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Technical note

COMPARATIVE ANALYSIS OF BUCKLING LOAD OF CIRCULAR AND CORRUGATED TUBES BY UTILIZING KEY PERFORMANCE INDICATORS

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The performance of buckling load of tubular structures under quasi-static axial loading is quite appreciable, numerous tubes of various cross-section have been extensively investigated and corrugated sections have been designed to further improve the performance. In this paper, a carefully designed set of key performance indicators (KPIs) is utilized to assess and compare the buckling load of circular and corrugated tubes. A series of diagrams related to KPIs with various parameters of tubes are presented to demonstrate the influence of sectional configuration on the performance of tubes as well as the effect of the material on the potential of the same. The work is inestimable to engineering designs and applications, and further studies on the buckling load of other configurations.

Key words: buckling load, tubular structures, quasi-static, key performance indicators (KPIs), circular and corrugated sections.

1. Introduction

Columns are the rudimentary components of the majority of structures and hence, a meticulous prediction of their capacity is crucial for overall structural efficiency. In recent years, tubular structures, specifically circular and square tubes under axial compression, have been explored by diversified experimental, theoretical and numerical means [1-5]. Not only circular and square tubes, numerous but also other cross-sectional tubes have also become the aim of research, including hexagonal tubes, octagonal tubes, 12-side star tubes, 16-side star tube s(also known as even side tubes)[6-12], as well as triangular tubes and pentagonal tubes (also known as odd side tubes)[13-15]. All these tubes possess their own features in axial loading and it is arduous for designers and engineers to compare and judge which tube has the best performance. Hollow sections are the most versatile and efficient form for the construction and mechanical applications. Many of the strongest and most impressive structures in the world today would not have been possible without hollow sections. The tubular form is inherently strong and efficient. It gives buildings a better strength-to-weight ratio than those using comparable steel, concrete or timber products. In construction, this strength-to-weight ratio reduces material usage and allows greater spans.

Elastic stability or buckling has been theoretically and experimentally analysed, in which buckling of non-prismatic members is of extreme significance. The elastic buckling analysis of open cross-sections, thin-walled columns usually exhibits at least three buckling modes: local, distortion and Euler. The distortion buckling was often intentionally barred in exploration and disregarded in design specifications.

In order to improve the buckling load of thin-walled tubes, various other approaches have been examined, including the use of corrugation especially, changing the cross-section from simple circular to the arc-tangent corrugation. The major benefit of using a corrugated cross-section is that there is a minimal

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increase in the weight of the structures and a noticeable rise in the buckling load of the tubes [16]. The main common feature of all corrugated structures is their exceedingly anisotropic behavior; high stiffness transverses to the corrugation direction in contrast to the compliance along the corrugation direction [17]. Due to this paramount trait, these structures have been vaguely accepted in academic research and industrial applications.

Clearly, it is obligatory for us to find a set of well- equipped criteria to suitably evaluate the buckling load of all kinds of tubular structures. Yu *et al*. [18] developed a set of Key Performance Indicators (KPIs) to assess and compare the performance of tubes with various configurations. Following the pioneering effort, in the present paper, we will study the performance of circular and corrugated tubes by using the set of key performance indicators (KPIs), thereby finding the effect of the cross-sectional shape and material on tubular sections.

2. The Key Performance Indicators (KPIs)

As shown in Eq.(2.1), various factors which affect the critical load for an axially compressed thin-walled tube, under quasi-static loading are dependent on the area moment of inertia, Young's modulus or modulus of elasticity and the effective length of the tube.

$$F = \frac{\pi^2 E I}{(KL)^2} \tag{2.1}$$

where
F= maximum critical load
E= modulus of elasticity
I= area moment of inertia of the cross-section of the tube
L= unsupported length of the column
K= column effective length factor, whose value depends on the conditions of end support of the column

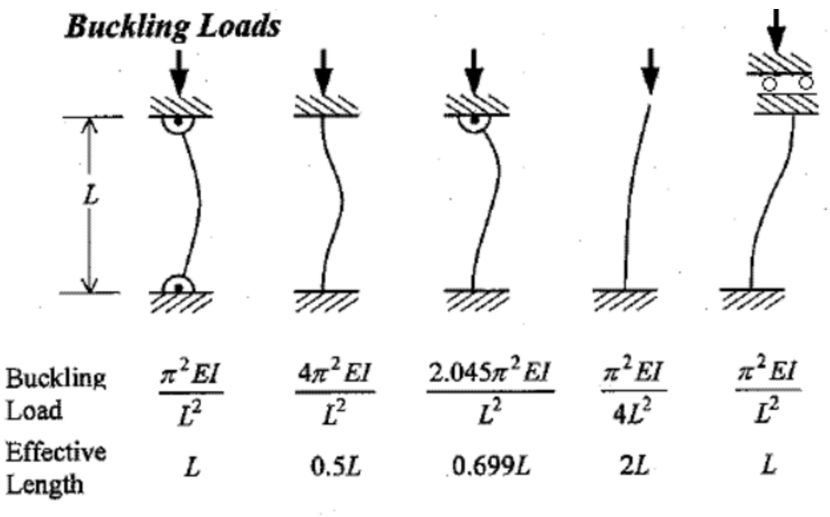

Fig.1. Buckling of columns [19].

For both ends pinned (hinged, free to rotate), $K = 1.0$.
For both ends fixed, $K = 0.50$.
For one end fixed and the other end pinned, $K \approx 0.699$.
For one end fixed and the other end free to move laterally, $K = 2.0$.

In this paper, the buckling loads are considered of columns or tubes whose one end is fixed and the other end is free to move laterally. To facilitate our analysis and comparison of corrugated and circular tubes by utilizing the KPIs, we will briefly illustrate the definitions of those KPIs as follows.

2.1. Area moment of inertia

It is a property of a two-dimensional plane which imparts impression of deflection under loading, when strained and also known as the second moment of inertia or second moment of area. The area moment of inertia has dimensions of length to the fourth power. It is a geometrical characteristic of an area which shows how its points are strewed with regard to an arbitrary axis.

The second moment of area about the *x*-axis is defined by

$$I_{x=}I_{yy=}\int y^2 dx\,dy\,, \tag{2.2}$$

while more generally, the "product" moment of area is defined by

$$I_{xy=}\int x\,y\,dx\,dy\,. \tag{2.3}$$

Here, the positive sign convention is used [20].

2.2. Young's modulus

Young's Modulus, also known as the elastic modulus, is a mechanical property of linear elastic solid materials. It is simply defined as the relationship between longitudinal stress (force per unit area) and strain (proportional deformation) in a material. It is a numerical constant that describes the elastic properties of a solid undergoing tension or compression in only one direction.

$$E=\frac{\text{stress}}{\text{strain}}\,, \tag{2.4}$$

$$E=\frac{F\,L_0}{\Delta L\,A} \tag{2.5}$$

where L_0 is the equilibrium length, ΔL is the length change under the applied stress, F is the force applied, and A is the area over which the force is applied.

Young's modulus is a measure of the ability of a material to withstand changes in length when under lengthwise tension or compression. The units of Young's modulus in the English system are pounds per square inch (psi), and in the metric system newtons per square meter (N/m^2).

3. Analysis on buckling load of circular and corrugated tubes

In order to compare the buckling load of circular and corrugated tubes, several parameters of thin-walled tubes made of steel and aluminum have been taken for evaluation. The parameters varied for the analysis are the mean diameter of the tubes (both circular and corrugated tubes), the number of loops in corrugation, pitch & depth of corrugation and thickness. All the parameters are so changed that the perimeter of both the kinds of tubes is kept same under any changes made in the parameters. Specific geometric dimensions of these tubes are listed in Tab.1 which are taken from previous papers [16]. The influence of the cross-section geometry and variation in the material is studied by analyzing and comparing the following data in the KPIs.

Table 1. Geometric dimensions of tubes.

Specimen	Mean Diameter (*mm*)	No. of Corrugation	Pitch (*mm*)	Amplitude/Depth (*mm*)	Thickness (*mm*)
ATMD01	*300*	*13*	*75*	*17.5*	*1.00*
ATMD02	*350*	*13*	*75*	*17.5*	*1.00*
ATMD03	*400*	*13*	*75*	*17.5*	*1.00*
ATMD04	*450*	*13*	*75*	*17.5*	*1.00*
ATNC01	*300*	*08*	*75*	*17.5*	*1.00*
ATNC02	*300*	*10*	*75*	*17.5*	*1.00*
ATNC03	*300*	*11*	*75*	*17.5*	*1.00*
	300	*13*			*1.00*
ATNC04			*75*	*17.5*	
ATPC01	*300*	*11*	*75*	*17.5*	*1.00*
ATPC02	*300*	*11*	*80*	*17.5*	*1.00*
ATPC03	*300*	*11*	*85*	*17.5*	*1.00*
	300	*11*	*90*	*17.5*	*1.00*
ATPC04					
ATT01	*300*	*13*	*75*	*17.5*	*0.25*
	300	*13*	*75*	*17.5*	*0.50*
ATT02	*300*	*13*	*75*	*17.5*	*0.75*
	300	*13*	*75*	*17.5*	*1.00*
ATT03					
	300	*11*	*75*	*17.5*	*1.00*
ATT04	*300*	*11*	*75*	*18.5*	*1.00*
	300	*11*	*75*	*19.5*	*1.00*
	300	*11*	*75*	*20.5*	*1.00*
ATDC01					
	300	-	-	-	*1.00*
ATDC02	*350*	-	-	-	*1.00*
	400	-	-	-	*1.00*
ATDC03	*450*	-	-	-	*1.00*
	300	-	-	-	*0.25*
ATDC04	*300*	-	-	-	*0.50*
	300	-	-	-	*0.75*
	300	-	-	-	*1.00*
BTMD01					
BTMD02					
BTMD03					
BTMD04					
BTT01					
BTT02					
BTT03					
BTT04					

3.1. Variation in mean diameter when number of loops in corrugation is kept constant

Based on the numerical data of the tubes, variations in the buckling load with the change in the mean diameter are depicted in Fig.2.

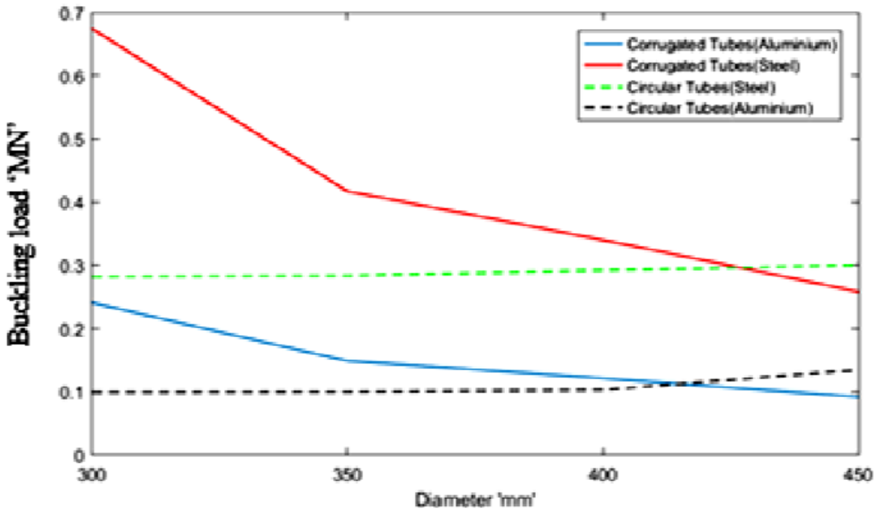

Fig.2. Effect of variations in the mean diameter when the number of loops in corrugation is kept constant.

Figure 2 compares corrugated tubes with circular tubes of two different materials namely steel and aluminium and also gives a view when there is a change in the mean diameter of circular and corrugated tubes of both the materials. The corrugated and circular tubes of steel have higher load bearing capacity than corrugated and circular tubes of aluminium, therefore the graph of corrugated and circular tubes of steel is higher. We can infer that the corrugated tubes of a material are better than circular tubes of the same material as from the graph buckling load capacity is higher of corrugated tubes than circular tubes. Further, it appears that the increase in the mean diameter of the corrugated tubes, when the number of loops in corrugation is kept constant, there is a decrease in the buckling load of the steel and aluminum tubes whereas in the case of circular tubes of both the materials no significant changes are observed.

3.2. Variation in the number of loops in corrugation when mean diameter is kept constant

As shown in Fig.3, an increase in the buckling load occurs when there is an increase in the number of loops in corrugation for the axially compressed tubes when the mean diameter is kept constant. It is noted here that initially, the buckling load of circular tubes was higher but eventually with the increase in the number of loops in corrugation there is a remarkable rise in the buckling load of the corrugated tubes. Moreover, there is hardly any change in the circular tubes for both the materials, therefore, their curve is almost a straight line . Hence, we can conclude that the steel tubes have higher load- bearing capacity than aluminium tubes.

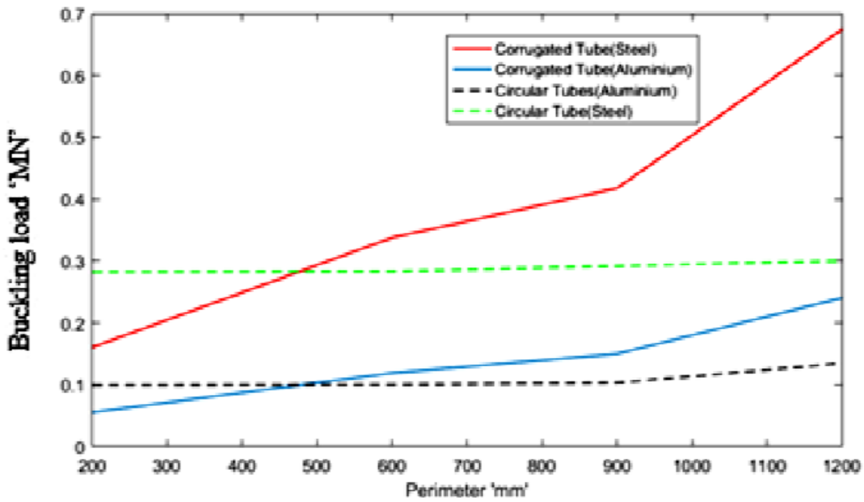

Fig.3. Effect of variation in the perimeter (i.e. number of loops in corrugation) when the mean diameter is kept constant.

3.3. Variation in the pitch of tubes

Variations in the buckling load when the pitch is altered in tubes are summarized in Fig.4. Incidentally, it is observed that corrugated tubes are more effective while changes are made in the perimeter of the tube by varying the values of pitch in the corresponding tubes. Furthermore, the results shown in Fig.4 indicate that no consequential effect is observed in circular tubes as compared to the corrugated tubes. The potential of steel tubes is better than aluminum tubes.

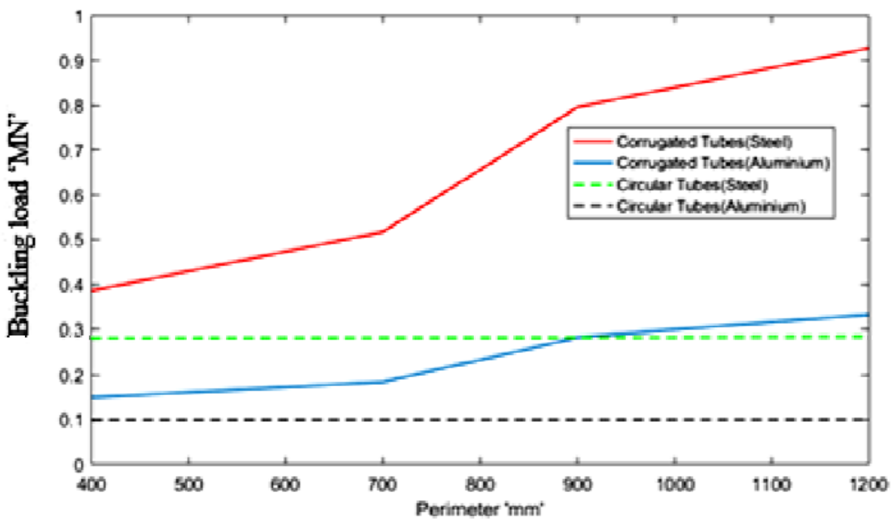

Fig.4. Effect of variation in the perimeter (i.e., pitch) of tubes.

3.4. Variation in the amplitude/depth of tubes

The buckling load of the tubes of two different cross-sections is plotted in Fig.5 with respect to the change in the perimeter whilst varying the pitch of the respective tubes. As seen in Fig.5, there is barely any noteworthy improvement in the buckling load of the corresponding tubes. Moreover, the rapid change in perimeter in pitch gives no remarkable change in the curve of the buckling load. It is inferred that though no significant variation is observed in the curve, still the steel tubes have shown a better result.

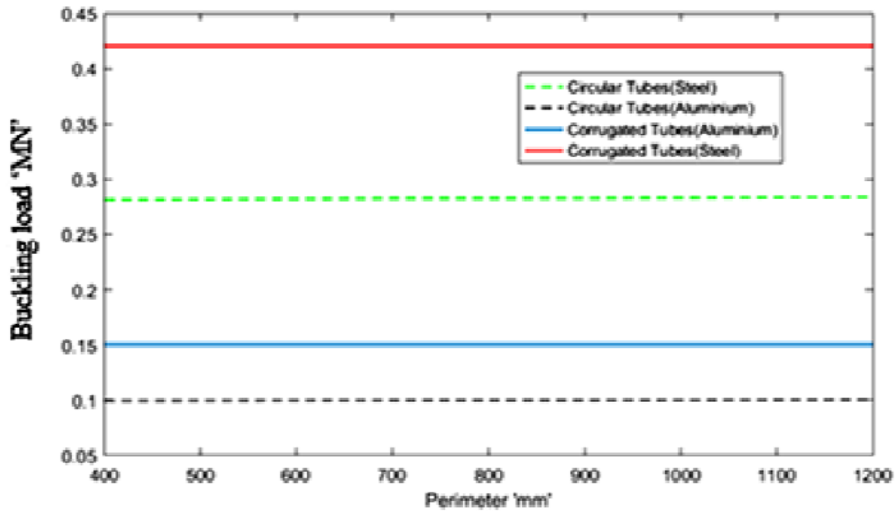

Fig.5. Effect of variation in the perimeter (i.e., amplitude/depth) of tubes.

3.5. Variation in thickness of the tubes

A comparison of the buckling load of corrugated and circular tubes is presented in Fig.6 and the parameter considered this time is the thickness of the tubes. It is evident that the buckling load increases with the increase in the thickness of the tubes whilst with the use of corrugation the buckling load of the tubes significantly increases and the performance of steel tubes is better than the aluminum ones.

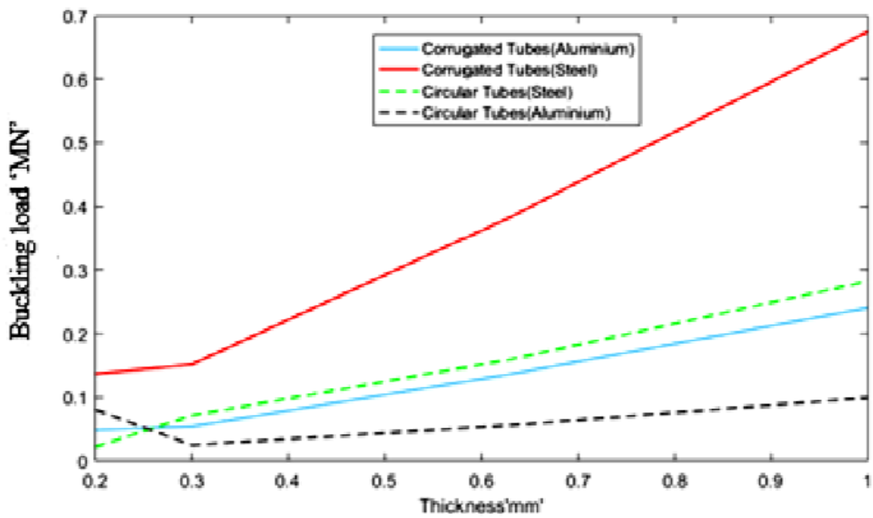

Fig.6 Effect of variation in thickness of the tubes.

4. Conclusion

The behavior of corrugated and circular tubes under quasi-static axial compression has been studied. The results of finite element models have been used for a comparative study of various parameters of the circular and corrugated tubes. Following validation of the models, the evaluation of tubular structures is greatly influenced with the mean diameter and perimeter variation. The changes in the perimeter are studied comprehensively by altering the pitch, depth, number of loops in corrugation and thickness of tubes.
The buckling load of hollow sections largely depends on the cross-section of the corresponding tubes. The buckling load are easily enhanced when switched from circular cross section to corrugated as seen in this study. On comparing the cross-sections of circular and corrugated tubes, it is inferred that the buckling load of the corrugated tubes is better than circular tubes. Furthermore, it was also contemplated observed that the tubes of steel have a better potential than aluminum tubes.

Certainly, various kinds of cross-sections such as trapezoidal, triangular, square corrugations, etc., as well as materials including composites can be evaluated by the proposed set of parameters and key performance indicators (KPIs), so as to guide their design and archive a certain degree of optimization.

Nomenclature

AT – arc-tangent corrugated tubes
BT – circular tubes
DC – depth of corrugation
MD – mean diameter
NC – number of loops in corrugation
PC – pitch of corrugation
T – thickness of the tubes

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