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EIGEN VALUE APPROACH TO GENERALIZED THERMOELASTIC INTERACTIONS IN AN UNBOUNDED BODY WITH CIRCULAR CYLINDRICAL CAVITY WITHOUT ENERGY DISSIPATION

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The theory of generalized thermoelasticity in the context of the Green-Naghdi model –II (thermoelasticity without energy dissipation) is studied for an infinite circular cylindrical cavity subjected to two different cases of thermoelastic interactions when the radial stress is zero for (a) maintaining constant temperature and (b) temperature is varying exponentially with time. The Laplace transform from time variable is used to the governing equations to formulate a vector matrix differential equation which is then solved by the eigen value approach. Numerical computations for the displacement component, temperature distribution and components of thermal stress have been made and presented graphically.

Key words: eigen value approach, Generalized thermoelasticity, Laplace transform and vector- matrix differential equation.

1. Introduction

The classical theory of coupled thermoelasticity, first developed by Biot [1], does not contain any elastic term and also is of parabolic type. Two contradictions are raised:
(1) Elastic changes produce thermal effects and
(2) the heat conduction equation is of hyperbolic type predicting finite speed of heat propagation.

To overcome this paradox, the conventional classical theory had been modified without violating Fourier's law of heat conduction equation which is called generalized thermoelasticity. Now, two different models of generalized thermoelasticity are mainly used. One is the Lord and Shulman [2] (L-S) theory, also known as extended thermoelasticity (ETE) and the other is the Green and Lindsay [3] (G-L) theory. The heat conduction equation associated with the L-S theory is of hyperbolic type, just introducing one thermal relaxation time parameter to the heat conduction equation without violating conventional Fourier's law, whereas the G-L theory modified not only the heat conduction equation but also the equation of motion in coupled theory introducing two relaxation time parameters. Three models (Model I, II and III) were developed by Green and Naghdi [5], [6], [7] for generalized thermoelasticity concerned to the theory of thermoelasticity with or without energy dissipation. This theory is called temperature rate-dependent thermoelasticity (TRDTE). These theories obviously removed the paradox of conventional classical theory of thermoelasticity.

Most of the problems of the thermoelasticity (classical, coupled, or generalized) have been solved by different ways such as:

(i) Potential function approach: In this approach, the boundary and initial conditions for physical problems are directly related to the physical quantities rather than the potential function. The solution of the physical consideration is convergent, while the potential function representations is not always convergent.

(ii) State-space approach: This is essentially an expansion in a series in terms of the coefficient matrix of the field variables in ascending powers, which is the extensive application of the Cayley–Hamilton theorem.

(iii) Eigenvalue approach: This method reduces the problem of a vector-matrix differential equation to an algebraic eigenvalue problem and the solutions for the resulting field equations are determined by solving these vector-matrix differential equations, which is the direct application of eigenvalues and the corresponding eigenvectors of the coefficient matrix. In the eigenvalue approach, the physical quantities such as material constants are directly involved in the formulation of the problem and as such the boundary and initial conditions can be applied directly.

Lahiri and Das [8] studied the problem of generalized thermoelastic interactions in an unbounded body with a circular cylindrical cavity without energy dissipation. Othman and Shulman [9] investigated the theory under the dependence of the modulus of elasticity on the reference temperature in two-dimensional generalized thermoelasticity.

The present investigation is devoted to study the problem of generalized thermoelastic interactions in an unbounded body with a circular cylindrical cavity without energy dissipation. The surface of the cavity is subjected to (a) maintaining constant temperature and (b) temperature is varying exponentially with time. The Laplace transform from time variable is used to the governing equations to formulate a vector matrix differential equation which is then solved by the eigen value approach. Numerical computations for the displacement component, temperature distribution and components of thermal stress have been made and presented graphically to analyse the different parameters used in this problem.

2. Basic-equation and formulation of the problem

We consider a homogeneous isotropic thermally conducting infinite elastic medium with a circular cylindrical cavity with radius ‘a’ at uniform absolute temperature θ_0 in an undisturbed state. Let the body forces and heat source be absent. The field equations for the linear generalized theory of thermoelasticity developed by Green and Naghdi are

$$(\lambda+\mu)\nabla \mathrm{div}\, u+\mu\nabla^2 u-\gamma\nabla\theta=\rho\ddot{u}\,, \tag{2.1}$$

$$\rho c\ddot{u}\theta+\gamma\theta_0 \mathrm{div}\,\ddot{u}=K\nabla^2\theta+K^*\nabla^2\theta\,. \tag{2.2}$$

Reducing Eqs (2.1) and (2.2) in cylindrical polar co-ordinates (r,ϕ,z) of any point of the body at any time t and assuming the displacement vector possesses only the displacement component $u=u(r,t)$, where r is the radial distance measured from the origin, we get

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}-\xi\alpha_0\frac{\partial\theta}{\partial r}=\frac{1}{\alpha^2}\frac{\partial^2 u}{\partial t^2}\,, \tag{2.3}$$

$$\frac{\partial^2\theta}{\partial t^2}+\frac{\varepsilon}{\xi\alpha_0}\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)=\frac{k}{\rho c}\frac{\partial}{\partial t}\left(\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}\right)+\frac{k^*}{\rho c}\left(\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}\right). \tag{2.4}$$

The stress components are given by

$$\sigma_r=(\lambda+2\mu)\frac{\partial u}{\partial r}+\lambda\frac{u}{r}-\gamma\theta, \tag{2.5}$$

$$\sigma_\theta = \lambda\frac{\partial u}{\partial r} + (\lambda + 2\mu)\frac{u}{r} - \gamma\theta. \tag{2.6}$$

The non-dimensional form of equations of motion (2.1) and the heat conduction equation of generalized thermoelasticity (2.2) and stresses are

$$\frac{\partial^2 U}{\partial R^2} + \frac{1}{R}\frac{\partial U}{\partial R} - \frac{U}{R^2} - \frac{\partial T}{\partial R} = \beta^2\frac{\partial^2 U}{\partial \tau^2}, \tag{2.7}$$

$$\frac{\partial^2 T}{\partial \tau^2} + \varepsilon\frac{\partial^2}{\partial \tau^2}\left(\frac{\partial U}{\partial R} + \frac{U}{R}\right) = \frac{\partial}{\partial \tau}\left(\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R}\right)T + B^*\left(\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R}\right)T, \tag{2.8}$$

$$\sigma_{RR} = \frac{\partial U}{\partial R} + \eta\frac{U}{R} - T, \tag{2.9}$$

$$\sigma_{\Theta\Theta} = \eta\frac{\partial U}{\partial R} + \frac{U}{R} - T \tag{2.10}$$

where the non- dimensional variables are

$$\begin{gathered} R = \frac{r}{a}, \qquad \tau = \frac{k}{\rho c a^2}t, \qquad U = \frac{(\lambda + 2\mu)}{a\gamma\theta_0}u, \qquad \beta^2 = \frac{k^2}{a^2\rho c \alpha^2}, \\ \sigma_{RR} = \frac{1}{\gamma\theta_0}\sigma_r, \qquad \sigma_{\Theta\Theta} = \frac{1}{\gamma\theta_0}\sigma_\theta, \qquad T = \frac{\theta}{\theta_0}, \qquad W = \frac{a^2\rho c}{k}\omega. \end{gathered} \tag{2.11}$$

3. Solution procedure

3.1. Formulation of the vector –matrix differential equation

We now apply the Laplace transform of the form

$$\bar{U}(R,p) = \int_0^\infty \bar{U}(R,t)\exp(-pt)\,dt, \qquad \bar{T}(R,p) = \int_0^\infty \bar{T}(R,t)\exp(-pt)\,dt, \tag{3.1}$$

to Eqs (2.7) and (2.8) to obtain

$$\frac{d^2\bar{U}}{dR^2} + \frac{1}{R}\frac{d\bar{U}}{dR} - \frac{\bar{U}}{R^2} - \frac{d\bar{T}}{dR} = \beta^2 p^2 \bar{U}, \tag{3.2}$$

$$p^2\bar{T} + \varepsilon p^2\left(\frac{d\bar{U}}{dR} + \frac{\bar{U}}{R}\right) = p\left(\frac{d^2\bar{T}}{dR^2} + \frac{1}{R}\frac{d\bar{T}}{dR}\right) + B^*. \tag{3.3}$$

Since at time $t = 0$, the body is at rest in an undeformed and unstressed state and is maintained at the reference temperature, so the following conditions hold

$$U\left(R,0\right)=\frac{\partial U(R,0)}{\partial t}=0, \qquad T\left(R,0\right)=\frac{\partial T(R,0)}{\partial t}=0 \ . \tag{3.4}$$

As in Das and Bhakta [4] Eqs (3.2) and (3.3) can be written as

$$L\underline{V}=A\underline{V} \tag{3.5}$$

where

$$L=\frac{d^2}{dR^2}+\frac{1}{R}\frac{d}{dR}-\frac{1}{R^2} \ ,$$

$$\underline{V} \ \ \underline{V}=\left[\bar{U},\frac{d\bar{T}}{dR}\right]^T$$

The matrix $\underline{A}$ is

$$\underline{A}=\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

where
$$C_{11}=\beta^2 p^2, \qquad C_{12}=1, \qquad C_{21}=\frac{\varepsilon\beta^2 p^4}{p+B^*}, \qquad C_{22}=\frac{p^2(1+\varepsilon)}{1+B^*} \ . \tag{3.6}$$

Following the method of Lahiri *et al.* (2009)[Appendix I] we substitute

$$\underline{V}(R)=\underline{X}\left(\alpha\right)J_1(\alpha R)\,, \tag{3.7}$$

in Eq.(3.5).

This leads to the algebraic eigen value problem

$$\underline{A}\,\underline{X}\left(\alpha\right)=\lambda\underline{X}\left(\alpha\right):\lambda=-\alpha^2 \tag{3.8}$$

where $\underline{X}\left(\alpha\right)$ is a scalar function of α .

3.2. Solution of the vector –matrix differential equation

The characteristic equation corresponding to the matrix $\underline{A}$ is given by

$$\lambda^2-\lambda\left(c_{11}+c_{22}\right)+c_{11}c_{22}-c_{12}c_{21}=0\,. \tag{4.1}$$

The roots of the characteristic equation are also the eigen values of the matrix $\underline{A}$ and are as follows

$$\lambda=\lambda_1 \qquad \text{and} \qquad \lambda=\lambda_2$$

where

$$\lambda_1+\lambda_2=c_1+c_2, \qquad \lambda_1\lambda_2=c_{11}c_{22}-c_{12}c_{21}. \tag{4.2}$$

The eigen vectors $X=\left[x_1,\, x_2\right]^T$ corresponding to the eigen values λ can be calculated as

$$X = \begin{bmatrix} C_{22} - \lambda \\ -C_{21} \end{bmatrix} . \tag{4.3}$$

From Eq.(4.3) we can easily calculate the eigen vectors X_i corresponding to the eigen values $\lambda = \lambda_i$.

For our further reference, we shall use the following notations

$$X_i = [X]_{\lambda=\lambda_i} \quad \text{for} \quad i = 1,2 . \tag{4.4}$$

Then the solution of Eq.(3.5) can be written as

$$\underline{V}(R) = C_1 \underline{X}_1 J_1(\alpha_1 R) + C_2 \underline{X}_2 J_1(\alpha_2 R) \tag{4.5}$$

where $\alpha_i^2 = - \lambda_i$, $i = 1,2$ and C_1 and C_2 are arbitrary contants to be determined from the boundary conditions.

The components of the space vector $\underline{V}(r)$ in Eq.(3.5) can be written as

$$\bar{U}(R,p) = -C_1 J_1(\alpha_1 R) - C_2 J_1(\alpha_2 R), \tag{4.6}$$

$$\frac{d\bar{T}}{dR} = C_1\left(C_{11} + \alpha_1^2\right) J_1(\alpha_1 R) + C_2\left(C_{11} + \alpha_2^2\right) J_1(\alpha_2 R), \tag{4.7}$$

$$\bar{T} = -\frac{C_1\left(C_{11} + \alpha_1^2\right)}{\alpha_1} J_0(\alpha_1 R) - \frac{C_2\left(C_{11} + \alpha_2^2\right)}{\alpha_2} J_0(\alpha_2 R). \tag{4.8}$$

Taking the Laplace transform of Eqs (2.9) and (2.10) and using Eqs (4.6) and (4.8) we get

$$\bar{\sigma}_{RR} = C_1\left[\frac{C_{11}}{\alpha_1} J_0(\alpha_1 R) + \frac{1-\eta}{R} J_1(\alpha_1 R)\right] + C_2\left[\frac{C_{11}}{\alpha_2} J_0(\alpha_2 R) + \frac{1-\eta}{R} J_1(\alpha_2 R)\right], \tag{4.9}$$

$$\begin{aligned}\bar{\sigma}_{\Theta\Theta} = {} & C_1\left[\frac{C_{11}}{\alpha_1} J_0(\alpha_1 R) - (\eta - 1)\alpha_1 J_0(\alpha_1 R) + \frac{\eta - 1}{R} J_1(\alpha_1 R)\right] + \\ & + C_2\left[\frac{C_{11}}{\alpha_2} J_0(\alpha_2 R) - (\eta - 1)\alpha_2 J_0(\alpha_2 R)\frac{\eta - 1}{R} J_1(\alpha_2 R)\right],\end{aligned} \tag{4.10}$$

C_1 and C_2 are arbitrary constants which are to be determined from the following boundary conditions.

Case (a)

We now consider a problem of thermoelastic interactions when the radial stress is zero and temperature is constant.

The boundary conditions at the surface of the cylinder $r = a$ in a dimensional form are given by

$$\sigma_{rr} = 0 \quad , \quad \theta = \theta_0 . \tag{4.11}$$

Using non-dimensional variables from Eq.(2.10) , we get

$$\sigma_{RR} = 0, \quad T = 1 \quad \text{at} \quad R = 1 . \tag{4.12}$$

Taking the Laplace transform of Eq.(4.12) and using Eqs (4.8) and (4.9) we can get the values of C_1 and C_2. Putting these values of C_1 and C_2 we get $\bar{U}(R,p)$, $\bar{T}$, $\bar{\sigma}_{RR}$, $\bar{\sigma}_{\Theta\Theta}$ in the Laplace transform domain for the above case.

Taking the Laplace transform of Eqr (4.12), we get

$$\bar{\sigma}_{RR} = 0,$$

$$\Rightarrow C_1\left[\frac{C_{11}}{\alpha_1}J_0(\alpha_1 R)+\frac{1-\eta}{R}J_1(\alpha_1 R)\right]+C_2\left[\frac{C_{11}}{\alpha_2}J_0(\alpha_2 R)+\frac{1-\eta}{R}J_1(\alpha_2 R)\right]=0.$$

Again $T=\dfrac{\theta}{\theta_0}$. Since $\theta=\theta_0$ we get $T=1$.

So, $$\bar{T}=\int_0^\infty 1.e^{-pt}dt=\frac{1}{p},$$

$$\Rightarrow \frac{1}{p}=-\frac{C_1\left(C_{11}+\alpha_1^2\right)}{\alpha_1}J_0(\alpha_1)-\frac{C_2\left(C_{11}+\alpha_2^2\right)}{\alpha_2}J_0(\alpha_2).$$

Now substituting $C_1=-\dfrac{\left[\dfrac{C_{11}}{\alpha_2}J_0(\alpha_2)+\dfrac{1-\eta}{R}J_1(\alpha_2)\right]}{\left[\dfrac{C_{11}}{\alpha_1}J_0(\alpha_1)+\dfrac{1-\eta}{R}J_1(\alpha_1)\right]}C_2$ in

$$\frac{1}{p}=-\frac{C_1\left(C_{11}+\alpha_1^2\right)}{\alpha_1}J_0(\alpha_1)-\frac{C_2\left(C_{11}+\alpha_2^2\right)}{\alpha_2}J_0(\alpha_2)$$

we get

$$C_2=\frac{\left[\dfrac{C_{11}}{\alpha_1}J_0(\alpha_1)+(1-\eta)J_1(\alpha_1)\right]}{p\left[\begin{array}{l}\dfrac{C_{11}\left(C_{11}+\alpha_1^2\right)}{\alpha_1\alpha_2}J_0(\alpha_1)J_0(\alpha_2)+(1-\eta)J_1(\alpha_2)J_0(\alpha_1)\\ -\dfrac{C_{11}\left(C_{11}+\alpha_2^2\right)}{\alpha_1\alpha_2}J_0(\alpha_1)J_0(\alpha_2)-\dfrac{\left(C_{11}+\alpha_2^2\right)(1-\eta)}{\alpha_2}J_0(\alpha_2)J_1(\alpha_1)\end{array}\right]}=$$

$$=\frac{\left[\dfrac{C_{11}}{\alpha_1}J_0(\alpha_1)+(1-\eta)J_1(\alpha_1)\right]}{p\left[\begin{array}{l}\dfrac{C_{11}\left(\alpha_1^2-\alpha_2^2\right)}{\alpha_1\alpha_2}J_0(\alpha_1)J_0(\alpha_2)+(1-\eta)J_1(\alpha_2)J_0(\alpha_1)\\ -\dfrac{\left(C_{11}+\alpha_2^2\right)(1-\eta)}{\alpha_2}J_0(\alpha_2)J_1(\alpha_1)\end{array}\right]}.$$

Hence

$$C_1 = \frac{\left[\dfrac{C_{11}}{\alpha_2} J_0(\alpha_2) + (1-\eta) J_1(\alpha_2)\right]}{p\left[\begin{array}{l} \dfrac{C_{11}\left(\alpha_1^2 - \alpha_2^2\right)}{\alpha_1 \alpha_2} J_0(\alpha_1) J_0(\alpha_2) + (1-\eta) J_1(\alpha_2) J_0(\alpha_1) \\ \qquad - \dfrac{\left(C_{11} + \alpha_2^2\right)(1-\eta)}{\alpha_2} J_0(\alpha_2) J_1(\alpha_1) \end{array}\right]}.$$

Case (b)

If we take $T = e^{i\omega t}, \sigma_r = 0, r = 1$ we get

$$R = \frac{1}{a}, \quad T = \frac{\theta}{\theta_0} \Rightarrow \theta = \theta_0 e^{i\omega t},$$

$$\sigma_{RR} = \frac{1}{\gamma\theta_0}\sigma_r = 0 \qquad \text{at} \qquad r = 1,$$

$$\Rightarrow \bar{\sigma}_{RR} = 0,$$

$$\Rightarrow C_1\left[\frac{C_{11}}{\alpha_1} J_0(\alpha_1 R) + \frac{1-\eta}{R} J_1(\alpha_1 R)\right] + C_2\left[\frac{C_{11}}{\alpha_2} J_0(\alpha_2 R) + \frac{1-\eta}{R} J_1(\alpha_2 R)\right] = 0,$$

$$\Rightarrow C_1\left[\frac{C_{11}}{\alpha_1} J_0\left(\frac{\alpha_1}{a}\right) + a(1-\eta) J_1\left(\frac{\alpha_1}{a}\right)\right] + C_2\left[\frac{C_{11}}{\alpha_2} J_0\left(\frac{\alpha_2}{a}\right) + a(1-\eta) J_1\left(\frac{\alpha_2}{a}\right)\right] = 0,$$

$$\Rightarrow C_1 = -\frac{\left[\dfrac{C_{11}}{\alpha_2} J_0\left(\dfrac{\alpha_2}{a}\right) + a(1-\eta) J_1\left(\dfrac{\alpha_2}{a}\right)\right]}{\left[\dfrac{C_{11}}{\alpha_1} J_0\left(\dfrac{\alpha_1}{a}\right) + a(1-\eta) J_1\left(\dfrac{\alpha_1}{a}\right)\right]} C_2 .$$

Again, $$\bar{T} = \int_0^\infty e^{i\omega t} e^{-pt}\, dt = \frac{p + i\omega}{p^2 + \omega^2},$$

$$\Rightarrow \frac{p + i\omega}{p^2 + \omega^2} = -\frac{C_1\left(C_{11} + \alpha_1^2\right)}{\alpha_1} J_0\left(\frac{\alpha_1}{a}\right) - \frac{C_2\left(C_{11} + \alpha_2^2\right)}{\alpha_2} J_0\left(\frac{\alpha_2}{a}\right) =$$

$$= C_2 \frac{\left[\dfrac{C_{11}}{\alpha_2} J_0\left(\dfrac{\alpha_2}{a}\right) + a(1-\eta) J_1\left(\dfrac{\alpha_2}{a}\right)\right]}{\left[\dfrac{C_{11}}{\alpha_1} J_0\left(\dfrac{\alpha_1}{a}\right) + a(1-\eta) J_1\left(\dfrac{\alpha_1}{a}\right)\right]} \frac{\left(C_{11} + \alpha_1^2\right)}{\alpha_1} J_0\left(\frac{\alpha_1}{a}\right) - \frac{C_2\left(C_{11} + \alpha_2^2\right)}{\alpha_2} J_0\left(\frac{\alpha_2}{a}\right),$$

$$\Rightarrow C_2 = \frac{p+i\omega}{p^2+\omega^2}\frac{\left[\frac{C_{11}}{\alpha_1}J_0\left(\frac{\alpha_1}{a}\right)+a(1-\eta)J_1\left(\frac{\alpha_1}{a}\right)\right]}{\left[\begin{array}{l}\frac{\left(C_{11}+\alpha_1^2\right)}{\alpha_1}\left\{\frac{C_{11}}{\alpha_2}J_0\left(\frac{\alpha_2}{a}\right)+a(1-\eta)J_1\left(\frac{\alpha_2}{a}\right)\right\}J_0\left(\frac{\alpha_1}{a}\right)\\ -\frac{\left(C_{11}+\alpha_2^2\right)}{\alpha_2}\left\{\frac{C_{11}}{\alpha_1}J_0\left(\frac{\alpha_1}{a}\right)+a(1-\eta)J_1\left(\frac{\alpha_1}{a}\right)\right\}J_0\left(\frac{\alpha_2}{a}\right)\end{array}\right]},$$

$$C_1 = \frac{p+i\omega}{p^2+\omega^2}\frac{\left[\frac{C_{11}}{\alpha_2}J_0\left(\frac{\alpha_2}{a}\right)+a(1-\eta)J_1\left(\frac{\alpha_2}{a}\right)\right]}{\left[\begin{array}{l}\frac{\left(C_{11}+\alpha_1^2\right)}{\alpha_1}\left\{\frac{C_{11}}{\alpha_2}J_0\left(\frac{\alpha_2}{a}\right)+a(1-\eta)J_1\left(\frac{\alpha_2}{a}\right)\right\}J_0\left(\frac{\alpha_1}{a}\right)\\ -\frac{\left(C_{11}+\alpha_2^2\right)}{\alpha_2}\left\{\frac{C_{11}}{\alpha_1}J_0\left(\frac{\alpha_1}{a}\right)+a(1-\eta)J_1\left(\frac{\alpha_1}{a}\right)\right\}J_0\left(\frac{\alpha_2}{a}\right)\end{array}\right]}.$$

Putting these values of C_1 and C_2, we get $U(R, p)$, T, σ_{RR}, $\sigma_{\Theta\Theta}$ in the Laplace transform domain for the above cases.

4. Numerical solution

In order to invert the Laplace transforms in the preceeding equations for deformation and temperature distribution, we use the Zakian algorithm technique (1969), [Appendix I], in which the time function $f(t)$ is computed as a sum of weighted evaluation of $F(p)$ where

$$F(p) = \int_0^\infty e^{-pt} f(t)dt \ .$$

The development of the Zakian algorithm is given in Rice and Do [10]. A significant feature of the derivation is the specification that the time function can be related to a finite series of exponential functions. This signification of the Zakian algorithm is very accurate for overdamped and slightly undamped system.

For this purpose, we take copper as thermoelastic material and the values for the parameters are

$$\rho = 8954,\quad c = 383.1,\quad \alpha = 0.0001,\quad \gamma = 1,\quad \varepsilon = 0.0168,\quad \beta = 2,\quad \omega = 1.$$

5. Conclusion

We have constructed a vector matrix differential equation concerning necessary field variables required to solve the problem of deformations of a homogeneous isotropic thermally conducting infinite elastic medium with a circular cylindrical cavity in an undisturbed state ,in the absence of body forces and heat source by the eigen value approach method.

1. Figure 1 exhibits the variations of u and R for fixed values time and B^*, case(b), we observe that: The amplitudes of u gradually decrease as R increases.

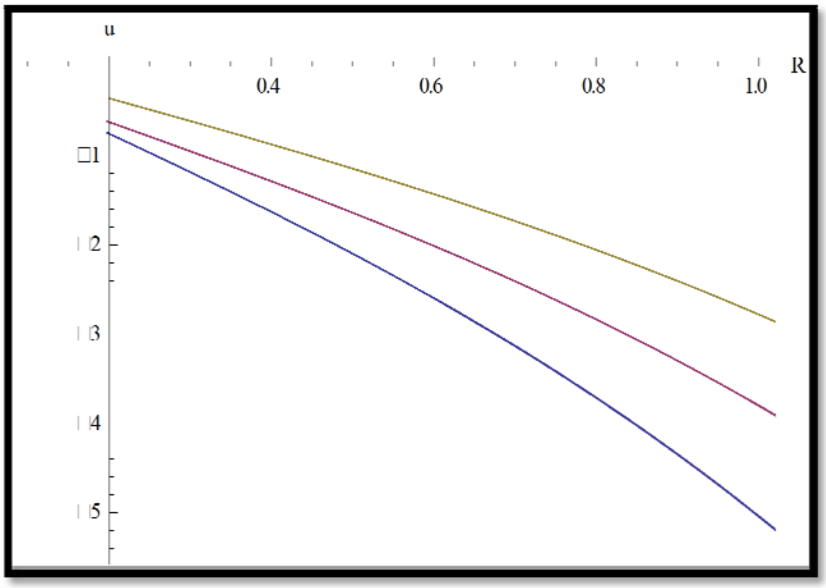

Fig.1. Distribution of u vs R for fixed time and B* = *1.25*, case (b).

2. Figure 2 exhibits the variations of stress and R for fixed values time and B^*, case(a), we observe that: The amplitudes of σ_{RR} gradually decrease as R increases.

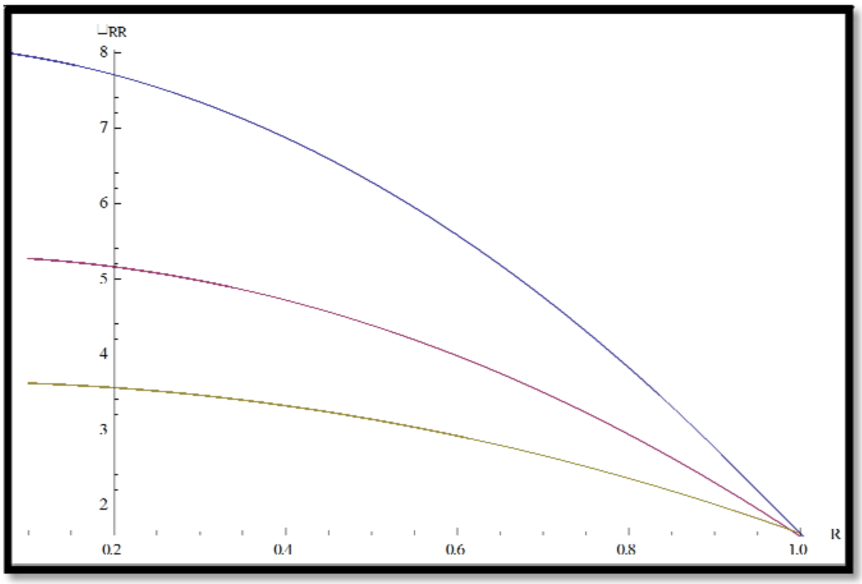

Fig.2. Distribution of σ_{RR} vs R for fixed time and B* = *1.25*, case (a).

3. Figure 3 depicts the variations of $\sigma_{\Theta\Theta}$ and B^* for fixed time, case(a), we observe that: The amplitudes of $\sigma_{\Theta\Theta}$ gradually decrease as R increases.

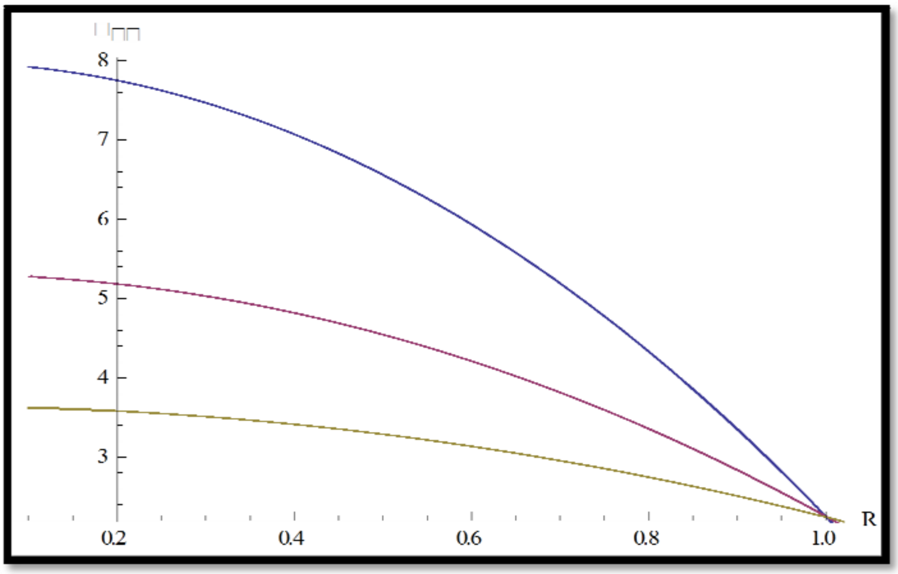

Fig.3. Distribution of $\sigma_{\Theta\Theta}$ vs R for fixed time and B* = *1.25*, case (a).

4. Figure 4 exhibits the variation of stresses and B^{*} for fixed time and *R*, case (a).

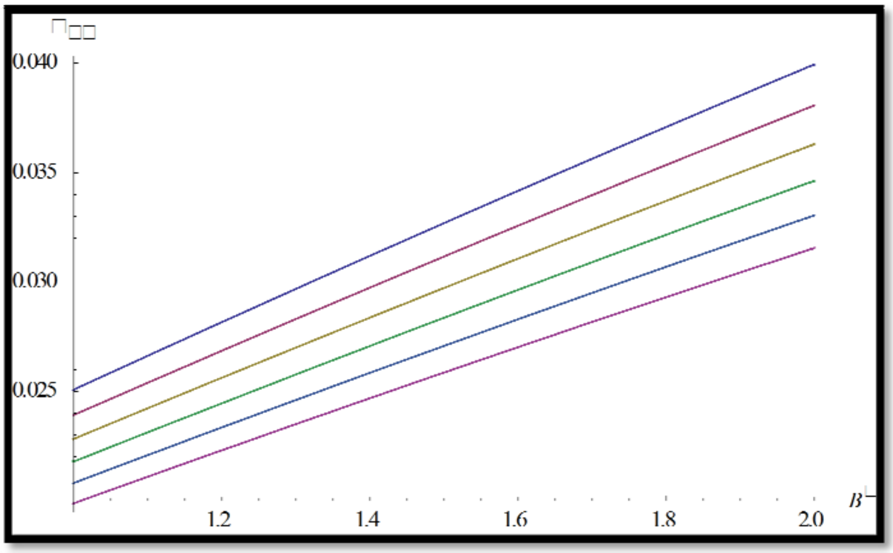

Fig.4. Distribution of $\sigma_{\Theta\Theta}$ vs B* for fixed time and R = *0.1*, case (a).

5. Figure 5 exhibits σ_{RR} vs *R* for fixed time and *B** = *1.25*, case (a).

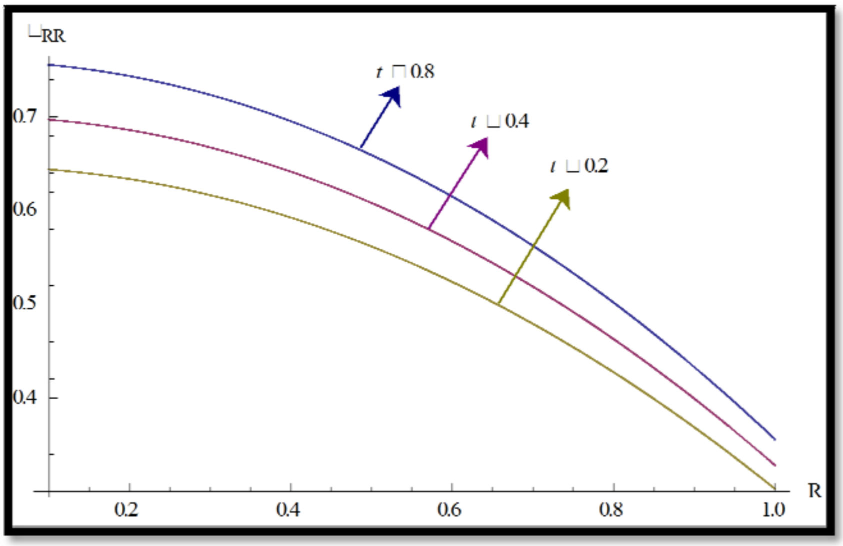

Fig.5. Distribution of σ_{RR} vs R for fixed time and B* = *1.25*, case (a).

6. Figure 6 exhibits distribution of σ_{RR} vs R for fixed time and $B^* = 1.25$, case (a).

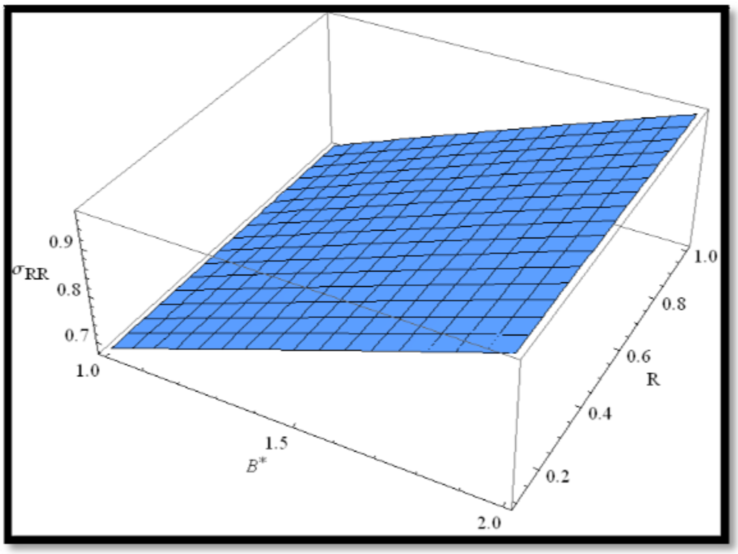

Fig.6. Distribution of σ_{RR} vs R for fixed time and B* = *1.25* ,case (a).

7. Figure 7 exhibits distribution of temperature for fixed values of ω and R ,case (b).

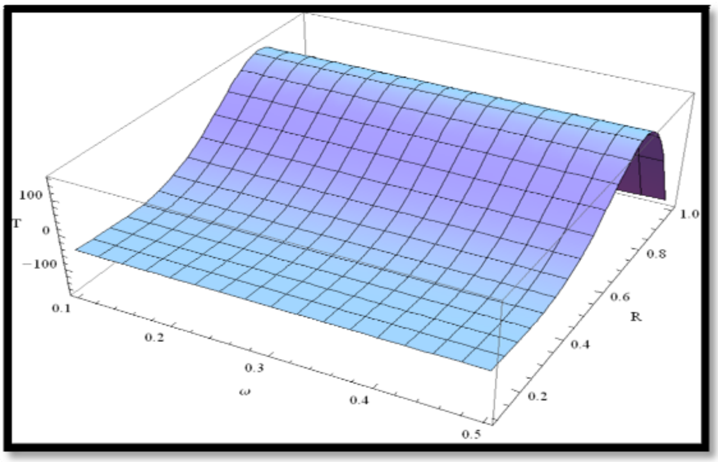

Fig.7. Distribution of temperature for different values of ω and R, case (b).

8. Figure 8 exhibits distribution of σ_{RR} for different values of B^* and ω ,case (b).

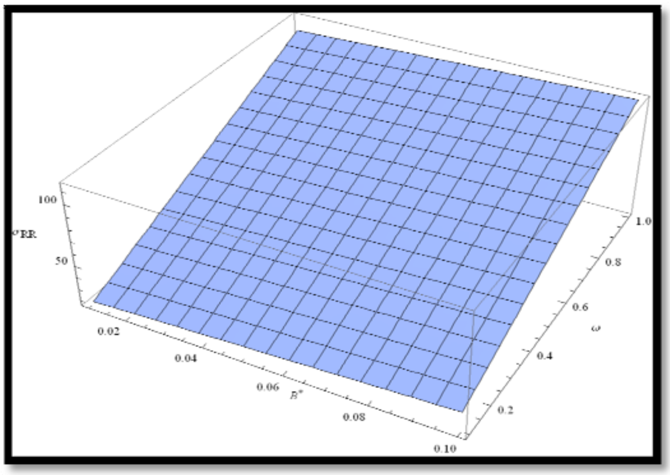

Fig.8. Distribution of σ_{RR} for different values of B* and ω ,case (b).

9. With Green and Naghdi 's parameter for the cases (a) and (b) for fixed values of *R* and time, we observe that: The amplitude gradually increases as Green and Neghdi's parameter increase and decrease as time increases.

Nomenclature

c − specific heat at constant deformation
div u − divergence of u
$J_n(\alpha)$ − Bessel function of order
k − thermal conductivity
k^* − parameter of Green and Neghdi's theory
p − Laplace transform parameter
u − displacement vector

α_0 – coefficient of volume expansion

$\gamma = (3\lambda + 2\mu)\alpha_0$

$\eta = \dfrac{\lambda}{\lambda + 2\mu}$

θ – temperature change above the reference temperature θ_0

λ, μ – Lame`s constants

ρ – mass density

τ – thermal relaxation parameter

$\nabla\phi$ – gradient of ϕ

Appendix I

Consider the differential equation in the form

$$L\underset{\sim}{v} = A\underset{\sim}{v} \qquad \text{where} \qquad L = \frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} - \frac{n^2}{x^2}. \tag{A.1}$$

This operator L is of frequent occurrence in problems on cylinders.

Let $$A = V\Lambda V^{-1}, \tag{A.2}$$

where $\Lambda = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$ is a diagonal matrix whose elements λ_1, λ_2, … λ_n are the distinct eigenvalues of A. Let $\underset{\sim}{V_1}$, $\underset{\sim}{V_2}$, ..., $\underset{\sim}{V_n}$ be the eigenvectors of A corresponding to λ_1, λ_2, …, λ_n respectively, and

$$V = \left[\underset{\sim}{V_1}, \quad \underset{\sim}{V_2}, \quad ....., \quad \underset{\sim}{V_n}\right] = (x_{ij}) \text{ (say)}; \; i, j = 1, 2, \ldots, n. \tag{A.3}$$

Substituting (A.2) in (A.1) and premultiplying by V^{-1}, we get

$$L\underset{\sim}{y} = \Lambda\underset{\sim}{y}, \qquad \text{where} \quad \underset{\sim}{y} = V^{-1}\underset{\sim}{v}, \tag{A.4}$$

as a system of decoupled equations.

A typical r^{th} Eq. of (A.4) is

$$Ly_r = \lambda_r y_r,$$

or, $$\frac{d^2 y_r}{dx^2} + \frac{1}{x}\frac{dy_r}{dx} - \left(\lambda_r + \frac{n^2}{x^2}\right)y_r = 0. \tag{A.5}$$

Case (i)

When $\lambda_r = \alpha_r^2$, the solution of Eq.(A.5) can be written as

$$y_r = A_r K_n(\alpha_r x) + B_r I_n(\alpha_r x), \tag{A.6}$$

n is an integer and A_r, B_r are constants. K_n, I_n are modified Bessel functions of the second kind of order n.

Case (ii)

When $\lambda_r = -\alpha_r^2$, the solution can be written as

$$y_r = A_r J_n(\alpha_r x) + B_r y_n(\alpha_r x),\ n \text{ is integral} \tag{A.7}$$

J_n, y_n are Bessel functions of the first kind of order n.

Hence the complete solution in this case can be written as $\underset{\sim}{v} = \sum_{r=1}^{n} V_r\, y_r$.

APPENDIX II

Numerical inversions of the Laplace transform:
Let the Laplace transform $F(p)$ of $u(t)$ be given by

$$F(p) = \int_0^\infty e^{-pt} u(t)\, dt, \qquad p \geq 0 \qquad\qquad (p = \text{transform parameter}) \tag{A.8}$$

For the Laplace inversion we use here the Zakian algorithm.

Zakian Algorithm

This algorithm is one of a class of algorithms in which $f(t)$ is computed as a sum of weighted evaluations of $F(p)$

$$f(t) = \sum_{i=1}^{N} K_i F(p_i)$$

where the values of K_i, p_i and N are dictated by a particular method. The development of Zakian's algorithm is given in Rice and Do [10] as well as in Zakian's original paper. A significant feature of the derivation is the specification that the time function can be related to a finite series of exponential functions

$$\sum_{i=1}^{N} K_i e^{\alpha_i t} .$$

This significance of this specification is that Zakian's algorithm is very accurate for over damped and slightly underdamped systems. But it is not accurate for systems with prolonged oscillations.

Given $F(p)$ and a value of time t, the following equation implements Zakian's algorithm and allows us to calculate the numerical value of $f(t)$.

$$f(t)=\frac{2}{t}\sum_{i=1}^{5} REAL\left(K_i F\left(\frac{\alpha_i}{t}\right)\right).$$

Table 1 gives the set of five complex constants for α_i and for K_i.

Table 1. Set of five constants for α_i and K for the Zakian method.

i	α_i	K_i
1	*12.83767675 +**i**1.666063445*	*-36902.08210 +**i**196990.4257*
2	*12.22613209 +**i**5.012718792*	*+61277.02524–**i**95408.62551*
3	*10.93430308 +**i**8.409673116*	*-28916.56288 +**i**18169.18531*
4	*8.776434715 +**i**11.92185389*	*+4655.361138 -**i**1.901528642*
5	*5.225453361 +**i**15.72952905*	*-118.7414011 -**i**141.3036911*

Zakian's algorithm is simple to implement and computes quickly. But note that the initial value, $f(t)$ at $t = 0$, cannot be computed. Also, when there are oscillatory systems, $f(t)$ becomes inaccurate after approximately the second cycle.

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