_{02}=L_{03}=7.5$ $t=1.0$, $\beta=60^0$	$E_S=1800$ MPa, $R_e=45$ MPa
2)	$L_{01}=8.0$ $L_{02}=L_{03}=16.0$, $t=1.0$, $\gamma=80^0$	$E_S=1800$ MPa, $R_e=45$ MPa

E_S material Young's modulus
R_e material rupture modulus

Material: PA6 polyamid

1) 2)

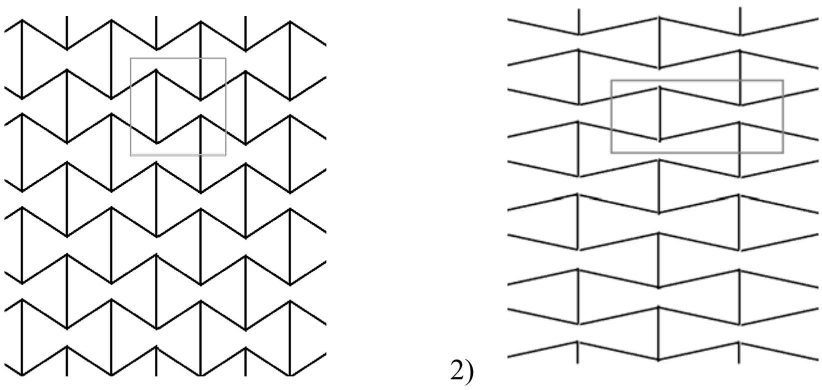


Fig.3. Two microstructures with repetitive structural segments.

a. b.

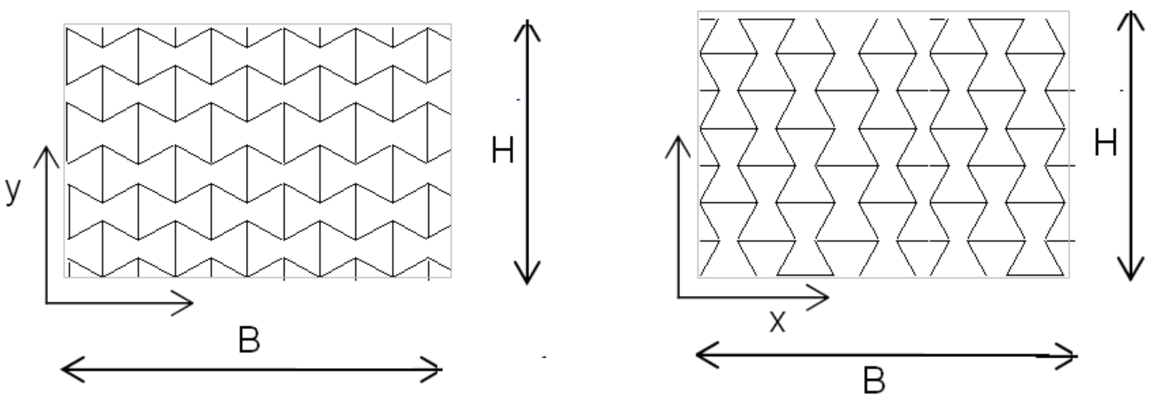


Fig.4. Two different structural sample orientations, a. horizontal orientation, b. vertical orientation.

5. Results

5.1. Simple shear

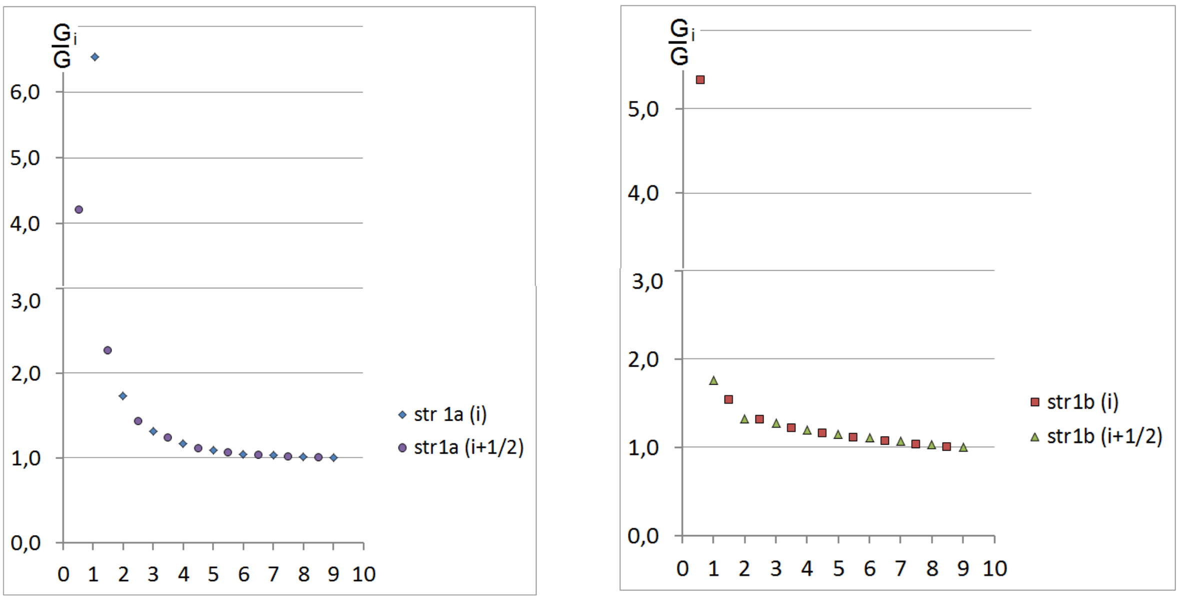
The specimen behavior is analyzed under the state of stress relevant to simple shear deformation shown in Fig.2a. The length of the specimens *B*, relative to the cell size is taken to be large enough to ensure that the stiffness is independent of *L*/*B*. The upper and lower sheets are assumed to be rigid and bonded to the cellular material. It means that translational and rotational degrees of freedom are constrained at the bottom of a sample. At the upper edge a horizontal displacement $u=\gamma H$ is applied, while the nodal vertical displacement and rotation are constrained.

The shear stress is calculated by dividing the sum of the reaction forces on the boundary nodes by the edge area. The macroscopic shear stress divided by shear strain gives the macroscopic shear stiffness.

Figure 5 shows the macroscopic shear stiffness normalized by the shear modulus *G* plotted against the number of unit cells in the specimen. Each dot corresponds to one finite element calculation. For each microstructure, *G* corresponds to the effective shear stiffness of an infinitely large block with the corresponding microstructure.

It can be observed that relative shear stiffness depends not only on the number of unit cells, but also on the length and the orientation of the cell walls at the boundaries, which we define as the boundary configuration.

G – effective shear stiffness of an infinitely large block with the auxetic microstructure
Gk – shear stiffness of sample with finite dimension



a. b.

Fig.5. The macroscopic shear stiffness normalized by the shear modulus versus the number of segments in the specimen for the auxetic microstructure 1 in two different orientations. a. b.

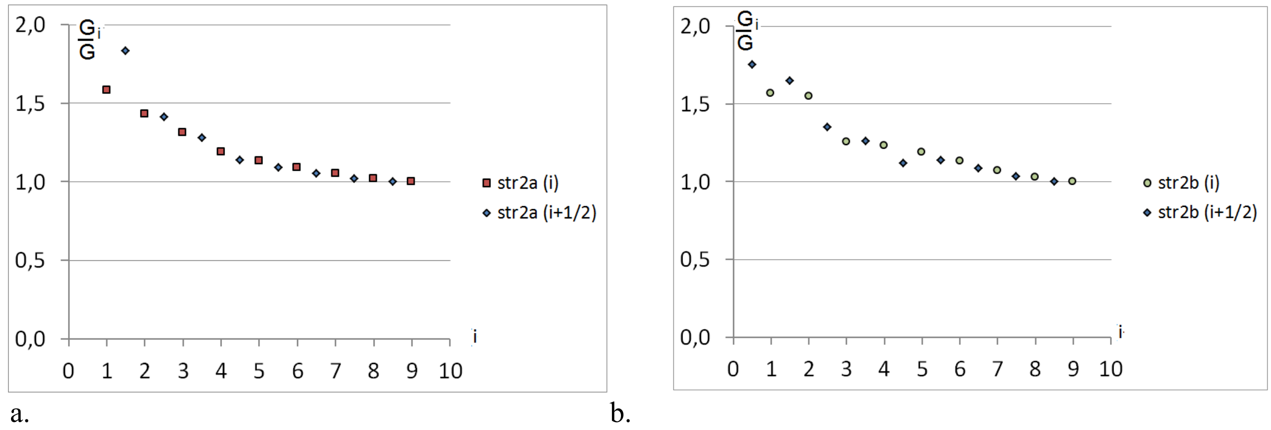


Fig.6. The macroscopic shear stiffness normalized by the shear modulus versus the number of segments in the specimen for the auxetic microstructure 2 in two different orientations. a. b.

5.2. Uniaxial compression

The material behavior is analyzed under the state of stress relevant to compression deformation shown in Fig.2b. Boundary conditions are applied on the upper and lower boundaries of the structure, to imitate an infinitely long material in the y direction. Nodes on opposite edges of the mesh on the left and right are force and moment free. The compressive stress is calculated by dividing the sum of the reaction forces on the boundary nodes by the edge area. The uniaxial compressive stiffness is calculated as the ratio of the compressive stress divided by the compressive strain. The width of the specimens relative to the cell size is taken to be large enough to ensure that the uniaxial compressive stiffness equals *E*.

E – effective compressive stiffness of an infinitely large block with the auxetic microstructure
Ek – longitudinal stiffness of sample with finite dimension

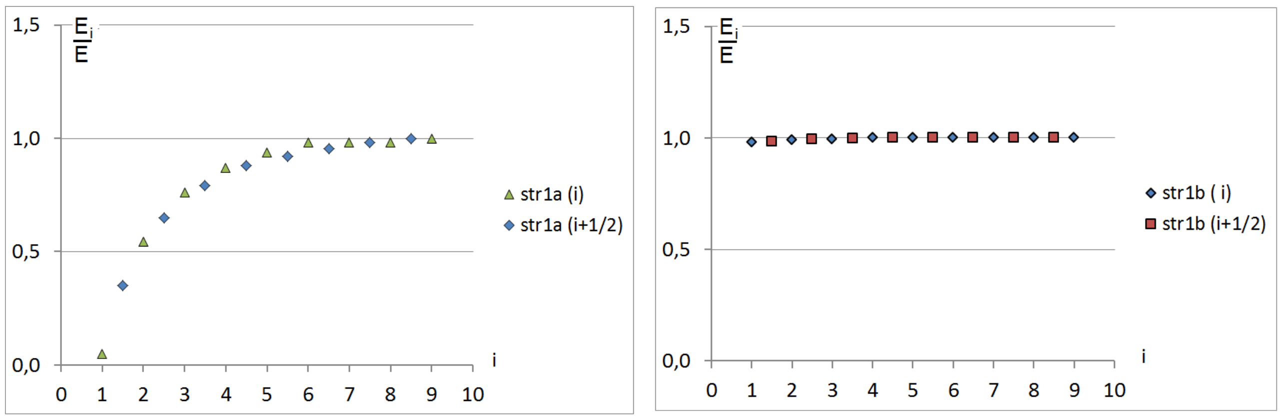


Fig.7. The macroscopic longitudinal stiffness normalized by the Young modulus versus the number of segments in the specimen for the auxetic microstructure 1 in two different orientations. a. b.

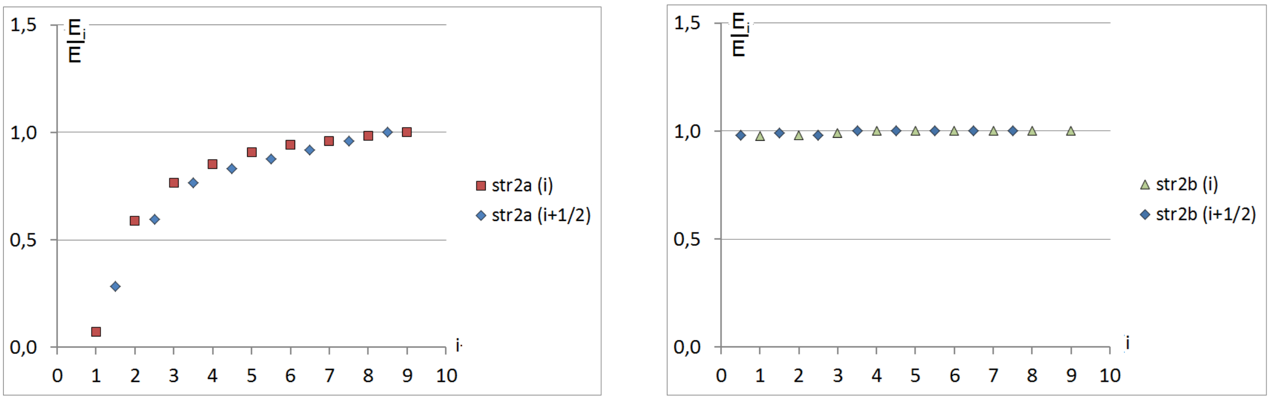


Fig.8. The macroscopic longitudinal stiffness normalized by the Young modulus versus the number of segments in the specimen for the auxetic microstructure 2 in two different orientations. a. b.

6. Conclusions

Numerical tests show the size effects in the mechanical behavior of two-dimensional cellular solids, for which the discreteness of the cellular morphology is taken into account by modelling individual cell walls as beam elements. The mechanical properties calculated this way can be compared with experiments.

For all of the tested microstructural materials, the macroscopic shear stiffness increases with decreasing sample size. The macroscopic (uniaxial) compressive stiffness decrease with decreasing sample size.

Stiffening under simple shear is associated with the strong boundary layers that form adjacent to the top and bottom boundaries, to which the cell walls are perfectly bonded. The smaller the sample size, the larger the area fraction of the strong boundary layers and thus the macroscopic shear stiffness.

Weakening in the compressive stiffness is a result of weak boundary layers that form adjacent to the traction free edges, where the cells are much more compliant compared to the bulk.

The size effect strongly depends on microstructural parameters and structure orientation with respect to compression direction or shear deformation due to anisotropy of the material.

The insight gained from the present analysis may be useful in designing new structures and also as a guide for future analysis. Such structures can be applied as cores of sandwich plates and cores of ergonomic seats [14].

The presented method is computationally expensive. The next step of investigation is to use the micropolar theory to capture the described size effects. The method is to fit the elastic constants of the micropolar continuum theory by comparing the analytical solution of the simple shear problem with the discrete analyses, in terms of the best agreement in the macroscopic shear stiffness of the samples.

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Received: December 12, 2016
Revised: May 30, 2017