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ON THERMAL INSTABILITY OF KUVSHINISKI FLUID WITH SUSPENDED PARTICLES SATURATED IN A POROUS MEDIUM IN THE PRESENCE OF A MAGNETIC FIELD

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The thermal instability of a Kuvshiniski viscoelastic fluid is considered to include the effects of a uniform horizontal magnetic field, suspended particles saturated in a porous medium. The analysis is carried out within the framework of the linear stability theory and normal mode technique. For the case of stationary convection, the Kuvshiniski viscoelastic fluid behaves like a Newtonian fluid and the magnetic field has a stabilizing effect, whereas medium permeability and suspended particles are found to have a destabilizing effect on the system, oscillatory modes are introduced in the system, in the absence of these the principle of exchange of stabilities is valid. Graphs in each case have been plotted by giving numerical values to the parameters, depicting the stability characteristics. Sufficient conditions for the avoidance of overstability are also obtained.

Key words: thermal convection, Kuvshiniski fluid, suspended particles, magnetic field and porous medium.

1. Introduction

Thermal convection in an electrically conducting layer of Newtonian fluids in the presence of a uniform magnetic field has been treated in detail by Chandrasekhar [1]. Scanlon and Segel [2] considered the effect of suspended particles on the onset of Benard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure fluid was supplemented by that of the particles. The derivation of the basic equations of a layer of fluid heated from below in a porous medium, using the Boussinesq approximation, has been given by Joseph [3]. The study of a layer of a fluid heated from below in porous media is motivated both theoretically and by practical applications in engineering disciplines. Among the applications in engineering disciplines one can find the food processing industry, chemical processing industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in a porous medium. There has been considerable interest in recent years in the study of the breakdown of the instability of a layer of a fluid subjected to a vertical temperature gradient in a porous medium and the possibility of convective flow. The stability of a flow of a single component fluid through a porous medium taking into account Darcy's resistance has been considered by Lapwood [4] and Wooding [5]. Darcy's equation describes the incompressible flow of a Newtonian fluid of viscosity μ through a macroscopically homogeneous and isotropic porous medium of permeability k_1. If $\boldsymbol{v}$ is the filter velocity of the fluid, the resistance term $-\left(\frac{\mu}{k_1}\right)\boldsymbol{v}$ replaces the usual viscous term in the equations of fluid motion. There is mounting evidence, both theoretical and experimental, that suggests that Darcy's equation sometimes provides an unsatisfactory description of the hydrodynamic conditions, particularly near boundaries of a porous medium. Beavers et al. [6] demonstrated experimentally the existence of shear within the porous medium near a surface where the porous medium is exposed to a freely flowing fluid, thus forming a zone of shear- induced fluid flow. Since viscoelastic fluids play an important role in polymers and electrochemical industry, the studies on waves and stability in different viscoelastic fluid dynamical configurations have been carried out by several researchers.

The present paper attempts to study thermal instability of a Kuvshiniski viscoelastic fluid with suspended particles in hydromagnetics in a porous medium. Chaudhary and Singh [7] considered the flow of a dusty viscoelastic (Kuvshiniski-type) fluid down an inclined plane. The effect of a magnetic field on the flow of a dusty viscoelastic (Kuvshiniski-type) fluid down an inclined plane was studied by Johari and Gupta [8]. Varshney and Dwivedi [9] studied the unsteady effect on MHD free convection and mass transfer flow of a Kuvshiniski fluid through a porous medium with constant suction and constant heat and mass flux. Kumar and Singh [10] studied a viscoelastic fluid heated from below in a porous medium and found that a Kuvshiniski fluid behaves like a Newtonian fluid in a stationary convection; rotation has a stabilizing effect, whereas medium permeability has both stabilizing and destabilizing effects. Also, Kumar and Singh [11] studied thermal instability of a Kuvshiniski viscoelastic fluid with fine dust in a porous medium and found that for a stationary convection medium permeability and suspended particles have destabilizing effects on the systems. Sharma and Sharma [12] studied adusty viscoelastic flow in a slip flow regime; the solution of the equations governing the flow was derived with the help of the Laplace transform technique. It was found that the effect of slip flow regime on the velocity fields is to increase them near the plate significantly which reduces the skin friction at the lower plate. Prakash *et al*. [13] studied MHD free convective flow of a viscoelastic (Kuvshiniski-type) dusty gas through a porous medium induced by the motion of a semi-infinite flat plate under the influence of radiative heat transfer moving with velocity decreasing exponentially with time. Kumar [14] studied magneto-rotatory stability of a two stratified fluid layers of a Kuvshiniski viscoelastic superposed fluid in a porous medium. Kumar and Kumar also [15] studied the effect of the magnetic field on an incompressible (Kuvshiniski-type) viscoelastic rotating fluid heated from below through a porous medium.

The knowledge regarding fluid-particle mixtures is not commensurate with their scientific and industrial importance. The analysis would be relevant to the stability of Kuvshiniski viscoelastic fluids. The present paper attempts to study the thermal instability of a Kuvshiniski viscoelastic fluid with suspended particles saturated in a porous medium in the presence of a magnetic field.

2. Formulation of the problem

Here we consider an infinite horizontal layer of an electrically conducting Kuvshiniski viscoelastic fluid permeated with suspended particles and bounded by the planes $z = 0$ and $z = d$ in a porous medium. The pressure p and density ρ are functions of the vertical coordinate z only. A uniform horizontal magnetic field $\boldsymbol{H}(H, 0, 0)$ pervades the whole system. This layer is heated from below so that a steady adverse temperature gradient $\beta\,(=\left|\frac{dT}{dz}\right|$ is maintained. As a consequence of Brinkman's equation, the resistance $-\left(\frac{\mu}{k_1}\right)\boldsymbol{v}$ will also occur with the usual viscous term in the equations of motion. The constitutive equation of a Kuvshiniski-type fluid, studied by Kuvshiniski [16] and Mandal *et al*. [17] is characterized as

$$\begin{aligned}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\rho}{\in}\left[\frac{\partial \boldsymbol{u}}{\partial t}+\frac{1}{\in}(\boldsymbol{u}.\nabla)\boldsymbol{u}\right] &= -\nabla p - g\rho\lambda + \\ &+\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{KN}{\in}(\boldsymbol{v}-\boldsymbol{u})+\frac{1}{4\pi}(\nabla\times\boldsymbol{H})\times\boldsymbol{H}-\frac{\mu}{k_1}\boldsymbol{u}\right],\end{aligned} \tag{2.1}$$

$$\nabla.\boldsymbol{u} = 0\,, \tag{2.2}$$

where $\boldsymbol{u}(u, v, w), \rho, \upsilon(\bar{x}, t)$ and $N(\bar{x}, t)$ denote the fluid velocity, the fluid density, suspended particle velocity and the suspended particle number density, respectively, where $\bar{x} = (x, y, z)$ and $\bar{\lambda} = (0, 0, 1)$, $K = 6\pi\mu\eta$, η being particle radius, is the Stokes drag coefficient, and $\in$ is the medium porosity. Assuming a uniform particle size, a spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term in the equations of motion (2.1), proportional to the velocity difference between the particles and the fluid.

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. Interparticle reactions are ignored because the distances between the particles are assumed to be quite large compared with their diameter. The effects due to pressure, gravity, Darcy's force, and magnetic field on the particles are small and so are ignored. If *mN* is the mass of particles per unit volume, then the equations of motion and continuity of the particles, under the above assumptions, are

$$mN\left[\frac{\partial \boldsymbol{v}}{\partial t} + \frac{1}{\in}(\boldsymbol{v}.\nabla)\boldsymbol{v}\right] = KN(\boldsymbol{u} - \boldsymbol{v}), \tag{2.3}$$

$$\frac{\partial N}{\partial t} + \nabla.(N\boldsymbol{v}) = 0. \tag{2.4}$$

If C, C_{pt}, T and q denote the heat capacity of the fluid, the heat capacity of the particles, the temperature and the " effective thermal conductivity" of the pure fluid, respectively, and when the fluid and the particles are in thermal equilibrium, the equation of heat conduction gives

$$\left[\rho C \in + \rho_s C_s (1 - \in)\right]\frac{\partial T}{\partial t} + \rho C(\boldsymbol{u}.\nabla)T + mNC_{pt}\left(\in\frac{\partial}{\partial t} + \boldsymbol{v}.\nabla\right)T = q\nabla^2 T \tag{2.5}$$

where ρ_s, C_s stand for the density and the heat capacity of the solid matrix, respectively.

The Maxwell equations yield

$$\in\frac{d\boldsymbol{H}}{dt} = (\boldsymbol{H}.\nabla)\boldsymbol{u} + \in\eta\nabla^2\boldsymbol{H}, \tag{2.6}$$

$$\nabla.\boldsymbol{H} = 0. \tag{2.7}$$

The equation of state for the fluid is

$$\rho = \rho_0\left[1 - \alpha(T - T_0)\right], \tag{2.8}$$

where α is the coefficient of thermal expansion and ρ_0, T_0 are , respectively, the mean density and temperature of the clean fluid at the bottom surface $z = 0$.

The initial state of the system is taken to be a quiescent layer (no setting) with a uniform particle distribution N_0. The initial state

$$\boldsymbol{u} = 0, \quad \boldsymbol{v} = 0, \quad T = -\beta z, \quad N_0 = \text{Constant}. \tag{2.9}$$

3. Perturbation equations and dispersion relation

Let $\theta, \delta p, \delta\rho, \boldsymbol{u}(u,v,w), \boldsymbol{v}(l,r,s), \boldsymbol{h}(h_x,h_y,h_z)$ and N denote, respectively, the perturbations in temperature T, pressure p, density ρ, fluid velocity (zero initially), particle velocity (zero initially), magnetic field $\boldsymbol{H}$ and number density N_0. Let us scale the physical variable using $d, \frac{d^2}{k}, \frac{k}{d}, \frac{\rho\upsilon k}{d^2}$, and βd as the length, time, velocity, pressure and temperature scale factors, respectively. The change in density $\delta\rho$, caused by the perturbation θ is given by

$$\delta\rho = -\rho_0\alpha\theta . \tag{3.1}$$

Then the linearized dimensionless hydromagnetic perturbation equations are

$$\begin{aligned} &\left(1+\lambda'\frac{\partial}{\partial t}\right)N_p^{-1}\frac{\partial \boldsymbol{u}}{\partial t} = -\nabla\delta p + N_R\theta\lambda + \\ &+\left(1+\lambda'\frac{\partial}{\partial t}\right)\left[\omega(\boldsymbol{v}-\boldsymbol{u}) + \frac{d^2}{4\pi\rho_0 k\upsilon}(\nabla\times\boldsymbol{h})\times\boldsymbol{H} - \frac{1}{p}\boldsymbol{u}\right], \end{aligned} \tag{3.2}$$

$$\nabla.\boldsymbol{u} = 0 , \tag{3.3}$$

$$\left(\tau\frac{\partial}{\partial t}+1\right)\boldsymbol{v} = \boldsymbol{u} , \tag{3.4}$$

$$\frac{\partial M}{\partial t} + \nabla.\boldsymbol{v} = 0 , \tag{3.5}$$

$$(E + h\in)\frac{\partial\theta}{\partial t} = (w + hs) + \nabla^2\theta , \tag{3.6}$$

$$\in k\frac{\partial \boldsymbol{h}}{\partial t} = k(\boldsymbol{H}.\nabla)\boldsymbol{u} + \in\eta\nabla^2\boldsymbol{h} , \tag{3.7}$$

$$\nabla.\boldsymbol{h} = 0 , \tag{3.8}$$

In writing Eq.(3.2), use has been made of the Boussinesq equation of state (3.1), w and s are the vertical fluid and particle velocities, υ is the kinetic viscosity of the fluid, k is the thermal diffusivity, $N_p = \frac{\in\upsilon}{k}$ is the modified Prandtl number, $N_R = \frac{g\alpha\beta d^4}{\upsilon k}$ is the Rayleigh number, $\omega = \frac{KN_0 d^2}{\rho_0\upsilon\in}$, $f = \frac{mN_0}{\rho_0} = \tau\omega N_p$ is the mass fraction, $\tau = mk / Kd^2, P = k_1 / d^2$, $h = fC_{pt} / C, M = \in N / N_0$, $\lambda' = \lambda k / d^2$ and $E = \in + (1-\in)\rho_s C_s / \rho C$.

Consider the case of two free surfaces having uniform temperatures. The case of two free surfaces is a little artificial (except in stellar atmospheres where it is most appropriate) but allows an analytical solution to the problem. The boundary conditions appropriate to the problem are

$$\left.\begin{array}{c} w=\dfrac{\partial^2 w}{\partial z^2}=\theta=0 \text{ at } z=0 \text{ and } z=1 \\ \text{and } h_x, h_y, h_z \text{ are continuous with an external} \\ \text{vacuum field on a nonconducting boundary} \end{array}\right\} \tag{3.9}$$

Eliminating $\boldsymbol{v}$ and δp, the fluid, heat and Maxwell equations become

$$\left(1+\lambda'\frac{\partial}{\partial t}\right)\left[L_1+\frac{L_2}{P}\right]\nabla^2 w = L_2 N_R \nabla_1^2\theta + \left(1+\lambda'\frac{\partial}{\partial t}\right)\frac{L_2 d^2 H}{4\pi\rho_0 \upsilon k}\nabla^2\left(\frac{\partial h_z}{\partial x}\right), \tag{3.10}$$

$$L_2\left[(E+h\in)\frac{\partial}{\partial t}-\nabla^2\right]\theta = \left(\tau\frac{\partial}{\partial t}+H\right)w, \tag{3.11}$$

$$\left[\frac{k}{\eta}\frac{\partial}{\partial t}-\nabla^2\right]h_z = \left(\frac{kH}{\in\eta}\right)\frac{\partial w}{\partial x}, \tag{3.12}$$

where

$$L_1 = N_p^{-1}\left(\tau\frac{\partial^2}{\partial t^2}+F\frac{\partial}{\partial t}\right), \qquad L_2=\tau\frac{\partial}{\partial t}+1, \qquad F=f+1, \qquad H=h=1$$

$$\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} \qquad \text{and} \qquad \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}.$$

Eliminating θ and h_z between Eqs (3.10) and (3.12), we obtain

$$\begin{aligned} &\left(1+\lambda'\frac{\partial}{\partial t}\right)\left[L_1+\frac{L_2}{P}\right]\left[(E+h\in)\frac{\partial}{\partial t}-\nabla^2\right]\left[\frac{k}{\eta}\frac{\partial}{\partial t}-\nabla^2\right]\nabla^2 w = \\ &= N_R\left(\tau\frac{\partial}{\partial t}+H\right)\left[\frac{k}{\eta}\frac{\partial}{\partial t}-\nabla^2\right]\nabla_1^2 w + \\ &+\frac{H^2 d^2}{4\pi\rho_0\upsilon\eta\in}\left(1+\lambda'\frac{\partial}{\partial t}\right)L_2\left[(E+h\in)\frac{\partial}{\partial t}-\nabla^2\right]\frac{\partial^2}{\partial x^2}\left(\nabla^2 w\right). \end{aligned} \tag{3.13}$$

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y, and t is given by

$$w = W(z)\exp(ik_x x + ik_y y + nt), \tag{3.14}$$

where $k=\left(k_x^2+k_y^2\right)^{\frac{1}{2}}$ is the wave number of the disturbance and n is the growth rate, which is, in general, a complex constant, k_x and k_y are wave numbers along the x and y directions, respectively. Using Eq.(3.14), Eq.(3.13) becomes

$$\left(1+\lambda' n\right)\left[L_1+\frac{L_2}{P}\right]\left[D^2-k^2-(E+h\in)n\right]\left[D^2-k^2-\frac{\kappa\, n}{\eta}\right]\left(D^2-k^2\right)W =$$
$$= N_R\left(\tau n+H\right)\left[D^2-k^2-\frac{\kappa\, n}{\eta}\right]k^2W+Q\left(1+\lambda' n\right)L_2\left[D^2-k^2-\left(E+h\in\right)n\right]k_x^{\,2}\left(D^2-k^2\right)W \tag{3.15}$$

where $L_1 = N_P^{-1}\left(\tau n^2+Fn\right), \qquad L_2 = (1+\tau n)\,,$

$$D=\frac{d}{dz}, \qquad Q=\frac{H^2 d^2}{4\pi\rho_0\upsilon\eta\in}\,.$$

4. The oscillatory mode

Here we examine oscillatory modes, if any and their impact on the flow stability due to the presence of the magnetic field, porous medium, suspended particles, and viscoelastic effects.

$$\text{Let}\, U=\left(D^2-k^2\right)W\,, \tag{4.1}$$

and

$$X=\left(1+\lambda' n\right)\left[L_1+\frac{L_2}{P}\right]. \tag{4.2}$$

The equation satisfied by W, in terms of X, is

$$\left[D^2-k^2-(E+h\in)n\right]\left[D^2-k^2-\frac{\kappa\, n}{\eta}\right]X = N_R\left(\tau n+H\right)\left[D^2-k^2-\frac{\kappa\, n}{\eta}\right]k^2W+$$
$$Qk_x^{\,2}\left(1+\lambda' n\right)L_2\left[D^2-k^2-(E+h\in)\right]\left(D^2-k^2\right)W. \tag{4.3}$$

Multiplying Eq.(4.3) by X^*, the complex conjugate of X, and integrating over the range of z and using boundary conditions (3.9), we obtain

$$I_1+\left[\left(E+h\in\right)-\frac{\kappa}{\eta}\right]nI_2+\left[\left(E+h\in\right)\frac{\kappa}{\eta}\right]n^2I_3 =$$
$$= k^2N_R\left(\tau n+H\right)\left(1+\lambda' n^*\right)\left[L_1+\frac{L_2}{P}\right]\left[I_4+\frac{\kappa\, n}{\eta}I_5\right]+ \tag{4.4}$$
$$+k_x^{\,2}QL_2\left(1+\lambda' n\right)\left(1+\lambda' n^*\right)\left[L_1^*+\frac{L_2^*}{P}\right]\left[I_6+k^2I_4+\left(E+h\in\right)nI_4\right]$$

where

$$I_1 = \int_0^1 \left(\left| D^2 X \right|^2 + 2k^2 \left| DX \right|^2 + k^4 \left| X \right|^2 \right) dZ,$$
$$I_2 = \int_0^1 \left(\left| DX \right|^2 + k^2 \left| X \right|^2 \right) dZ,$$
$$I_3 = \int_0^1 \left(\left| X \right|^2 \right) dZ,$$
$$I_4 = \int_0^1 \left(\left| D\ W \right|^2 + 2k^2 \left| DW \right|^2 + k^4 \left| W \right|^2 \right) dZ, \tag{4.5}$$
$$I_5 = \int_0^1 \left(\left| DW \right|^2 + k^2 \left| W \right|^2 \right) dZ,$$
$$I_6 = \int_0^1 \left(\left| D^3 W \right|^2 + 2k^2 \left| D^2 W \right|^2 + k^4 \left| DW \right|^2 \right) dZ,$$

which all are positive definite.

Putting $n = in_0$, where n_0 is real, into Eq.(4.4) and equating the imaginary parts, we obtain

$$n_0 = 0, \tag{4.6}$$

or

$$\begin{aligned}
&\left[\left(E + h \in \right) - \frac{\kappa}{\eta} \right] I_2 = n_0^4 \left[-k^2 N_R \left\{ N_P^{-1} \tau^2 \lambda' \frac{\kappa}{\eta} I_5 \right\} - k_x^2 Q \left\{ -N_P^{-1} \tau^2 \lambda'^2 \left(k^2 I_4 + I_6 \right) + \right. \right. \\
&\left. \left. + N_P^{-1} \tau \lambda'^2 \left(E + h\varepsilon \right) \times \left(F - 1 \right) I_4 + \frac{\tau^2 \lambda' \left(E + h\varepsilon \right) I_4}{P} \right\} \right] + \\
&+ n_0^2 \left[k^2 N_R \left\{ \frac{-\tau^2 \lambda' I_4}{P} - N_P^{-1} \tau I_4 \left(\tau + H\lambda' + F\lambda' \right) + \frac{N_P^{-1} \kappa}{\eta} I_5 (\tau (H + F) + FH) + \right. \right. \\
&\left. + \frac{\kappa \tau I_5}{\eta P} \left(\tau + \lambda' (1 + H) \right) \right\} - k_x^2 Q \left\{ \frac{\left(\lambda' k \right)^2 I_4}{P} \left(1 - \tau \right) - \left(N_P^{-1} I_6 + N_P^{-1} k^2 I_4 \right) \left(\tau^2 + F \lambda'^2 \right) + \right. \\
&\left. \left. + N_P^{-1} \tau I_4 \left(F - 1 \right) \times \left(E + h\varepsilon \right) + \frac{\left(E + h\varepsilon \right) I_4}{P} \left(\lambda'^2 + \tau^2 \right) \right\} \right] + \\
&+ \left[k^2 N_R \left\{ N_P^{-1} FHI_4 + \frac{I_4}{P} \left(\tau + H \left(\lambda' + \tau \right) \right) - \frac{H \kappa I_5}{\eta P} \right\} - k_x^2 Q \left\{ -N_P^{-1} F \left(k^2 I_4 + I_6 \right) \right. \right. \\
&\left. \left. + \frac{\left(E + h\varepsilon \right) I_4}{P} \right\} \right].
\end{aligned} \tag{4.7}$$

Now we consider some special cases.

In the absence of the magnetic field, suspended particles and electrically non-conducting fluid $(\eta \to \infty)$, we obtain

$$n_0 = 0, \tag{4.8}$$

or

$$EI_2 = n_0^2\left[k^2 N_R\left(-N_P^{-1}\tau^2 I_4 - \frac{\tau^2\lambda' I_4}{P}\right)\right] + \left[k^2 N_R\left(\frac{I_4\tau}{P}\right)\right]. \tag{4.9}$$

Equation (4.9) gives

$$n_0{}^2 = -\left[EI_2 - \frac{k^2 N_R I_4 \tau}{P}\right]\left[k^2 N_R\left(N_P^{-1}\tau^2 I_4 + \frac{\tau^2\lambda' I_4}{P}\right)\right]^{-1}. \tag{4.10}$$

It is evident from Eq.(4.10) that the values of n_0 are imaginary, which is impossible as n_0 is real. Therefore $n_0 = 0,$ and the principle of exchange of stabilities is valid.

Now, in the absence of the magnetic field and presence of suspended particles and electrically non - conducting fluid, we have

$$n_0 = 0, \tag{4.11}$$

or

$$\begin{aligned}(E + h\in)I_2 = n_0^2&\left[k^2 N_R\left\{-N_P^{-1}\tau I_4\left(\tau + \lambda'(H+F)\right) - \frac{\tau^2\lambda' I_4}{P}\right\}\right] + \\ &+\left[k^2 N_R\left\{N_P^{-1} FHI_4 + \frac{I_4}{P}\left(\tau + H(\lambda' + \tau)\right)\right\}\right].\end{aligned} \tag{4.12}$$

Equation (4.12) gives

$$\begin{aligned}n_0{}^2 = -&\left[(E + h\varepsilon)I_2 - k^2 N_R\left\{N_P^{-1} FHI_4 + \frac{I_4}{P}\left(\tau + H(\lambda' + \tau)\right)\right\}\right]\times \\ &\times\left[k^2 N_R\left\{N_P^{-1}\tau I_4\left(\tau + \lambda'(H+F)\right) + \frac{\tau^2\lambda' I_4}{P}\right\}\right]^{-1}\end{aligned} \tag{4.13}$$

Both the values of n_0 are imaginary, which is impossible as n_0 is real. Therefore, $n_0 = 0$ and the principle of exchange of stabilities is valid.

However, in the presence of the magnetic field, suspended particles and for finite electrically conducting fluid, it clear from Eqs (4.11) and (4.13) that and $n_0 = 0$, meaning thereby that the modes will be oscillatory. The presence of the magnetic field, suspended particles, viscoelasticity and porous medium effects brings about oscillatory modes in the system, which did not exist in their absence.

5. Stationary convection

Here we consider the case of two free boundaries. It can be shown that all the even order derivatives and hence the proper solution of Eq.(3.15) characterizing the lowest mode is

$$W = W_0 \sin \pi z \tag{5.1}$$

where W_0 is constant. Substituting the solutions (5.1) in (3.15), we obtain

$$N_R = \frac{(1+\lambda' n)\left[L_1 + \frac{L_2}{P}\right]\left[\pi^2 + k^2 + (E + h \in) n\right]\left[\pi^2 + k^2\right]}{k^2(\tau n + H)} + \\ + \frac{Q\cos^2\theta(1+\lambda' n)(1+\tau n)\left(\pi^2 + k^2\right)\left[\pi^2 + k^2 + (E + h\in)n\right]}{(\tau n + H)\left[\pi^2 + k^2 + \frac{\kappa n}{\eta}\right]} \tag{5.2}$$

where $k_x / k = \cos\theta$.

When instability sets in as stationary convection, the marginal state will be characterized by $n = 0$ and Eq.(5.2) reduces to

$$N_R = \frac{\left[\pi^2 + k^2\right]^2}{k^2 HP} + Q\cos^2\theta \frac{\left[\pi^2 + k^2\right]}{H}. \tag{5.3}$$

Thus for stationary convection, the Kuvshiniski viscoelastic fluid behaves like an ordinary Newtonian fluid as the stress relaxation time and strain retardation time parameters vanish with *n*. To investigate the effects of medium permeability, suspended particles, and the magnetic field, we examine the nature of dN_R/dP, dN_R/dH, and dN_R/dQ analytically. Equation (5.3) yields

$$\frac{dN_R}{dP} = -\frac{\left[\pi^2 + k^2\right]^2}{k^2 HP^2}, \tag{5.4}$$

which is always negative. Thus medium permeability, therefore, has a destabilizing effect on the system. Equation (5.3) also yields

$$\frac{dN_R}{dH} = -\frac{\left[\pi^2 + k^2\right]}{H^2}\left[\frac{\left(\pi^2 + k^2\right)}{k^2 P} + Q\cos^2\theta\right], \tag{5.5}$$

which is always negative, therefore, the suspended particles have a destabilizing effect on the system. It is evident from Eq.(5.3) that

$$\frac{dN_R}{dQ} = \cos^2\theta \frac{\left(\pi^2 + k^2\right)}{H}, \tag{5.6}$$

which implies that the magnetic field has a stabilizing effect on the system. We now examine the dispersion relation (5.3) numerically. We have plotted the Rayleigh number N_R versus the medium permeability *P*, suspended particles parameter *H* and magnetic field *Q* in Figs 1, 2 and 3, respectively.

In Fig.1: the Rayleigh number N_R is plotted against P = *25*, *50*, *75*, *100*, *125*, for fixed values of H = *20*, Q = *10*, $\theta = 45^0$ and k = *1*, *2*, *3*, *4*, *5*. The Rayleigh number decreases with the increase in the permeability parameter showing its destabilizing effect on the system in the presence of suspended particles, horizontal magnetic field in a Kuvshiniski viscoelastic fluid through a porous medium.

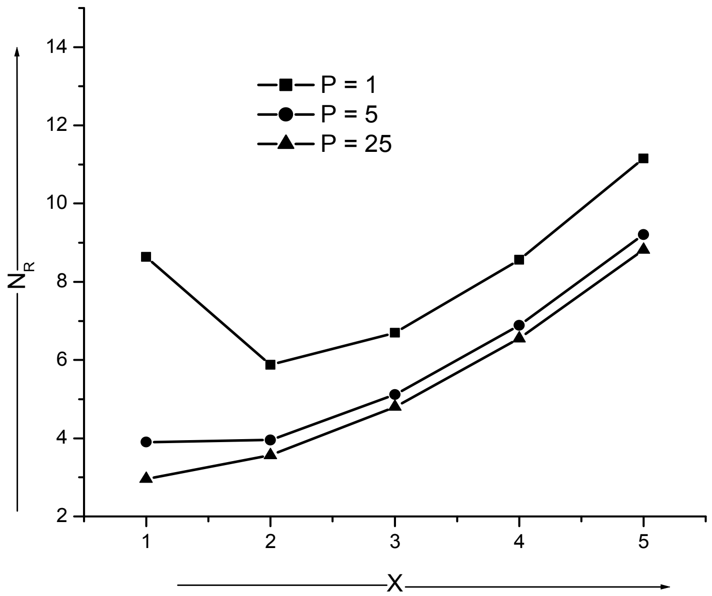

Fig.1. Variation of the Rayleigh number N_R, with the wave numbers $k(=1, 2, 3, 4, 5)$ for $P(=1, 5, 25)$ when Q=*10*, H=*20* and $\theta = 45^0$.

In Fig.2: the Rayleigh number N_R is plotted against H = *20*, *40*, *60*, for fixed values of P = *25*, Q = *10*, $\theta = 45^0$ and k = *1*, *2*, *3*, *4*, *5*. As the value of the suspended particles parameter increases, the corresponding value of the Rayleigh number decreases, showing its destabilizing effect on the system.

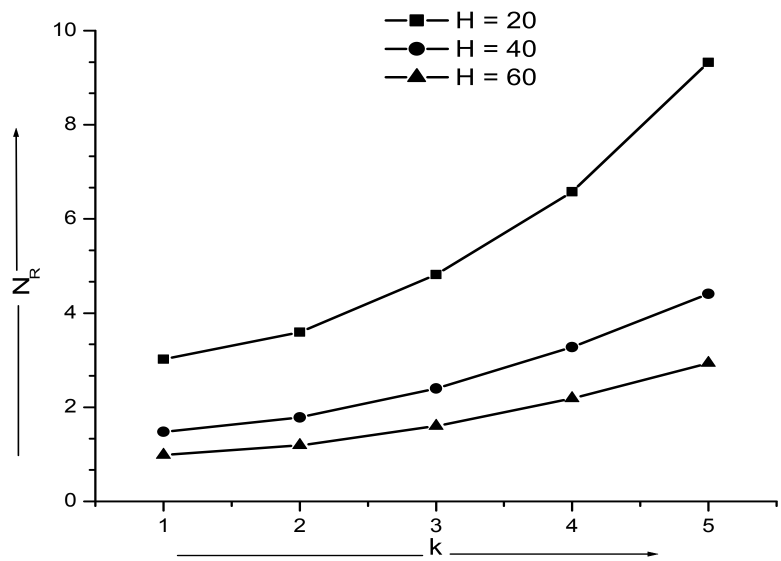

Fig.2. Variation of the Rayleigh number N_R, with the wave numbers $K(=1, 2, 3, 4, 5)$ for fixed values $P = 25$, Q=*10* and $\theta = 45^0$ for $H(=20, 40, 60)$.

In Fig.3: the Rayleigh number N_R is plotted against $Q = 10, 20, 30, 40, 50$, for a fixed value of $P = 25$, $H = 20$, $\theta = 45^0$ and $k = 1, 2, 3, 4, 5$. As the value of magnetic field increases, the corresponding value of the Rayleigh number increases, showing its stabilizing effect on the system.

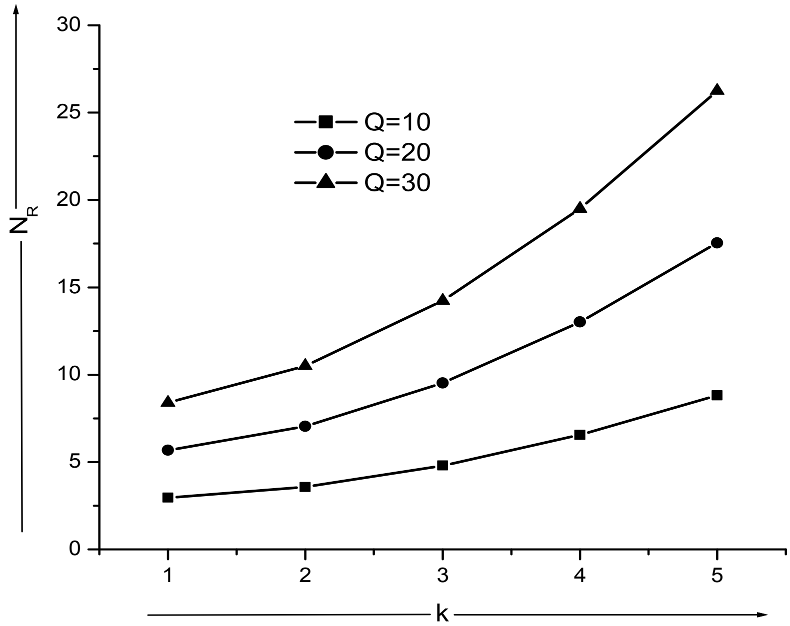

Fig.3. Variation of the Rayleigh number (N_R), with the wave numbers $k(=1, 2, 3, 4, 5)$ for $Q(=10, 20, 30)$ when $H=20$, $P=25$ and $\theta = 45^0$.

Thus, the medium permeability and suspended particles have destabilizing effects, whereas the magnetic field has a stabilizing effect on the system for stationary convection.

6. The overstable case

Here we consider the possibility of an overstability. Put $n = in_0$, where n_0 is real, in Eq.(5.2). Since for overstability we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which Eq.(5.2) has solutions. If we equate real and imaginary parts of Eq.(5.2) and eliminate N_R between them, we obtain

$$A_1 n^4{}_0 + A_2 n_0^3 + A_3 n_0^2 + A_4 n_0 + A_5 = 0, \tag{6.1}$$

where

$$A_1 = \frac{N_P{}^{-1}\tau^2\lambda'\kappa I_5 k^2 N_R}{\eta} + N_P{}^{-1}\tau(E + h\in)I_4\lambda'^2 k_x{}^2 Q(F-1) +$$
$$+ N_P{}^{-1}\tau^2\lambda'^2 I_6 k_x{}^2 Q + \tau^2\lambda'^2 I_4 k_x{}^2 Q\left(N_P{}^{-1}k^2 - \frac{(E + h\in)}{P}\right),$$

$$A_2 = k_x{}^2 Q N_P{}^{-1} F k^2 I_4 \lambda',$$

$$A_3 = k^2 N_R N_P^{-1} \tau\lambda' I_4 (F-H) + \frac{k^2 N_R N_P^{-1} \tau\kappa I_5}{\eta}(F-H) + \frac{k^2 N_R \tau\lambda'\kappa I_5}{\eta P}(H-1) +$$
$$+ k_x^2 Q \tau^2 I_4 \left(N_P^{-1} k^2 - \frac{(E+h\in)}{P} \right) + k_x^2 Q N_P^{-1} \tau I_4 (E+h\in)(F-1) + k^2 N_R N_P^{-1} \tau^2 I_4 +$$
$$+ \frac{\tau^2 k^2 N_R (I_4 + I_5)}{P} \left(\lambda' - \frac{\kappa}{\eta} \right) + \frac{k^2 N_R N^{-1} F \lambda' H \kappa I_5}{\eta} + k_x^2 Q \lambda'^2 \left(N_P^{-1} F - \frac{(E+h\in)}{P} \right)(I_4 + I_6) +$$
$$+ k_x^2 Q N_P^{-1} \tau^2 I_6 , \tag{6.2}$$
$$A_4 = k_x^2 Q N_P^{-1} F k^2 I_4 ,$$
$$A_5 = \frac{k^2 N_R \tau I_4 (H-1)}{P} + k^2 N_R N_P^{-1} F H I_4 + \frac{k^2 N_R H}{P}(I_4 + I_5)\left(\lambda' - \frac{\kappa}{\eta} \right) +$$
$$+ k_x^2 Q (I_4 + I_6) \left(N_P^{-1} - \frac{(E+h\in)}{P} \right) + \left[(E+h\in) - \frac{\kappa}{\eta} \right] I_2 ,$$

since n_0 is real for overstability, the four values of n_0 are positive. If all the coefficients $A_1 - A_5 > 0$, i.e., if

$$F > 1, \quad N_{Pk}^{-1} k^2 > \frac{E+h\in}{P}, \quad F > H, \quad H > 1, \quad \lambda' > \frac{\kappa}{\eta}, \quad N_P^{-1} F > \frac{E+h\in}{P}$$
$$\text{and} \quad N_P^{-1} > \frac{E+h\in}{P} \tag{6.3}$$
$$\text{i.e. } F > H > 1, \quad N_P^{-1} > \max . \frac{(E+h\in)}{P}\left\{ \frac{1}{k^2}, \frac{1}{F}, 1 \right\}, \text{ and } \lambda' > \frac{\kappa}{\eta},$$

are the sufficient conditions for the non-existence of overstability for the thermal instability of a Kuvshiniski viscoelastic fluid with suspended particles in hydromagnetics in a porous medium.

7. Concluding remarks

In the present paper, we have investigated the effect of suspended particles on an electrically conducting Kuvshiniski viscoelastic fluid layer heated from below in the presence of a horizontal magnetic field saturated in a porous medium. The dispersion relation governing the effects of suspended particles, the magnetic field, Kuvshiniski fluid and a porous medium is derived. The main results obtained are as follows:

(i): Medium permeability and suspended particles has destabilizing effects as well as magnetic field has stabilizing effect on the system, in the presence of suspended particles, magnetic field and porous medium on thermal instability of Kuvshiniski viscoelastic fluid.

(ii): Graphically observations of the problem is analyzed by the Figs 1 – 3.

(iii): Absence of magnetic field, suspended particles and electrically non- conducting fluid and absence of magnetic field, presence of suspended particles and electrically non – conducting fluid observed separately, by Eqs (4.8), (4.10) and (4.11), (4.13).

(iv): The presence of magnetic field, suspended particles, viscoelasticity and porous medium effects brings oscillatory modes in the system, in the absence of these principle of exchange of stabilities is valid.

(v): The sufficient conditions for the avoidance of overstability are obtained in Eq.(6.3), for thermal instability of Kuvshiniski viscoelastic fluid with suspended particles in hydromagnetics saturated in a porous medium.

Nomenclature

C_p – specific heat at constant pressure $\left(Jkg^{-1}K\right)$

C_{pt} – heat capacity of the particles $(-)$

D – mass diffusion coefficient $\left(m^2s^{-1}\right)$

F – dimensionless couple-stress parameter

$\boldsymbol{H}(H, 0, 0)$ – magnetic field intensity vector

K – Stoke's drag coefficient $\left(kgs^{-1}\right)$

$k = \left(k^2{}_x + k^2{}_y\right)^{1/2}$ – wave number $\left(m^{-1}\right)$

$N(\boldsymbol{x}, t)$ – number density of suspended particles

N_p – modified Prandtl number

N_R – Rayleigh number

p – fluid pressure (pa)

α – thermal diffusivity $\left(m^{-2}s^{-1}\right)$

β – steady adverse temperature gradient $\left(Km^{-1}\right)$

$\in$ – medium porosity $\left(m^0s^0k^0\right)$

η – electrical resistivity $(-)$

θ – dimensionless temperature

μ – dynamic viscosity $\left(km^{-1}s^{-1}\right)$

ρ – density $\left(kgm^{-3}\right)$

Subscripts

w - conditions at the wall

References

[1] Chandrasekhar S. (1961): *Hydrodynamic and Hydromagnetic Stability*. – Oxford: Claredon Press.

[2] Scanlon J.W. and Segel L.A. (1973): *Some effect of suspended particles on the onset of Benard convection*. – Physics Fluids, vol.16, pp.1573-1578.

[3] Joseph D.D. (1976): *Stability of Fluid Motions, Vol. I and II*. – Berlin: Springer-Verlag.

[4] Lapwood E.R. (1948): *Convection of a fluid in a porous medium.* – Proc. Camb. Phil. Soc., vol.44, pp.508-554.

[5] Wooding R.A. (1960): *Rayleigh instability of a thermal boundary layer in flow through a porous medium.* – J. Fluid Mech., vol.9, pp.183-192.

[6] Beavers G.S., Sparrow E.M. and Magnuson R.A. (1970): *Experiments on coupled parallel flows in a channel and a bounding porous medium*. – J. Basic Engin. Trans. ASME, D92, 843.

[7] Chaudhary R.K.S. and Singh K.K. (1991): *Flow of a dusty visco- elastic (Kuvshiniski type) liquid down an inclined plane.* – Proc. Nat. Acad., India, vol.61A (II), 223.

[8] Johari R. and Gupta G.D. (1999): *MHD flow of a dusty visco-elastic (Kuvshiniski type) liquid past an inclined plane.* – Acta Cin. Ind. XXVM (3), 275.

[9] Varshney N.K. and Dwivedi R.K. (2004): *Unsteady effect on MHD free convection and mass transfer flow of Kuvshiniski fluid through porous medium with constant suction and constantheat and mass flux*. – Acta Ciencia Indica, vol.2, pp.271-280.

[10] Kumar P. and Singh M. (2006): *On a viscoelastic fluid heated from below in a porous medium.* – J. Non-Equilib. Thermodyn., vol.31, pp.189-203.

[11] Kumar P. and Singh M. (2008): *Thermal instability of Kuvshiniski viscoelastic fluid with finite dust in a porous medium*. – Int. J. of Applied Mechanics and Engineering, vol.13, No.1, pp.193- 201.

[12] Sharma D.S. and Sharma R.G. (1991): *Dusty viscoelastic flow in a slip-flow regime*. – Proc. Indian Natn. Sci. Acad., 57A (5), pp.623-630.

[13] Prakash O., Kumar D. and Dwivedi Y.K. (2011): *MHD free convective flow of a visco-elastic (Kuvshiniski-type) dusty gas through a semi infinite plate moving with velocity decreasing exponentially with time and radiative heat transfer*. – AIP Advances, vol.1, 022132-9.

[14] Kumar P. (2011): *Magneto-rotatory stability of two stratified Kuvshiniski viscoelastic superposed fluids in porous medium*. – G.J.P and A. Sc. and Tech., vol.1, pp.28-35.

[15] Kumar V. and Kumar P. (2013): *Thermal convection in a (Kuvshiniski-type) viscoelastic rotating fluid in the presence of magnetic field through porous medium*. – IJE Transactions A: Basic, vol.26, No.7, pp.753-760.

[16] Kuvshiniski E.V. (1951): *Flow of dusty visco – elastic fluid (Kuvshiniski type) between two oscillating plates*. – J. Experiment Theoretical Physics (USSR), vol.21, 88.

[17] Mandal G.C., Mukherjee S.K. and Mukherjee S. (1986): *Unsteady flow of dusty visco-elastic liquid between two oscillating plates.* – J. Indian Institute of Science, vol.66, pp.77-83.

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