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Brief note

INERTIA AND COUPLE-STRESS EFFECTS IN A CURVILINEAR THRUST HYDROSTATIC BEARING

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The flow of a couple-stress lubricant in a clearance of a curvilinear thrust hydrostatic bearing with impermeable walls is considered. The flow in the bearing clearance is considered with inertia forces. The equations of motion are solved by an averaged inertia method. As a result , the formulae for pressure distributions without and with inertia effects were obtained. Radial thrust bearings and spherical bearings are discussed as numerical examples. It is shown that inertia effects influence the bearing performance considerably.

Key words: couple-stress lubricant, inertia effects, hydrostatic thrust bearing.

1. Introduction

Steady state radial flows and time- dependent squeezing flows of Newtonian fluids are encountered in a variety of fields. These flows are found in fabrication operations such as stamping, injection molding and sheet forming. In addition, such flows are encountered in lubrication systems.

The flows of Newtonian fluids in the clearance between two impermeable walls have been examined theoretically and experimentally. The clearance walls have been modelled as two disks, two conical or spherical surfaces (Agrawal [1]; Gould [2]; Murti [3]; Vora [4]). A more general case is established by a flow in the clearance formed by two surfaces of revolution (Walicka [5]).

Classical lubrication theory assumes a laminar flow and neglects inertia terms in the equations of motion governing a lubricant film flow (Möller and Boor [6]; Myshkin *et al.* [7, 8], 2002; Szeri [9]; Walicka [10, 11]; Walicki [12]). Although these assumptions are justified for small values of Reynolds numbers, they are valid in the majority of bearing applications.

With the development of modern machine elements the increasing use of complex fluids as lubricants has become of great interest. Many experimental works have also shown that complex fluids can improve the lubrication properties.

The presence of small amounts of additives in a lubricant can improve bearing performance by enhancing the lubricant viscosity and thus producing an increase in the load capacity. They also reduce the coefficient of friction and increase the temperature range in which the bearing can operate.

The additives are long-chain organic compounds, e.g., the length of the polymer chain may be a million times the diameter of a water molecule. Thus couple stresses might be expected to appear in noticeable magnitudes in liquids containing additives with these large molecules. These couple stresses may be significant particularly under lubrication conditions where thin films usually exist. A number of theories of the

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microcontinuum have been postulated and applied (see, e.g. Walicka [10, 11]; Walicki and Walicka [13]). Amongst them, the Stokes theory [14] allows for polar effects such as the presence of couple stresses.

The purpose of this study is to investigate the inertia and couple-stress effects on the pressure distribution and load-carrying capacity in a non-Newtonian lubricant flow in the clearance of a bearing formed by two coaxial surfaces of revolution, shown in Fig. 1. The analysis is based on the averaged inertia method (Walicka [15]) modified by Walicka and Wojnarowski [16].

2. Analysis of a lubricant flow in a bearing clearance

Let us consider a thrust bearing with a curvilinear profile of the working surfaces shown in Fig. 1. The lower surface is described by the functions $R(x)$ which denotes the radius of this surface; the bearing clearance thickness is given by the function $h(x)$. An intrinsic curvilinear orthogonal coordinate system x, ϑ, y linked with the lower bearing surface is also presented in Fig.1.

Taking into account the considerations of the works (Walicka [10, 11]) one may present the equations of a couple-stress lubricant motion for axial symmetry in the form:

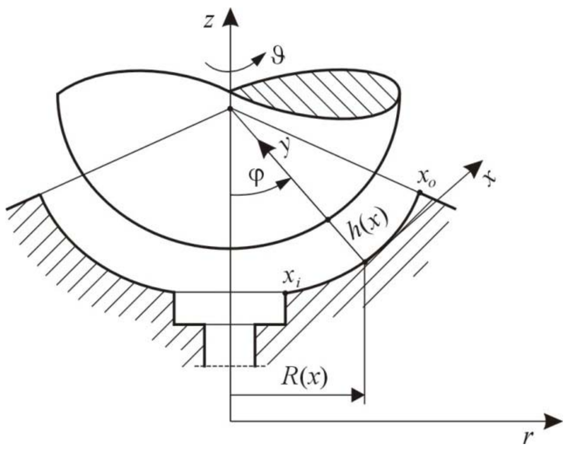


Fig.1. Configuration of a thrust hydrostatic bearing.

$$\frac{1}{R}\frac{\partial(R\upsilon_x)}{\partial x}+\frac{\partial \upsilon_y}{\partial y}=0\,, \tag{2.1}$$

$$\rho\left[\frac{1}{R}\frac{\partial}{\partial x}\left(R\upsilon_x^2\right)+\frac{\partial}{\partial y}\left(\upsilon_x\upsilon_y\right)\right]+\frac{dp}{dx}=\mu\frac{\partial^2\upsilon_x}{\partial y^2}-\eta\frac{\partial^4\upsilon_x}{\partial y^4}\,; \tag{2.2}$$

here μ is the classical viscosity, but η is a new material constant responsible for the couple-stress lubricant property. The problem statement is complete after specification of boundary conditions which are

$$\upsilon_x=0\,,\qquad \frac{\partial^2\upsilon_x}{\partial y^2}=0\,,\qquad \upsilon_y=0\qquad \text{for}\qquad y=0\,, \tag{2.3}$$

$$\upsilon_x=0\,,\qquad \frac{\partial^2\upsilon_x}{\partial y^2}=0\,,\qquad \upsilon_y=0\qquad \text{for}\qquad y=h\,, \tag{2.4}$$

$$p(x_i)=p_i\,,\qquad\qquad p(x_o)=p_o\,. \tag{2.5}$$

Averaging the left-hand side of Eq.(2.2) across the clearance thickness and taking into account boundary conditions (2.3) and (2.4), we may write

$$\frac{\partial^2 \upsilon_x}{\partial y^2} - l^2 \frac{\partial^4 \upsilon_x}{\partial y^4} = F(x) \qquad \text{and} \qquad F(x) = \frac{1}{\mu}\left[\frac{dp}{dx} + \frac{\rho}{Rh}\frac{\partial}{\partial x} R \int_0^h \upsilon_x^2 \, dy\right]; \tag{2.6}$$

here l – having a dimension of length – denotes the square root of the ratio η to μ, i.e., $l = (\eta/\mu)^{1/2}$.
Integrating Eq.(2.6)$_1$ with respect to y in the interval $0 \le y \le h$ and determining the arbitrary constants from the boundary conditions (2.3), we obtain

$$\upsilon_x = \frac{F}{2} V(y), \qquad V(y) = y^2 - hy^2 + 2l^2\left(1 - ch\frac{y}{l} + th\frac{h}{2l} sh\frac{y}{l}\right). \tag{2.7}$$

By substituting Eqs (2.7) into Eq.(2.1) one obtains

$$\frac{1}{R}\frac{\partial}{\partial x}\left[Rh^3 F f(l,h)\right] = 0, \tag{2.8}$$

a general form of the modified Reynolds equation in which

$$f(l,h) = 1 - 12\left(\frac{l}{h}\right)^2 + 24\left(\frac{l}{h}\right)^3 th\frac{h}{2l}. \tag{2.9}$$

3. Solution to the Reynolds equation

The solution to the Reynolds equation (2.8) has a form

$$F = \frac{C_1}{Rh^3 f(l,h)}, \tag{3.1}$$

while the pressure distribution is given by the solution of the following equation

$$\frac{dp}{dx} = \mu F - \frac{\rho}{Rh}\frac{\partial}{\partial x} R \int_0^h \upsilon_x^2 dy. \tag{3.2}$$

To solve this equation we assume that the velocity υ_x of a lubricant flow with inertia is approximately equal to the velocity without inertia υ_{xR}: $\upsilon_x \approx \upsilon_{xR}$.
For an approximation without inertia (the Reynolds approximation) there is

$$\frac{dp_R}{dx} = \mu F_R = \frac{C_{1R}}{Rh^3 f(l,h)} \tag{3.3}$$

and

$$\upsilon_{xR} = \frac{1}{2\mu}\frac{dp_R}{dx} V(y). \tag{3.4}$$

Substituting Eq.(3.3) into the Reynolds equation (2.8), integrating and taking into account the boundary conditions (2.5), one finds

$$p_R = \frac{p_o A_i - p_i A_o}{A_i - A_o} + \frac{p_i - p_o}{A_i - A_o} A(x) \tag{3.5}$$

where

$$A(x) = \int \frac{dx}{Rh^3 f(l,h)}, \qquad A_i = A(x_i), \qquad A_o = A(x_o). \tag{3.6}$$

Calculating dp_R/dx from Eq.(3.5) we have

$$\frac{dp_R}{dx} = \frac{p_i - p_o}{A_i - A_o} \frac{1}{Rh^3 f}, \qquad C_{1R} = \frac{p_i - p_o}{A_i - A_o} \quad \text{and} \quad \upsilon_{xR} = \frac{C_{1R}}{2\mu Rh^3 f} V(y). \tag{3.7}$$

Substituting Eqs (3.1) and (3.7) into Eq.(3.2), integrating and taking into account the boundary conditions (2.5) we get

$$\begin{gathered} p(x) = \frac{(p_o + P_l B_o) A_i - (p_i + P_l B_i) A_o}{A_i - A_o} + \\ + \frac{(p_i + P_l B_i) - (p_o + P_l B_o)}{A_i - A_o} A(x) - P_l B(x) \end{gathered} \tag{3.8}$$

where

$$P_l = \frac{\rho}{240\mu^2} \left(\frac{p_i - p_o}{A_i - A_o} \right)^2, \qquad B(x) = 2\int \left\{ \frac{1}{Rh} \frac{\partial}{\partial x} \left[\frac{g(l,h)}{Rhf^2} \right] \right\} dx. \tag{3.9}$$

Here

$$g(l,h) = 1 - 20\left(\frac{l}{h}\right)^2 + 24\left(\frac{l}{h}\right)^3 \left(th\frac{h}{2l} - \frac{h}{2l} \right) + O\left(\frac{l}{h}\right)^4. \tag{3.10}$$

The load-carrying capacity is defined by

$$N(x_i) = \pi R_i^2 p_i + 2\pi \int_{x_i}^{x_o} pR\cos\varphi dx \tag{3.11}$$

whereas the sense of angle φ arises from Fig.1.
This formula may also be presented in the form

$$N(x_i) = \pi R_o^2 p_o - 2\pi \left[G(x_o) - G(x_i) \right] \tag{3.12}$$

where

$$G(x) = \int \frac{dp}{dx} \left(\int R\cos\varphi dx \right) dx \tag{3.13}$$

and

$$\frac{dp}{dx} = \frac{C_1}{Rh^3 f} + 2P_l \left\{ \frac{1}{Rh} \frac{\partial}{\partial x} \left[\frac{g(l,h)}{Rhf^2} \right] \right\}; \qquad C_1 = \frac{(p_i + P_l B_i) - (p_o + P_l B_o)}{A_i - A_o}. \tag{3.14}$$

In practical applications there is $0 \le l/h \le 0.2$; that allows us to reduce formulae (2.9) and (3.10) to the following form

$$f(l,h) \approx 1 - 12\left(\frac{l}{h}\right)^2, \qquad g(l,h) \approx 1 - 20\left(\frac{l}{h}\right)^2. \tag{3.15}$$

4. Examples of thrust bearings

To consider examples of thrust bearings we will present the results (3.8) and (3.11) in dimensionless forms. To this aim let us introduce the following parameters

$$\tilde{x} = \frac{x}{x_o}, \qquad \tilde{R} = \frac{R}{R_o}, \qquad \epsilon = \frac{x_i}{x_o}, \qquad \tilde{h} = \frac{h}{h_o}, \qquad \tilde{p} = \frac{p}{p_o},$$
$$\delta = \frac{p}{p_o}, \qquad \tilde{A}(\tilde{x}) = A(x)\frac{h_o^3 R_o}{x_o}, \qquad \tilde{B}(\tilde{x}) = B(x) R_o^2 h_o^2, \qquad \tilde{N} = \frac{N - \pi R_o^2 p_o}{\pi R_o^2 p_o}. \tag{4.1}$$

The dimensionless pressure distribution and load-carrying capacity are given in the forms

$$\tilde{p}(\tilde{x}) = -P_l \tilde{B}(\tilde{x}) + \frac{\left[\tilde{A}(\tilde{x}) - \tilde{A}_o\right]\left(\delta + P_l \tilde{B}_i\right) - \left[\tilde{A}(\tilde{x}) - \tilde{A}_i\right]\left(1 + P_l \tilde{B}_o\right)}{\tilde{A}_i - \tilde{A}_o}, \tag{4.2}$$

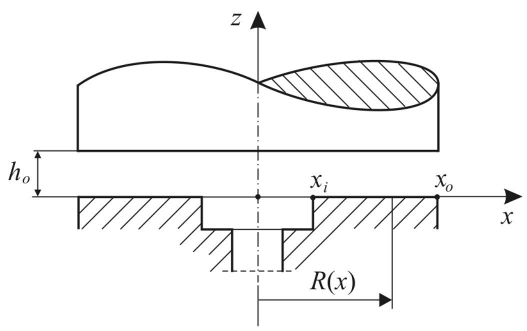
$$\tilde{N}(\tilde{x}_i) = 2\left[\tilde{G}(\tilde{x}_i) - \tilde{G}(\tilde{x}_o)\right] \tag{4.3}$$

where now

$$P_l = P_{l0}\left(\frac{\delta - 1}{\tilde{A}_i - \tilde{A}_o}\right)^2 = \frac{\rho h_o^4 p_o}{240 \mu^2 R_o^2}\left(\frac{\delta - 1}{\tilde{A}_i - \tilde{A}_o}\right)^2 \tag{4.4}$$

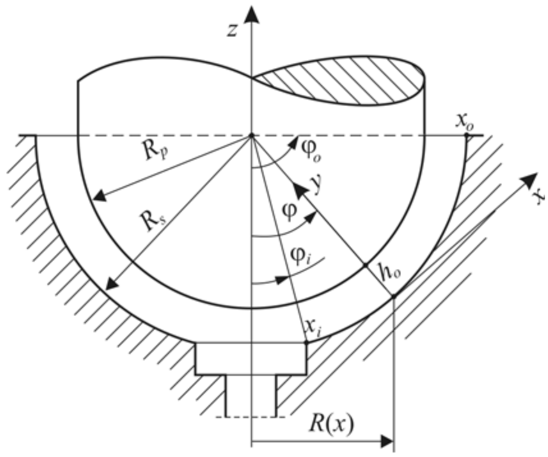
is the inertia effects parameter.

The coefficient P_{l0} is dependent on the bearing geometry and physical lubricant properties. Its practical values are contained in the interval: $0 \le P_{l0} \le 1.0$. The value equal to 0 denotes the lubricant flow without inertia.



$$h(x) = h_o,$$
$$R(x) = x.$$

Fig.2. Configuration of a radial thrust bearing.



$$C = R_s - R_p \,,$$

$$h(x) = h_o \,,$$

$$R = R_s \sin\varphi \,,$$

$$\varphi = \frac{x}{R_s} \,.$$

Fig.3. Configuration of a spherical bearing.

Let us consider two bearings as examples, the first of them is a radial thrust bearing shown in Fig.2, whereas the other one is a spherical bearing shown in Fig.3.
Mechanical parameters of the radial thrust bearing are given as follows

$$\tilde{p}(\tilde{x}) = 1 + \frac{g}{f^2} P_l \left(1 - \frac{1}{\tilde{x}^2}\right) + \left[(\delta - 1) - \frac{g}{f^2} P_l \left(1 - \frac{1}{\text{€}^2}\right)\right] \frac{\ln \tilde{x}}{\ln \text{€}}, \tag{4.5}$$

$$\tilde{N}(\varepsilon) = \frac{(\delta - 1) - \frac{g}{f^2} P_l \left(1 - \frac{1}{\text{€}^2}\right)}{2 \ln \text{€}} \left(\text{€}^2 - 1\right) + 2 \frac{g}{f^2} P_l \ln \text{€} \,, \tag{4.6}$$

where

$$P_l = P_{l0} \left(\frac{\delta - 1}{\ln \text{€}}\right)^2, \qquad f = 1 - 12\left(l^*\right)^2, \qquad g = 1 - 20\left(l^*\right)^2 ; \tag{4.7}$$

P_l denotes the Newtonian inertia effects parameter (Walicka and Wojnarowski [16]).
The pressure distribution depends on the relationship g/f^2 which is presented in Fig.4.

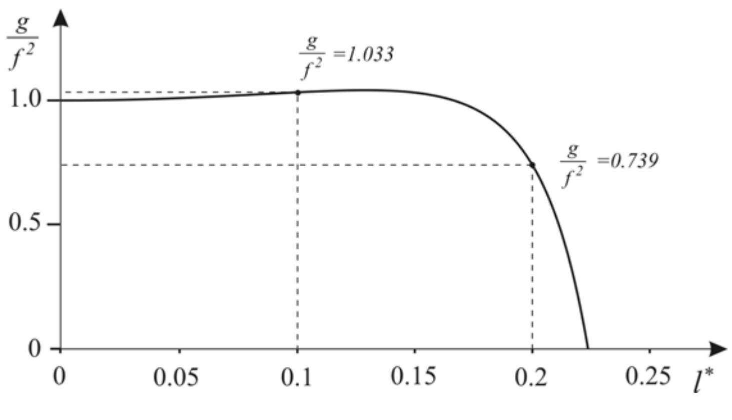


Fig.4. Graph of the function g/f^2 versus l^*.

Figures 5 and 6 present the dimensionless pressure distributions and load-carrying capacity, respectively.

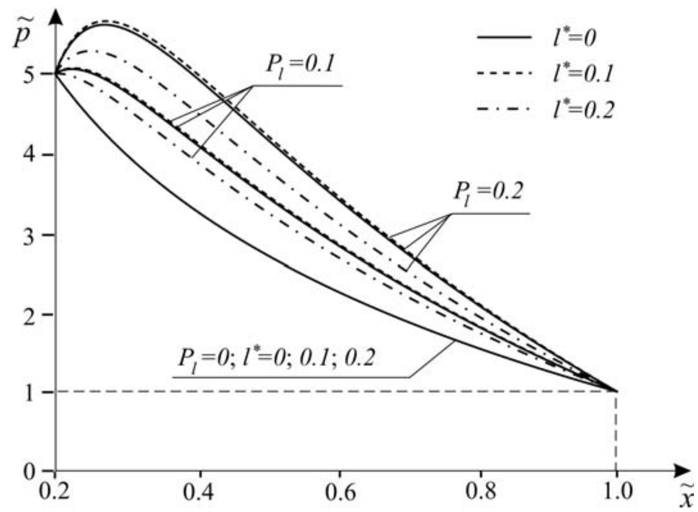


Fig.5. Dimensionless pressure distributions for the radial thrust bearing.

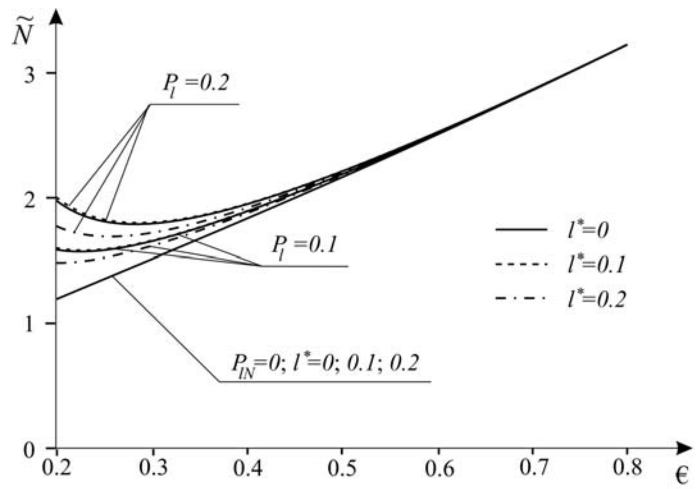


Fig.6. Dimensionless load-carrying capacity for the radial thrust bearing.

Mechanical parameters of the spherical bearing for a clearance of constant thickness, $h = h_o = const$ are given as follows

$$\tilde{A}(\varphi) = \ln \mathrm{tg}\frac{\varphi}{2}, \qquad \tilde{B}(\varphi) = \frac{1}{\sin^2 \varphi}, \qquad P_l = \frac{\rho h_o^2 p_o}{240 \mu^2 R_s^2}\left(\frac{\delta - 1}{\tilde{A}_i - \tilde{A}_o}\right)^2, \tag{4.8}$$

$$\tilde{p}(\varphi) = -\frac{g P_l}{f^2 \sin^2 \varphi} + \frac{1}{\tilde{A}_i - \tilde{A}_o}\left\{\left[\tilde{A}(\varphi) - \tilde{A}_o\right]\left(\delta + \frac{g P_l}{f^2 \sin^2 \varphi_i}\right) + \right.$$
$$\left. -\left[\tilde{A}(\varphi) - \tilde{A}_i\right]\left(1 + \frac{g P_l}{f^2 \sin^2 \varphi_o}\right)\right\}, \tag{4.9}$$

$$\tilde{N}(\varphi_i) = -\int_{\varphi_i}^{\varphi_o} \frac{\partial \tilde{p}}{\partial \varphi} \sin^2 \varphi d\varphi = -\left[\tilde{G}(\varphi_o) - \tilde{G}(\varphi_i)\right] \tag{4.10}$$

where

$$\tilde{G}(\varphi) = \int \frac{\partial \tilde{p}}{\partial \varphi} \sin^2 \varphi d\varphi ; \tag{4.11}$$

here

$$\tilde{G}(\varphi) = -\frac{(\delta - 1) - \dfrac{g}{f^2} P_l \left(\dfrac{1}{\sin^2 \varphi_o} - \dfrac{1}{\sin^2 \varphi_i} \right)}{\ln \operatorname{tg} \dfrac{\varphi_i}{2} - \ln \operatorname{tg} \dfrac{\varphi_o}{2}} \cos \varphi + 2 \frac{g}{f^2} P_l \ln \sin \varphi , \tag{4.12}$$

and the load carrying capacity may be obtained from Eq.(4.10).

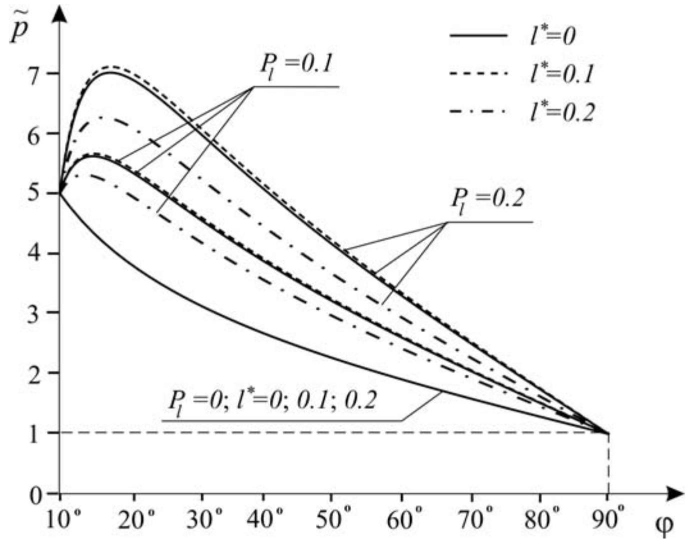


Fig.7. Dimensionless pressure distributions for the spherical bearing with a constant clearance thickness.

Figures 7 and 8 present the dimensionless pressure distributions and load-carrying capacity for a spherical bearing for a clearance of constant thickness, $h = h_o = const$.

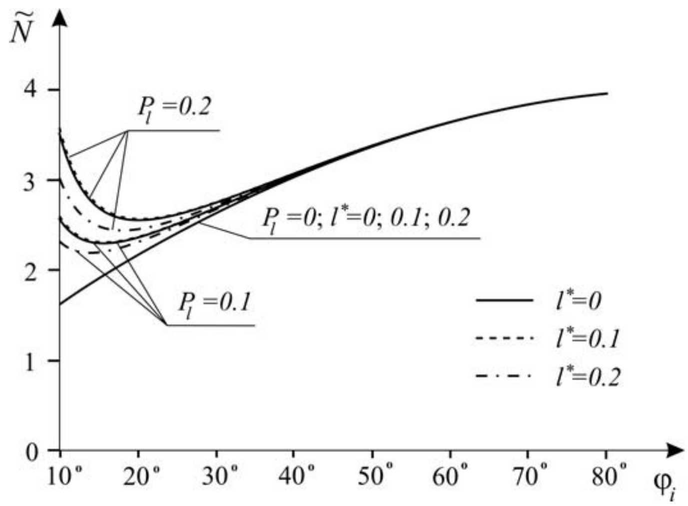


Fig.8. Dimensionless load-carrying capacity for the spherical bearing with a constant clearance thickness.

5. Conclusions

The inertia and couple-stress effects in the lubricant flow through a clearance of a curvilinear thrust hydrostatic bearing are considered.

The equations of motion of a couple-stress lubricant are solved by an averaged inertia method. As a result of the considerations the formulae for pressure distribution and load-carrying capacity are obtained. As examples two particular cases of the bearings are discussed, namely: a radial thrust bearing and spherical bearing.

It is found that the values of the inertia parameter P_{lN} has an essential influence on the bearing performance. With an increase of P_{lN} the values of pressure distributions and load-carrying capacities increase, too. This increase of bearings performance is greater for spherical bearings than that one for step bearings. The effects of couple-stresses are very small and in practice one may neglect these effects.

References

[1] Agrawal V.K. (1970): *Effect of lubricant inertia on squeeze film in spherical bearing.* – Jap. J. Appl. Phys., vol.9, No.7, pp.831-833.

[2] Gould F. (1975): *High-pressure spherical squeeze films.* – J. Lubric. Technol. Trans. ASME, ser. F, vol.97, No.1, pp.207-208.

[3] Murti P.R.K. (1975): *Squeeze film in curved circular plates.* – J. Lubric. Technol. Trans. AME, ser F, vol.97, No.4, pp.650-654.

[4] Vora K.H. (1980): *Behaviour of squeeze film between curved circular plates with a concentric circular pocket.* – Wear, vol.65, pp.35-38.

[5] Walicka A. (1989): *Accurate and Asymptotic Solutions of Simplified Sets of Equations Describing the Motion of Viscous Fluids in a slot Bounded by Two Coaxial Surfaces of Revolution* (in Polish). – Warszawa WNT.

[6] Möller U.J., Boor U. (1996): *Lubricants in Operations*. – Dűseldorf: VDI Verlag.

[7] Myshkin N.K., Kim C.K. and Petrokovets M.I. (1997): *Introduction to Tribology*. – Seoul, Cheong Moon Gak.

[8] Myshkin N.K. and Petrokovets M.I. (2002): *Tribology. Principles and Applications* (in Russian). – Gomel, IMMS NANB.

[9] Szeri A.Z. (1998): *Fluid Film Lubrication. Theory and Design*. – Cambridge University Press.

[10] Walicka A. (2002a): *Rheodynamics of Non-Newtonian Fluids Flow in Straight and Curved Channels* (in Polish), – Zielona Gora: University Press.

[11] Walicka A. (2002b): *Rotational Flows of Rheological Complex Media in Narrow Annular Channels* (in Russian). – Zielona Góra: University Press.

[12] Walicki E. (2005): *Rheodynamics of Slide Bearings Lubrication* (in Polish). – Zielona Góra: University Press.

[13] Walicki E. and Walicka A. (2000): *Mathematical modelling of some biological bearings*. – J. Smart Mat. Struct., vol.9, No.1, pp.280-285.

[14] Stokes V.K. (1966): *Couple stresses in fluids*. – Phys. Fluids, vol.9, No.9, pp.1709-1715.

[15] Walicka A. (1993): *Averaged inertia method for the flow of viscous fluid in a slot between fixed surfaces of revolution*. – Sci. Rep. TU of Zielona Gora, No.18, pp.153-163.

[16] Walicka A. and Wojnarowski J. (2010): *Inertia effects in a curvilinear thrust hydrostatic bearing*. – Int. J. of Applied Mechanics and Engineering, vol.15, No.3, pp.909-920.

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