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MASS TRANSFER EFFECTS ON AN UNSTEADY MHD FREE CONVECTIVE FLOW OF AN INCOMPRESSIBLE VISCOUS DISSIPATIVE FLUID PAST AN INFINITE VERTICAL POROUS PLATE

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In this paper, a numerical solution of mass transfer effects on an unsteady free convection flow of an incompressible electrically conducting viscous dissipative fluid past an infinite vertical porous plate under the influence of a uniform magnetic field considered normal to the plate has been obtained. The non-dimensional governing equations for this investigation are solved numerically by using the Ritz finite element method. The effects of flow parameters on the velocity, temperature and concentration fields are presented through the graphs and numerical data for the skin-friction, Nusselt and Sherwood numbers are presented in tables and then discussed.

Key words: free convection flow, MHD, non-linear system, partial differential equation, Ritz FEM.

1. Introduction

Simultaneous heat and mass transfer from different geometrics embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation and underground energy transport. The change in wall temperature causing the free convection flow could be a sudden or a periodic one, leading to a variation in the flow. In nuclear engineering, cooling of the medium is more important the point of view of safety and during this cooling process the plate temperature starts oscillating about a non-zero constant mean temperature. Further, an oscillatory flow has applications in industrial and aerospace engineering. Soundalgekar and Ganesan [1] studied transient free convection with mass transfer on an isothermal vertical flat plate by using the finite difference method. Agrawal *et al*. [2] presented the effects of Hall currents on hydro-magnetic free convection with mass transfer in a rotating fluid. Jang and Ni [3] studied transient free convection with mass transfer from an isothermal vertical plate embedded in a porous medium. Jha [4] studied MHD free convection and mass transfer flow through a porous medium. Soundalgekar and Wavre [5] studied an unsteady free convection flow past an infinite vertical plate with constant suction and mass transfer. Ganesan and Palani [6] presented mass transfer effects on impulsively started semi-infinite inclined plate with constant heat flux. Ibrahim *et al*. [7] studied an unsteady magneto-hydrodynamic micro-polar fluid flow and heat transfer over a vertical porous plate through a porous medium in the presence of thermal and mass diffusion with constant heat source. Chaudhary and Jain [8] presented the combined heat and mass transfer effects on an MHD free convection flow past an oscillating plate embedded in a porous medium. Chaudhary and Jain [9] studied heat and mass diffusion flow by natural convection past a surface embedded in a porous medium. Muthucumaraswamy *et al*. [10] studied on unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion.

Viscous mechanical dissipation effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. Gebhart [11] showed the importance of viscous dissipative heat in a free convection flow in the case of isothermal and constant heat flux at the plate. Gebhart and Mollendorf [12] considered the effects of viscous dissipation for external natural convection flow over a surface. Viscous dissipation heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate was studied by Soundalgekar [13]. Gokhale and Samman [14] presented effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi-infinite vertical plate with constant heat flux. Cookey *et al*. [15] studied the influence of viscous dissipation and radiation on an unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Chen [16] studied combined heat and mass transfer in MHD free convection from a vertical surface with Ohmic heating and viscous dissipation.

The aim of the present work is to study the effects of mass transfer on an unsteady free convection flow of viscous dissipative fluid past an infinite vertical porous plate under the influence of a uniform magnetic field applied normal to the plate. As the problem is governed by a coupled non-linear system of partial differential equations, whose exact solutions are difficult to obtain, if possible, so, the Ritz finite element method has been adopted for its solution, which is more economical from the computational point of view. The behavior of the velocity, temperature, concentration, skin-friction, Nusselt and Sherwood numbers has been discussed for variations in the governing parameters.

2. Mathematical analysis

Consider an unsteady free convection flow of an incompressible electrically conducting viscous dissipative fluid past an infinite vertical porous plate. Let the x'-axis be chosen along the plate in the vertically upward direction and the y'-axis is chosen normal to the plate. A uniform magnetic field of intensity H_0 is applied transversely to the plate. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is taken to be very small. Initially, the temperature of the plate T' and the fluid T'_∞ are assumed to be the same. The concentration of species at the plate C'_w and in the fluid through out C'_∞ are assumed to be the same. At time $t'>0$, the plate temperature is changed to T'_w, which is then maintained constant, causing convection currents to flow near the plate and mass is supplied at a constant rate to the plate. Under these conditions the flow variables are functions of time y' and t' alone. The problem is governed by the following equations

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right) + g\beta^\bullet\left(C' - C'_\infty\right) - \frac{\sigma\mu_e^2 H_0^2 u'}{\rho} - \frac{\nu u'}{K'}, \tag{2.1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + \mu\left(\frac{\partial u'}{\partial y'}\right)^2, \tag{2.2}$$

$$\frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2}. \tag{2.3}$$

The corresponding initial and boundary conditions are

$$\begin{aligned}
&t' \le 0; \quad u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \text{for all} \quad y', \\
&t' > 0; \quad u' = 0, \quad T' = T'_w, \quad C' = C'_w \quad \text{at} \quad y' = 0, \\
&u' = 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y' \to \infty.
\end{aligned} \tag{2.4}$$

It is convenient to introduce the following non-dimensional quantities into the basic equations and initial and boundary conditions in order to make them dimensionless.

$$U_0 = \left(\nu g\beta\Delta T\right)^{\frac{1}{3}}, \quad L = \left(\frac{g\beta\Delta T}{\nu^2}\right)^{-1/3}, \quad T_R = \frac{\left(g\beta\Delta T\right)^{-2/3}}{\nu^{-1/3}},$$

$$\Delta T = T'_w - T'_\infty, \quad t = \frac{t'}{T_R}, \quad y = \frac{y'}{L},$$

$$u = \frac{u'}{U_0}, \quad K = \frac{K'}{\nu T_R}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \tag{2.5}$$

$$\mathrm{Pr} = \frac{\mu C_p}{k}, \quad \mathrm{Sc} = \frac{\nu}{D_M}, \quad \mathrm{Ec} = \frac{U_0^2}{C_p \Delta T},$$

$$N = \frac{\beta^\bullet \left(C'_w - C'_\infty\right)}{\beta\left(T'_w - T'_\infty\right)}, \quad M = \frac{\sigma\mu_0^2 H_0^2 T_R}{\rho}.$$

On substitution of Eqs (2.5) into Eqs (2.1)-(2.4), the following governing equations in non-dimensional form are obtained

$$\frac{\partial u}{\partial t} = \theta + \frac{\partial^2 u}{\partial y^2} + N\phi - Mu - \frac{1}{K}u, \tag{2.6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} + \mathrm{Ec}\left(\frac{\partial u}{\partial y}\right)^2, \tag{2.7}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{\mathrm{Sc}}\frac{\partial^2 \phi}{\partial y^2}. \tag{2.8}$$

The corresponding initial and boundary conditions are

$$t \le 0; \quad u = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{for all} \quad y,$$

$$t > 0; \quad u = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad y = 0, \tag{2.9}$$

$$u = 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad y \to \infty.$$

3. Solution of the problem

Equations (2.6)-(2.8) are a coupled non-linear system of partial differential equations to be solved under the initial and boundary conditions (2.9). However, whose exact or approximate solutions are not possible to solve the set of these equations. Hence, these equations are solved by the Ritz finite element method.

The linear functional for Eq.(2.6) over the typical line segment element $(e),\left(y_j \le y \le y_k\right)$ is

$$J^{(e)}(u) = \frac{1}{2}\int_{y_j}^{y_k}\left\{\left(\frac{\partial u^{(e)}}{\partial y}\right)^2 + M_1 u^{(e)^2} + 2u^{(e)}\frac{\partial u^{(e)}}{\partial t} - 2u^{(e)}(\theta + N\phi)\right\}dy = \text{Minimum} \tag{3.1}$$

where $\quad M_1 = M + (1/K)$.

The finite element model may be obtained from Eq.(3.1) by substituting the finite element approximation over the two nodded elements $(e),\left(y_j \le y \le y_k\right)$ of the form

$$u^{(e)} = \psi^{(e)}\chi^{(e)} . \tag{3.2}$$

Here $\quad \psi^{(e)} = \left[\psi_j, \psi_k\right] \quad$ and $\quad \chi^{(e)} = \left[u_j, u_k\right]^T$

where u_j, u_k are the velocity components at j^{th} and k^{th} nodes of the typical element (e) and ψ_j, ψ_k are the basis functions defined as

$$\psi_j = \frac{y_k - y}{y_k - y_j} \qquad \text{and} \qquad \psi_k = \frac{y - y_j}{y_k - y_j} .$$

On substitution of Eq.(3.2) into Eq.(3.1), assembling the element equations for two consecutive elements $y_{i-1} \le y \le y_i$ and $y_i \le y \le y_{i+1}$, then putting the row corresponding to the node i equal to zero, we obtain the difference schemes with $h = y_k - y_j$ is the length of the element (e)

$$\left(\dot{u}_{i-1} + 4\dot{u}_i + \dot{u}_{i+1}\right) = \frac{1}{h^2}\left(6 - M_1 h^2\right)\left(u_{i-1} + u_{i+1}\right) - \frac{1}{h^2}\left(12 + 4M_1 h^2\right) + 6(\theta + N\phi) . \tag{3.3}$$

Applying the trapezoidal rule to Eq.(3.3) and using the Crank-Nicholson method we have

$$\begin{gathered}\left(1 - 3r + \frac{1}{2}rM_1h^2\right)u_{i-1}^{j+1} + \left(4 + 6r + 2rM_1h^2\right)u_i^{j+1} + \left(1 - 3r + \frac{1}{2}rM_1h^2\right)u_{i+1}^{i+1} = \\ = \left(1 + 3r - \frac{1}{2}rM_1h^2\right)u_i^{j-1} + \left(4 - 6r - 2rM_1h^2\right)u_i^j + \left(1 + 3r - \frac{1}{2}rM_1h^2\right)u_{i+1}^j + 6k\left(\theta_i^j + N\phi_i^j\right).\end{gathered} \tag{3.4}$$

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Now from Eqs (2.7) and (2.8), the following are obtained

$$\begin{gathered}\left(\Pr - 3r\right)\theta_{i-1}^{j+1} + \left(4\Pr + 6r\right)\theta_i^{j+1} + \left(\Pr - 3r\right)\theta_{i+1}^{j+1} = \left(\Pr + 3r\right)\theta_{i-1}^j + \left(4\Pr - 6r\right)\theta_i^j + \\ + \left(\Pr + 3r\right)\theta_{i+1}^j + 6r\Pr\text{Ec}\left(u_{i+1}^j - u_i^j\right)^2 ,\end{gathered} \tag{3.5}$$

$$\left(\mathrm{Sc}-3r\right)\varphi_{i-1}^{j+1}+\left(4\mathrm{Sc}-6r\right)\varphi_{i}^{j+1}+\left(\mathrm{Sc}-3r\right)\varphi_{i+1}^{j+1}=\left(\mathrm{Sc}+3r\right)\varphi_{i-1}^{j}+ \\ +\left(4\mathrm{Sc}+6r\right)\varphi_{i}^{j}+\left(\mathrm{Sc}+3r\right)\varphi_{i+1}^{j}. \tag{3.6}$$

Here $r=k/h^2$ and h,k are mesh sizes along the $y-$direction and $t-$direction, respectively. The index i refers to the space and j refers to the time. The mesh system is divided by taking $h=0.5$ and $k=0.0625$. Taking $i=1(1)n$ in the above Eqs (3.4)-(3.6) using initial and boundary conditions Eq.(2.9), the following tri-diagonal system of equations is obtained

$$AU=B,$$

$$D\theta=E,$$

$$F\varphi=G$$

where A,D and F are tri-diagonal matrices of order-n whose elements are given by

$$a_{i,i}=4+6r+2rM_1h^2, \qquad d_{i,i}=4\mathrm{Pr}+6r, \qquad f_{i,i}=4\mathrm{Sc}+6r, \qquad i=1(1)n,$$

$$a_{i-1,i}=a_{i,i-1}=1-3r+2rM_1h^2, \quad d_{i-1,i}=d_{i,i-1}=\mathrm{Pr}-3r, \quad f_{i-1,i}=f_{i,i-1}=\mathrm{Sc}-3r, \quad i=2(1)n.$$

Here u,θ,ϕ and B,E,G are column matrices having the n-components $u_i^{j+1},\theta_i^{j+1},\phi_i^{j+1}$ and $u_i^j,\theta_i^j,\phi_i^j$, respectively. The solutions of the above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by C – program. In numerical computations, special attention is given to specify h and k in order to achieve convergence and stability of the solution procedure and to obtain steady state solution. To judge the accuracy of the convergence and stability of the Ritz finite element method, the computations have been carried out by making very small changes in the values of h and k. For slightly changed values of h and k, no significant change is observed in the values of u,θ and ϕ. Hence, the Ritz finite element method is convergent and stable.

Skin-friction, rate of heat and mass transfer

We present the computational results for the major physical quantities, such as the skin-friction (τ), the heat transfer coefficient (Nu), and the mass transfer coefficient (Sh) defined by

$$\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}, \qquad \mathrm{Nu}=-\left(\frac{\partial \theta}{\partial y}\right)_{y=0}, \qquad \mathrm{Sh}=-\left(\frac{\partial \phi}{\partial y}\right)_{y=0}.$$

4. Numerical results and discussion

The problem of an unsteady free convection mass transfer flow of an incompressible electrically conducting viscous fluid past an infinite vertical porous plate with viscous dissipation is addressed in this study. Numerical calculations have been carried out for the non-dimensional velocity (u), temperature (θ), concentration (ϕ), skin-friction (τ), Nusselt number (Nu) and Sherwood number (Sh) for various values of the material parameters encountered in the problem under the investigation. The numerical calculations of these results are presented through graphs and tables.

Figure 1 depicts the effects of the Prandtl number Pr on the temperature filed for $\Pr = 0.71$ which corresponds to air, $\Pr = 1.0$ which corresponds to electrolytic solution, $\Pr = 7.0$ which corresponds to water and $\Pr = 11.4$ which corresponds to water at 4^0C respectively. The numerical results show that an increasing value of the Prandtl number decreases the temperature field. The effects of viscous dissipation parameter, i.e., the Eckert number Ec and time parameter t on the temperature field are shown in Fig.2. It is observed that an increase in the Eckert number and time parameter increases the temperature field. Figure 3 depicts the effects of the Schmidt number Sc and time parameter t on the species concentration for $\mathrm{Sc} = 0.22, 0.60$ and 0.78 as would correspond to hydrogen, water-vapour and ammonia respectively. It is observed that an increase in the Schmidt number decreases the species concentration whereas an increase in the time parameter increases the species concentration.

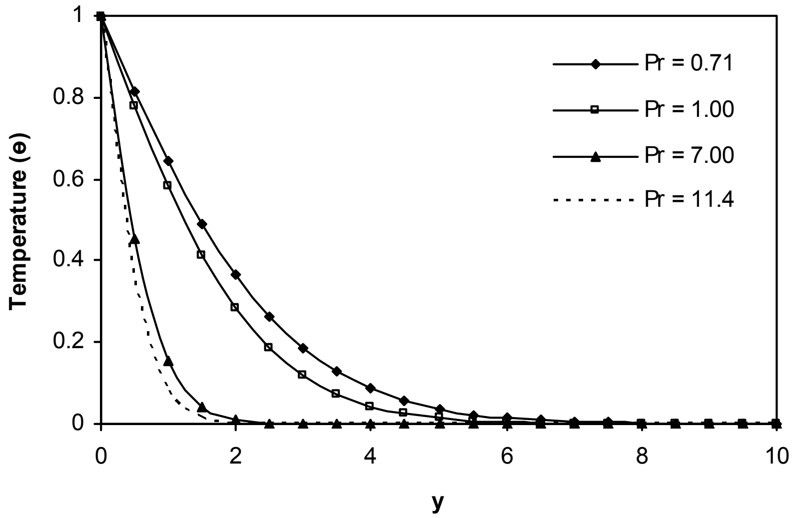

Fig.1. Temperature profiles for different values of Pr for $\mathrm{Sc} = 0.22, M = 0.5, K = 0.5, N = 0.5, \mathrm{Ec} = 0.1$ at $t = 1.0$.

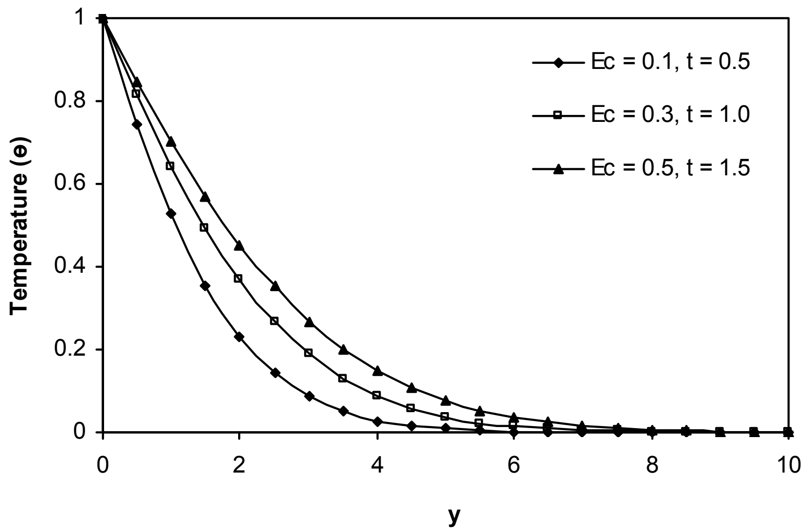

Fig.2. Temperature profiles for different values of Ec and t for $\Pr = 0.71, S_c = 0.22, M = 0.5, K = 0.5, N = 0.5$.

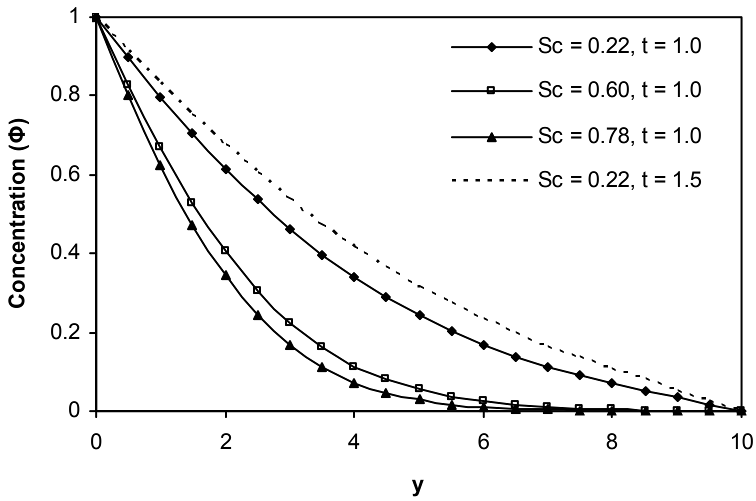

Fig.3. Concentration profiles for different values of Sc and t.

Figure 4 shows the velocity profiles in the boundary layer for various values of the Prandtl number Pr. It is observed that an increase in the Prandtl number decreases the velocity field. The effects of the magnetic parameter M on the velocity field are presented in Fig.5. It is seen that an increase in the magnetic parameter decreases the velocity field. This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow. The magnetic field controls the flow characteristics. Figure 6 depicts the effects of the porosity parameter K on the velocity field. It can be seen that an increase in the porosity parameter increases the velocity field. The effects of the ratio of mass transfer parameter N on the velocity field are presented in Fig.7. It is clear that an increase in N leads to an increase in the velocity field. Figures 8 to 11 shows the effects of the Eckert number Ec and time parameter t on the velocity field for a fixed value of the Schmidt number Sc. From these figures, it can clearly be seen that an increase in the Eckert number and time parameter increases the velocity field. Also, observe that an increase in the Schmidt number decreases the velocity field.

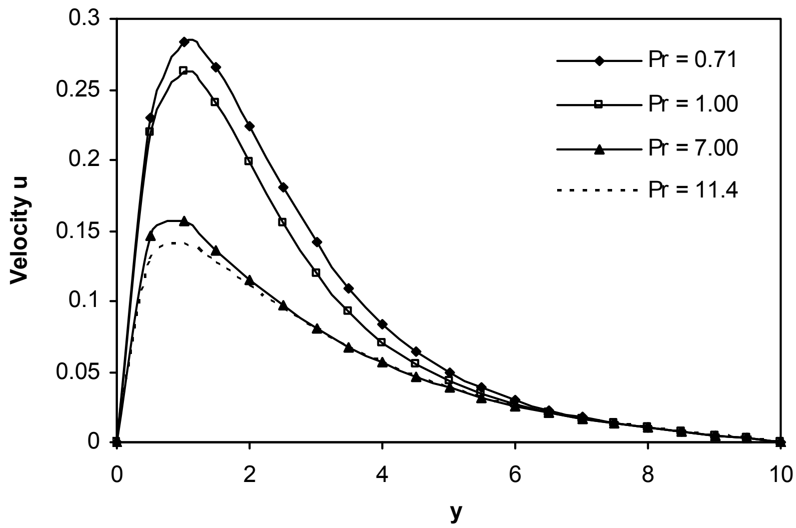

Fig.4. Velocity profiles for different values of the Prandtl number Pr for $\mathrm{Sc} = 0.22, M = 0.5, K = 0.5, N = 0.5, \mathrm{Ec} = 0.1$ at $t = 1.0$.

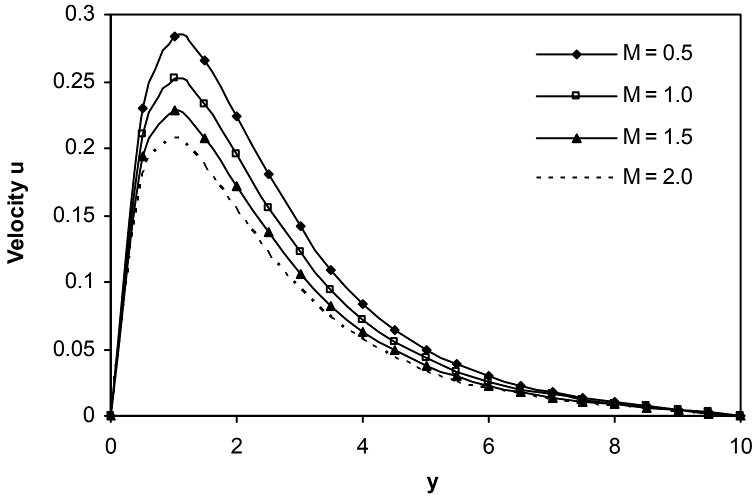

Fig.5. Velocity profiles for different values of the magnetic parameter M for $\mathrm{Pr}=0.71, \mathrm{Sc}=0.22$, $M=0.5, N=0.5, \mathrm{Ec}=0.1$ at $t=1.0$.

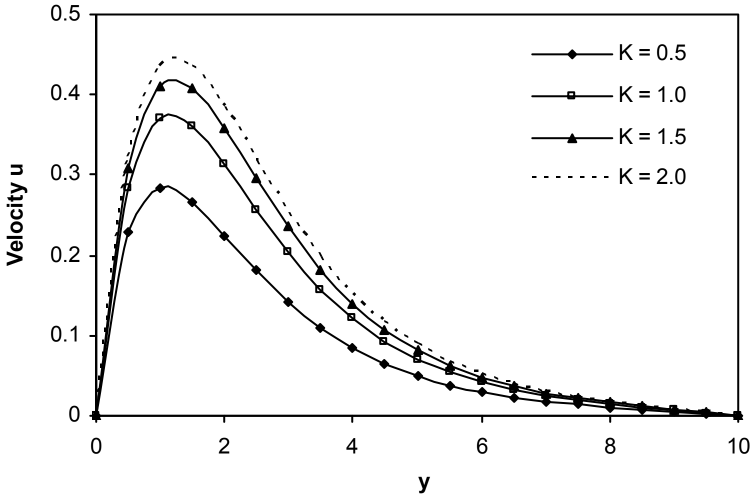

Fig.6. Velocity profiles for different values of the porosity parameter K for $\mathrm{Pr}=0.71, \mathrm{Sc}=0.22, M=0.5$, $N=0.5, \mathrm{Ec}=0.1$ at $t=1.0$.

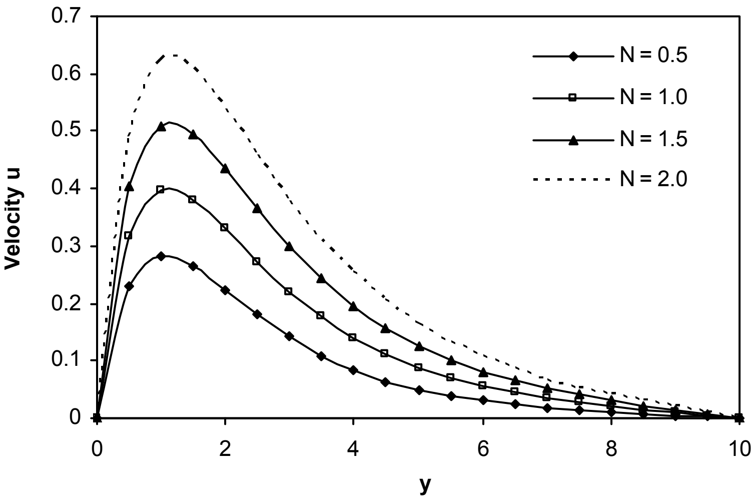

Fig.7. Velocity profiles for different values of N for $\mathrm{Pr}=0.71, \mathrm{Sc}=0.22, M=0.5, K=0.5, \mathrm{Ec}=0.1$ at $t=1.0$.

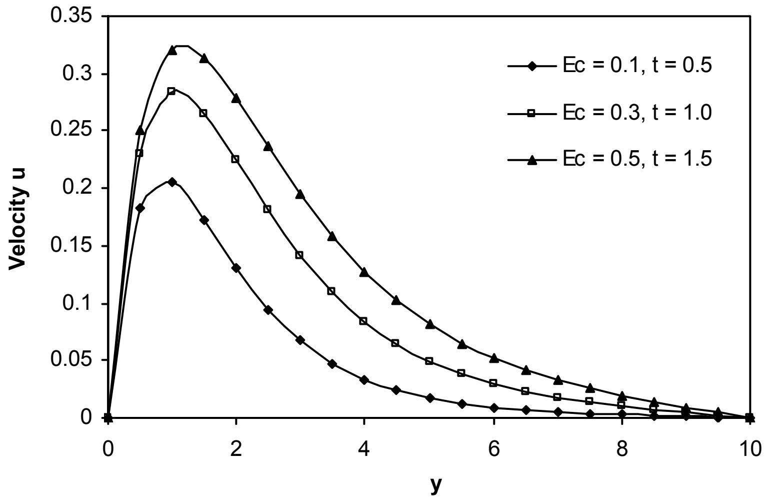

Fig.8. Velocity profiles for different values of Ec and t for $\Pr = 0.71, M = 0.5, K = 0.5, N = 0.5$ when $\mathrm{Sc} = 0.22$.

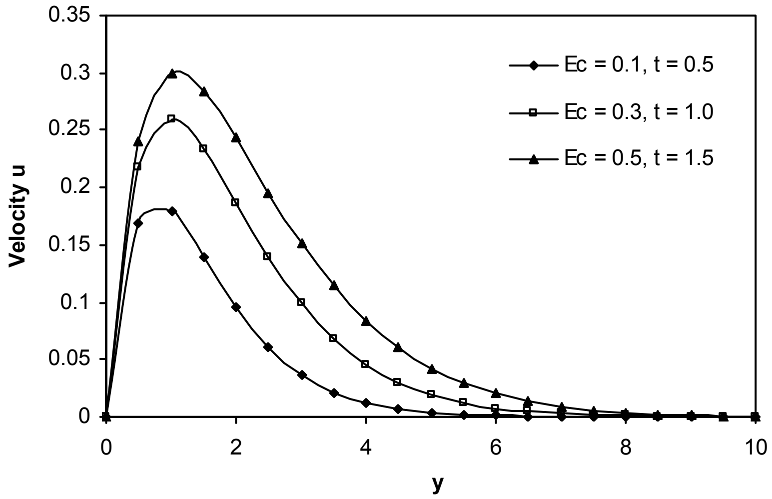

Fig.9. Velocity profiles for different values of Ec and t for $\Pr = 0.71, M = 0.5, K = 0.5, N = 0.5$ when $\mathrm{Sc} = 0.60$.

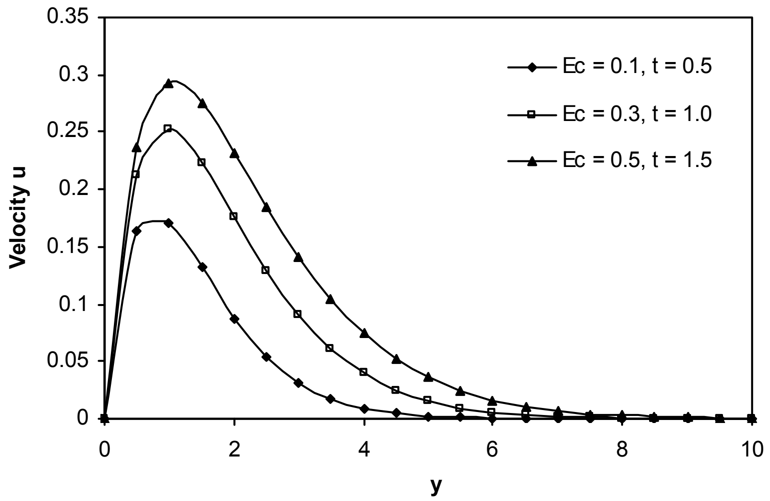

Fig.10. Velocity profiles for different values of Ec and t for $\Pr = 0.71, M = 0.5, K = 0.5, N = 0.5$ when $\mathrm{Sc} = 0.78$.

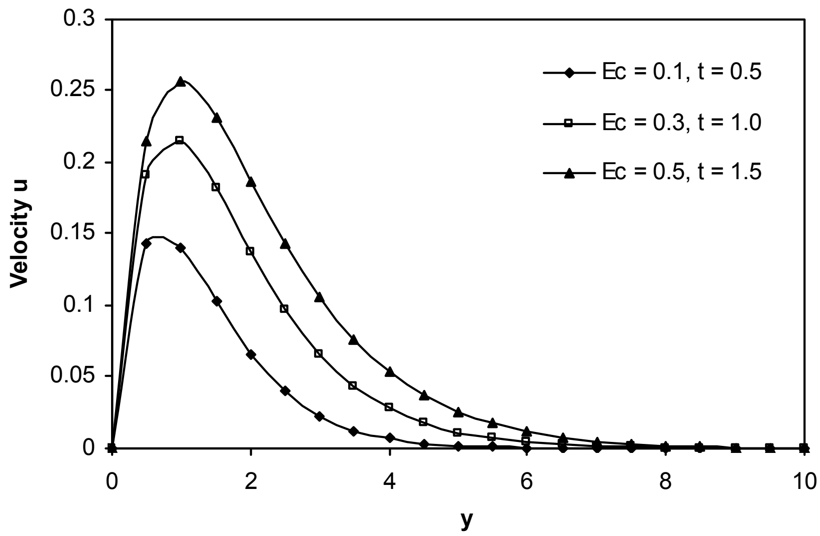

Fig.11. Velocity profiles for different values of Ec and t for $\mathrm{Pr} = 0.71, M = 0.5, K = 0.5, N = 0.5$ when $\mathrm{Sc} = 2.62$.

The numerical values of the skin-friction coefficient (τ), Nuesslt number (Nu) and Sherwood number (Sh) are presented in Tabs 1, 2 and 3 respectively. From the Tab.1, it is observed that an increase in the Prandtl number or Schmidt number or the magnetic parameter leads to decrease in the value of the skin-friction coefficient whereas an increase in the porosity parameter or ratio of mass transformation parameter or Eckert number and time parameter leads to an increase in the value of the skin-friction coefficient. From Tab.2, it is observed that an increase in the Prandtl number Pr leads to an increase in the value of the Nusselt number whereas an increase in the Eckert number Ec and time parameter t leads to a decrease in the Nusselt number. From Tab.3, it is observed that an increase in the Schmidt number Sc leads to an increase in the Sherwood number whereas an increase in the time parameter decreases the Sherwood number.

Table 1. Numerical data for the skin-friction coefficient (τ).

Pr	Sc	M	K	N	Ec	t	τ
0.71	0.22	0.5	0.5	0.5	0.1	1.0	0.459896
7.00	0.22	0.5	0.5	0.5	0.1	1.0	0.292022
0.71	0.78	0.5	0.5	0.5	0.1	1.0	0.426130
0.71	0.22	1.0	0.5	0.5	0.1	1.0	0.420418
0.71	0.22	0.5	1.0	0.5	0.1	1.0	0.569360
0.71	0.22	0.5	0.5	1.0	0.1	1.0	0.633770
0.71	0.22	0.5	0.5	0.5	0.3	1.0	0.460070
0.71	0.22	0.5	0.5	0.5	0.1	2.0	0.500918

Table 2. Numerical data for the Nusselt number (Nu).

Pr	t / Ec	*0.1*	*0.3*	*0.5*
0.71	*1.0*	*0.372742*	*0.371802*	*0.370858*
	1.5	*0.307862*	*0.306288*	*0.304712*
	2.0	*0.268000*	*0.265918*	*0.263824*
7.00	*1.0*	*1.087840*	*1.087678*	*1.087516*
	1.5	*0.924670*	*0.924206*	*0.923738*
	2.0	*0.816074*	*0.815140*	*0.814200*

Table 3. Numerical data for the Sherwood number (Sh).

Sc	t	Sh
0.22	*1.0*	*0.207852*
0.60	*1.0*	*0.343342*
0.78	*1.0*	*0.390970*
0.22	*2.0*	*0.179094*

5. Conclusions

In this study we have examined the governing equations for mass transfer effects on a free convection flow of an incompressible viscous dissipative fluid past an infinite vertical porous plate under the influence of a magnetic field considered normal to the plate. The leading equations for the investigation have been solved numerically by using the Ritz finite element method. The present results illustrate the flow characteristics for the velocity, temperature, concentration, skin-friction, Nusselt and Sherwood numbers and show how the flow fields are influenced by the material parameters of the flow problem. We can conclude that an increase in the Prandtl number leads to a decrease in the velocity and temperature fields. An increase in the Schmidt- number decreases the velocity and concentration of the fluid. Also, an increase in the Eckert number and time parameter increases the velocity and temperature of the fluid. These results are in very good agreement with those in the literature.

Nomenclature

C_p − specific heat at constant pressure
C' − species concentration near the plate
C'_∞ − species concentration in the fluid far away from the plate
D_M − chemical molecular diffusivity
Ec − Eckert number
g − acceleration due to gravity
K − thermal conductivity

N – ratio of mass transformation
Pr – Prandtl number
Sc – Schmidt number
T' – temperature of the fluid near the plate
T'_w – temperature of the plate
T'_∞ – temperature of the fluid far away the plate
t' – time
β – co-efficient of volume expansion
$\beta^\bullet$ – co-efficient of species expansion
μ – viscosity
ν – kinematic viscosity
ρ – density

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