Int. J. of Applied Mechanics and Engineering, 2016, vol.21, No.2, pp.341-358
DOI: 10.1515/ijame-2016-0021

EFFECT OF SLIP CONDITION ON VERTICAL CHANNEL FLOW IN THE PRESENCE OF RADIATION

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An analysis is made on the three dimensional flow of a viscous incompressible fluid through a vertical channel in the presence of radiation in slip flow regime. The right plate is subjected to an uniform injection and the left plate to a periodic suction velocity distribution. The velocity and temperature fields have been derived using the perturbation technique. It is found that the velocity decreases with the increase of the slip parameter. It is also found that the velocity decreases with the increase of the radiation parameter but near the right plate it increases. For cooling of the plate, the velocity increases with the increase of the Grashoff number and decreases near the right plate but the reverse effect is observed for heating the plate.

Key words: three-dimensional, injection, suction, buoyancy, transpiration cooling, free convection.

1. Introduction

The study of viscous fluid flows through a vertical channel is of practical importance due to its application to transpiration cooling of reentry vehicles and rocket boosters etc. and in gaseous diffusion. In boundary layer control, the decelerated fluid particles in the boundary layer are removed through slits in the wall into the exterior of the body. With sufficiently strong suction, separation can be prevented. Also suction is applied in chemical processes to remove reactants where blowing is used to add reactants, prevent corrosion and reduce drag. Hydrodynamic combined convective flow as well as free convective hydrodynamic flows in a vertical channel were studied by many authors. Mention may be made of the works of Aung [1], Sparrow *et al.* [2] and Aung and Worku [3]. Kettleborough [4] studied the transient laminar two dimensional motion generated by a temperature gradient perpendicular to the direction of the body force of a fluid between two heated vertical plates. Wang and Skalak [5] obtained the solution for a three dimensional problem of fluid injection through one side of a long vertical channel for a Newtonian fluid. An extension of this problem was studied by Sharma and Chaudhary [6] for a visco-elastic fluid, Baris [7] for a Newtonian thermodynamically compatible fluid of second grade and Baris [8] for Walter's B' fluid. Singh *et al*. [9] studied the three dimensional free convection flow and heat transfer past a vertical plate with periodic suction. If the fluid temperature is rather high, radiation effects play an important role and this situation does exist in space technology. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites are examples of such engineering areas. In this cases, one has to take into account the effects of radiation and free convection. When the temperature of the plate is high, the radiation effects are not negligible. Takhar *et al.* [10] studied the radiation effects on an MHD free convection flow of a gas past a semi-infinite vertical plate. Pathak *et al*. [11] studied the radiation effects on an unsteady free convective flow through a porous medium bounded by an oscillating plate with a variable wall temperature. Perdikis and Rapti [12] also studied the unsteady MHD free convection flow in the presence of radiation. Sharma *et al*. [13] presented an approximated solution for the radiation effect on the temperature distribution in a three-dimensional Couette flow with periodic suction. Guria and Jana [14] also have studied the effect of periodic suction on a three dimensional vertical channel flow. Due to the periodic suction the flow becomes three dimensional. Recently, Guria *et al.* [15] investigated the effect of radiation on a three dimensional vertical channel flow.

The phenomenon of slip-flow regime has attracted the attention of a large number of scholars due to its wide ranging application. The problem of the slip flow regime is very important in this era of modern science, technology and vast ranging industrialization. In practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle at the surface has a finite tangential velocity, it slips along the surface. The flow regime is called the slip flow regime and its effect cannot be neglected. The fluid slippage phenomenon at the solid boundaries appears in many applications such as micro channels or nano channels. In a geothermal region, a situation may arise when a slip of particles at the boundary may ocour. Gupta and Goyal [16], Jothimani and Devi [17], Jain and Tanija [18] solved the problems considering first order velocity slip conditions. Recently Jain and Gupta [19] studied the effects of the transverse sinusoidal injection velocity distribution on the free convective flow of a viscous incompressible fluid in slip flow regime under the influence of a heat source. The aim of our present paper is to study the effect of the radiation parameter and slip parameter on a three dimensional flow of a viscous incompressible fluid past a vertical channel. An approximate solution has been obtained by the series expansion method.

2. Basic equations

Consider a steady flow of a viscous, incompressible fluid between vertical parallel porous plates separated by a distance d. Here the x^* - axis is chosen along the direction of the flow, the y^* - axis is perpendicular to the plate of the channel and the z^* - axis is normal to the $x^* y^*$ - plane [see Fig.1]. The temperature at the plates $y^* = 0$ and $y^* = d$ are T_w and T_0 $(T_w > T_0)$ respectively.

The plate $y^* = d$ is subjected to a uniform injection V_0 and the plate $y^* = 0$ to a periodic suction velocity distribution of the form

$$v^* = -V_0\left[1 + \varepsilon\cos\left(\frac{\pi z^*}{d}\right)\right] \tag{2.1}$$

where $\varepsilon(\ll 1)$ is the amplitude of the suction velocity.

The velocity and temperature fields are independent of x since the channel is infinitely long along the $x-$ direction. The flow itself will be three dimensional due to cross flow.

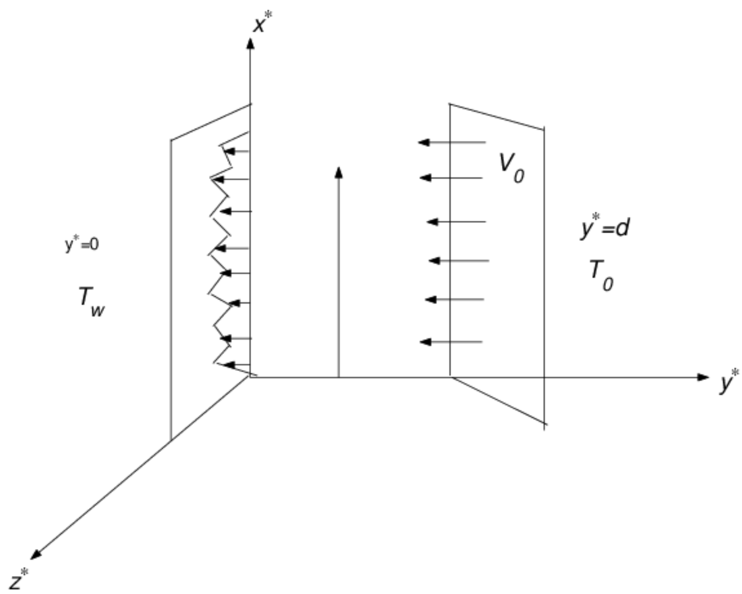

Fig.1. Physical model and co-ordinates system.

Let u^*, v^*, w^* be the velocity components in the direction of the $x^* -, y^* -, z^* -$ axes respectively. The problem is governed by the following equations

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0, \tag{2.2}$$

$$v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = \nu \left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}} \right) + g\beta \left(T^* - T_0 \right), \tag{2.3}$$

$$v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}} \right), \tag{2.4}$$

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*} + \nu \left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}} \right), \tag{2.5}$$

$$v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} = \nu \left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}} \right) - \frac{1}{\rho C_p} \frac{\partial q_r^*}{\partial y^*} \tag{2.6}$$

where ν is the kinematic coefficient of viscosity, ρ is the density, p^* is the fluid pressure, g is the acceleration due to gravity, β is the thermal expansion and C_p is the specific heat at constant pressure.

The equation of conservation of radiative heat transfer per unit volume for all wavelengths is

$$\nabla . q_r^* = \int_0^\infty K_\lambda \left(T^* \right) \left(4 e_{\lambda h} \left(T^* \right) - G_\lambda \right) d\lambda$$

where $e_{\lambda h}$ is Plank's function and the incident radiation G_λ is defined as

$$G_\lambda = \frac{1}{\pi} \int_{\Omega = 4\pi} e_\lambda (\Omega) d\Omega$$

$\nabla . q_r^*$ is the radiative flux divergence and Ω is the solid angle. Now, for an optically thin fluid exchanging radiation with an isothermal flat plate at temperature T_0 and according to the above definition for the radiative flux divergence and Kirchhoffs law, the incident radiation is given by $G_\lambda = 4 e_{\lambda h} \left(T_0 \right)$ then

$$\nabla . q_r^* = 4 \int_0^\infty K_\lambda \left(T^* \right) \left(e_{\lambda h} \left(T^* \right) - e_{\lambda h} \left(T_0 \right) \right) d\lambda .$$

Expanding $K_\lambda \left(T^* \right)$ and $e_{\lambda h} \left(T_0 \right)$ in a Taylor series around T_0, for small $\left(T^* - T_0 \right)$, we can rewrite the radiative flux divergence as

$$\nabla . q_r^* = 4\left(T^* - T_0\right)\int_0^\infty K_{\lambda_0}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_0 d\lambda$$

where $K_{\lambda_0} = K_{\lambda(T_0)}$.

Hence an optical thin limit for a non-gray gas near equilibrium yields, the following relation

$$\nabla . q_r^* = 4\left(T^* - T_0\right) I ,$$

and hence

$$\frac{\partial q_r^*}{\partial y^*} = 4\left(T^* - T_0\right) I$$

where

$$I = \int_0^\infty K_{\lambda_0}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_0 d\lambda .$$

The boundary conditions of the problem are

$$u^* = 0, \quad v^* = -V_0\left[1 + \varepsilon \cos\left(\frac{\pi}{d} z^*\right)\right], \quad w^* = 0, \quad T^* = T_w \quad \text{at} \quad y^* = 0 ,$$
$$u^* = U_0 + L_1 \frac{\partial u^*}{\partial y^*}, \quad v^* = -V_0, \quad w^* = 0, \quad T^* = T_0, \quad p^* = p_\infty \quad \text{at} \quad y^* = d. \tag{2.7}$$

Introducing the non dimensional variables

$$y = \frac{y^*}{d}, \quad z = \frac{z^*}{d}, \quad p = \frac{p^*}{\rho V_0^2}, \quad u = \frac{u^*}{U_0}, \quad v = \frac{v^*}{V_0}, \quad w = \frac{w^*}{V_0}, \quad \theta = \frac{\left(T^* - T_0\right)}{\left(T_w - T_0\right)}, \tag{2.8}$$

Eqs (2.2)-(2.6) become

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 , \tag{2.9}$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \mathrm{Gr}\theta , \tag{2.10}$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right), \tag{2.11}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right), \tag{2.12}$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{\mathrm{RePr}}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)-F\theta \tag{2.13}$$

where $\mathrm{Re}=V_0 d/\nu$, the Reynolds number, $\mathrm{Pr}=\nu/\rho$, the Prandtl number and $\mathrm{Gr}=dg\beta\left(T_w-T_0\right)/V_0^2$, the Grashoff number, $F=4Id/\rho C_p V_0$, the radiation parameter, $h=L_1/d$, the slip parameter. Using Eq.(2.8), the boundary conditions (2.7) become

$$\begin{aligned}&u=0,\quad v=-\left[1+\varepsilon\cos(\pi z)\right],\quad w=0,\quad \theta=1,\quad \text{at}\quad y=0,\\ &u=1+h\frac{\partial u}{\partial y},\quad v=-1,\quad w=0,\quad \theta=0,\quad p=\frac{p_\infty}{\rho V^2}\quad \text{at}\quad y=1.\end{aligned} \tag{2.14}$$

3. Solution of the problem

In order to solve the differential Eqs (2.9)-(2.13), we assume the solution of the following form

$$\begin{aligned}&u(y,z)=u_0(y)+\varepsilon u_1(y,z)+\varepsilon^2 u_2(y,z)+\cdots,\\ &v(y,z)=v_0(y)+\varepsilon v_1(y,z)+\varepsilon^2 v_2(y,z)+\cdots,\\ &w(y,z)=w_0(y)+\varepsilon w_1(y,z)+\varepsilon^2 w_2(y,z)+\cdots,\\ &p(y,z)=p_0(y)+\varepsilon p_1(y,z)+\varepsilon^2 p_2(y,z)+\cdots,\\ &\theta(y,z)=\theta_0(y)+\varepsilon\theta_1(y,z)+\varepsilon^2\theta_2(y,z)+\cdots.\end{aligned} \tag{3.1}$$

On substituting Eq.(3.1) in Eqs (2.9)-(2.13) and equating the terms independent of ε, we get the following system of differential equations

$$v_0'=0, \tag{3.2}$$

$$u_0''-\mathrm{Re}\,v_0 u_0'=-\mathrm{Re}\,\mathrm{Gr}\,\theta_0, \tag{3.3}$$

$$\theta_0''-\mathrm{Re}\,\mathrm{Pr}\,v_0\theta_0'-F\mathrm{Re}\,\mathrm{Pr}\,\theta_0=0 \tag{3.4}$$

where primes denote differentiation with respect to y and the corresponding boundary conditions become

$$u_0 = 0, \quad v_0 = -1, \quad \theta_0 = 1 \quad \text{at} \quad y = 0, \quad \text{and} \quad u_0 = 1 + h\frac{\partial u_0}{\partial y},$$

$$v_0 = -1, \quad \theta_0 = 0 \quad \text{at} \quad y = 1. \tag{3.5}$$

The solution of Eqs (3.2) - (3.4), subject to the boundary conditions (3.5) are

$$v_0(y) = -1, \tag{3.6}$$

$$\theta_0(y) = \frac{1}{\left(e^{-m_2} - e^{-m_1}\right)}\left[e^{-m_2}e^{-m_1 y} - e^{-m_1}e^{-m_2 y}\right], \tag{3.7}$$

$$u_0(y) = A_1 e^{-m_1 y} + A_2 e^{-m_2 y} + A_3 + A_4 e^{-\mathrm{Re}\, y} \tag{3.8}$$

where

$$A_1 = \frac{-\mathrm{Gr}\,\mathrm{Re}\, e^{-m_2}}{\left(e^{-m_2} - e^{-m_1}\right) m_1 \left(m_1 - \mathrm{Re}\right)}, \tag{3.9}$$

$$A_2 = \frac{\mathrm{Gr}\,\mathrm{Re}\, e^{-m_1}}{\left(e^{-m_2} - e^{-m_1}\right) m_2 \left(m_2 - \mathrm{Re}\right)},$$

$$A_3 = \frac{\left[1 + A_1\left\{e^{-\mathrm{Re}}\left(1 + \mathrm{Re}\, h\right) - e^{-m_1}\left(1 + m_1 h\right)\right\} + A_2\left\{e^{-\mathrm{Re}}\left(1 + \mathrm{Re}\, h\right) - e^{-m_1}\left(1 + m_1 h\right)\right\}\right]}{1 - e^{-Re}\left(1 + \mathrm{Re}\, h\right)},$$

$$A_4 = -\frac{\left[1 + A_1\left\{1 - e^{-m_1}\left(1 + m_1 h\right)\right\} + A_2\left\{1 - e^{-m_1}\left(1 + m_1 h\right)\right\}\right]}{1 - e^{-\mathrm{Re}}\left(1 + \mathrm{Re}\, h\right)}.$$

On substituting Eq.(3.1) in Eqs (2.9)-(2.13) and equating the coefficient of ε, we get the following system of differential equations

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \tag{3.10}$$

$$v_0\frac{\partial u_1}{\partial y} + v_1\frac{\partial u_0}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) + \mathrm{Gr}\theta_1, \tag{3.11}$$

$$v_0\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right), \tag{3.12}$$

$$v_0 \frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right), \tag{3.13}$$

$$v_0 \frac{\partial \theta_1}{\partial y} + v_1 \frac{\partial \theta_0}{\partial y} = \frac{1}{\mathrm{RePr}}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) - F\theta_1 . \tag{3.14}$$

The corresponding boundary conditions become

$$\begin{aligned} &u_1 = 0, \quad v_1 = -\cos(\pi z), \quad w_1 = 0, \quad \theta_1 = 0 \quad \text{at} \quad y = 0, \\ &u_1 = h\frac{\partial u_1}{\partial y}, \quad v_1 = 0, \quad w_1 = 0, \quad \theta_1 = 0 \quad \text{at} \quad y = 1. \end{aligned} \tag{3.15}$$

These are the linear partial differential equations describing the three dimensional flow. To solve Eqs (3.10)-(3.14), we assume velocity components and pressure in the following form

$$\begin{aligned} &u_1(y,z) = u_{11}(y)\cos(\pi z), \\ &v_1(y,z) = v_{11}(y)\cos(\pi z), \\ &w_1(y,z) = -\frac{1}{\pi} v'_{11}(y)\sin(\pi z), \\ &p_1(y,z) = p_{11}(y)\cos(\pi z), \\ &\theta_1(y,z) = \theta_{11}(y)\cos(\pi z) \end{aligned} \tag{3.16}$$

v_1 and w_1 are so chosen that the continuity Eq.(3.10) is satisfied automatically.

Substituting Eq.(3.16) in Eqs (3.11)-(3.14) and comparing the coefficients of harmonic terms, we obtain the following set of differential equations

$$v''_{11} + \mathrm{Re} v'_{11} - \pi^2 v_{11} = \mathrm{Re} p'_{11}, \tag{3.17}$$

$$v'''_{11} + \mathrm{Re} v''_{11} - \pi^2 v'_{11} = \mathrm{Re}\pi^2 p_{11}, \tag{3.18}$$

$$\theta''_{11} + \mathrm{RePr}\theta'_{11} - \left(F\mathrm{RePr} + \pi^2\right)\theta_{11} = \mathrm{RePr} v_{11}\theta'_0, \tag{3.19}$$

$$u''_{11} + \mathrm{Re} u'_{11} - \pi^2 u_{11} = \mathrm{Re} v_{11} u'_0 - \mathrm{GrRe}\theta_{11}. \tag{3.20}$$

When $F = 0$, Eqs (3.17)-(3.20) coincide with Eqs (3.17)-(3.20) of Guria and Jana [19].

The corresponding boundary conditions are

$$u_{11} = 0, \quad v_{11} = -1, \quad v'_{11} = 0, \quad \theta_{11} = 0 \quad \text{at} \quad y = 0,$$

$$u_{11} = h\frac{\partial u_{11}}{\partial y}, \quad v_{11} = 0, \quad v_{11}' = 0, \quad \theta_{11} = 0 \quad \text{at} \quad y = 1. \tag{3.21}$$

Solutions of Eqs (3.17)-(3.20) subject to Eq.(3.21) and on using Eqs (3.6)-(3.8) yield

$$v_1(y,z) = \left[A_5 e^{-m_3 y} + A_6 e^{-m_4 y} + A_7 e^{\pi y} + A_8 e^{-\pi y}\right]\cos(\pi z), \tag{3.22}$$

$$w_1(y,z) = \frac{1}{\pi}\left[A_5 m_3 e^{-m_3 y} + A_6 m_4 e^{-m_4 y} - A_7 \pi e^{\pi y} + A_8 \pi e^{-\pi y}\right]\sin(\pi z), \tag{3.23}$$

$$p_1(y,z) = \left[A_7 e^{\pi y} + A_8 e^{-\pi y}\right]\cos(\pi z), \tag{3.24}$$

$$\begin{aligned}\theta_1(y,z) = \Big[& B_1 e^{-\lambda_1 y} + B_2 e^{-\lambda_2 y} + K_1\Big\{B_3 e^{-(m_1+m_3)y} + B_4 e^{-(m_1+m_4)y} + \\ & + B_5 e^{(\pi-m_1)y} + B_6 e^{-(\pi+m_1)y}\Big\} + K_2\Big\{B_7 e^{-(m_2+m_3)y} + B_8 e^{-(m_2+m_4)y} + \\ & B_9 e^{(\pi-m_2)y} + B_{10} e^{-(\pi+m_2)y}\Big\}\Big]\cos\pi z,\end{aligned} \tag{3.25}$$

$$\begin{aligned}u_1(y,z) = \Big[& A e^{-m_3 y} + B e^{-m_4 y} + C e^{-\lambda_1 y} + D e^{-\lambda_2 y} + \\ & + C_1 e^{-(m_1+m_3)y} + C_2 e^{-(m_1+m_4)y} + C_3 e^{(\pi-m_1)y} + C_4 e^{-(\pi+m_1)y} + \\ & + C_5 e^{-(m_2+m_3)y} + C_6 e^{-(m_2+m_4)y} + C_7 e^{(\pi-m_2)y} + C_8 e^{-(\pi+m_2)y} + \\ & + C_9 e^{-(m_3+\mathrm{Re})y} + C_{10} e^{-(m_4+\mathrm{Re})y} + C_{11} e^{(\pi-\mathrm{Re})y} + C_{12} e^{-(\pi+\mathrm{Re})y}\Big]\cos\pi z\end{aligned} \tag{3.26}$$

where

$$m_{1,2} = \frac{1}{2}\left\{\mathrm{Re}\,\mathrm{Pr} \pm \sqrt{\mathrm{Re}^2\mathrm{Pr}^2 + 4F\mathrm{Re}\,\mathrm{Pr}}\right\},$$

$$m_{3,4} = \frac{1}{2}\left\{\mathrm{Re} \pm \sqrt{\mathrm{Re}^2 + 4\pi^2}\right\},$$

$$\lambda_{1,2} = \frac{1}{2}\left\{\mathrm{Re}\,\mathrm{Pr} \pm \sqrt{\mathrm{Re}^2\mathrm{Pr}^2 + 4\left(F\mathrm{Re}\,\mathrm{Pr} + \pi^2\right)}\right\},$$

$$r_1 = e^{-m_3} - \frac{1}{2\pi}\left[e^{\pi}(\pi - m_3) + e^{-\pi}(\pi + m_3)\right],$$

$$r_2 = e^{-m_4} - \frac{1}{2\pi}\left[e^{\pi}(\pi - m_4) + e^{-\pi}(\pi + m_4)\right],$$

$$r_3 = m_3 e^{-m_3} + \frac{1}{2}\left[e^{\pi}\left(\pi - m_3\right) - e^{-\pi}\left(\pi + m_3\right)\right],$$

$$r_4 = m_4 e^{-m_4} + \frac{1}{2}\left[e^{\pi}\left(\pi - m_4\right) - e^{-\pi}\left(\pi + m_4\right)\right], \tag{3.27}$$

$$A_5 = \left[\pi r_2\left(e^{\pi} - e^{-\pi}\right) + r_4\left(e^{\pi} + e^{-\pi}\right)\right] \Big/ 2\left(r_1 r_4 - r_2 r_3\right),$$

$$A_6 = -\left[\pi r_1\left(e^{\pi} - e^{-\pi}\right) + r_3\left(e^{\pi} + e^{-\pi}\right)\right] \Big/ 2\left(r_1 r_4 - r_2 r_3\right),$$

$$A_7 = -\frac{1}{2\pi}\left[\pi + A_5\left(\pi - m_3\right) + A_6\left(\pi - m_4\right)\right],$$

$$A_8 = -\frac{1}{2\pi}\left[\pi + A_5\left(\pi + m_3\right) + A_6\left(\pi + m_4\right)\right],$$

$$K_1 = \frac{-\mathrm{Re}\,\mathrm{Pr}\, m_1 e^{-m_2}}{\left(e^{-m_2} - e^{-m_1}\right)}, \qquad K_2 = \frac{\mathrm{Re}\,\mathrm{Pr} m_2 e^{-m_1}}{\left(e^{-m_2} - e^{-m_1}\right)}.$$

The other constants are not given here to save space. $v_1(y,z)$, $w_1(y,z)$ and $p_1(y,z)$ are the same as obtained by Guria and Jana [14] and [15]. This is due to fact that the velocity components $v_1(y,z)$, $w_1(y,z)$ and pressure $p_1(y,z)$ are independent of the radiation effect. $\theta(y,z)$ is same as obtained by Guria and Jana [15]. This is due to fact that $\theta(y,z)$ is independent of the slip effect.

4. Results and discussion

In order to get a physical insight into the problem the velocity field, temperature field, shear stresses and Nusselt number have been discussed for various non dimensional parameters. We have plotted the non-dimensional velocity u in Figs 2-5 for different values of the slip parameter, radiation parameter, Grashoff number and Reynolds number for $\mathrm{Pr} = 0.71$, $\varepsilon = 0.05$, $z = 0.0$. It is evident from Fig.2 that the velocity decreases with the increase of the slip parameter. It is found that the velocity decreases with the increase of the radiation parameter but near the right plate it increases. For cooling of the plate $(\mathrm{Gr} > 0)$, the velocity increases with the increase of the Grashoff number and decreases near the right plate but the reverse effect is observed for heating of the plate $(\mathrm{Gr} < 0)$.

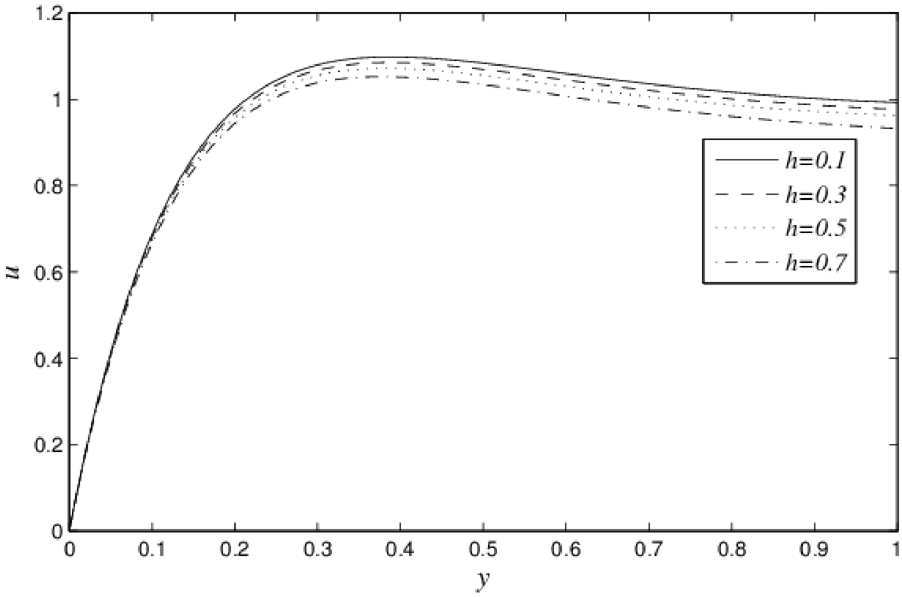

Fig.2. Primary velocity u for Gr=*5.0*, Re=*5.0*, Pr=*0.71*, $\varepsilon = 0.05$, *z=0.0*.

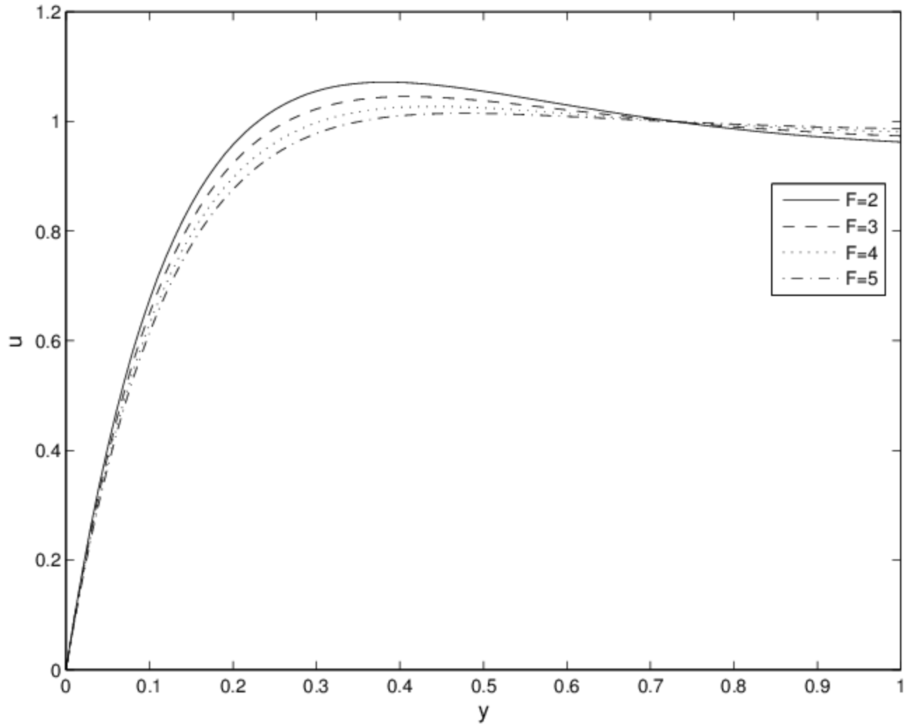

Fig.3. Primary velocity u for Gr=*5.0*, Re=*5.0*, Pr=*0.71*, $\varepsilon = 0.05$, *z=0.0*.

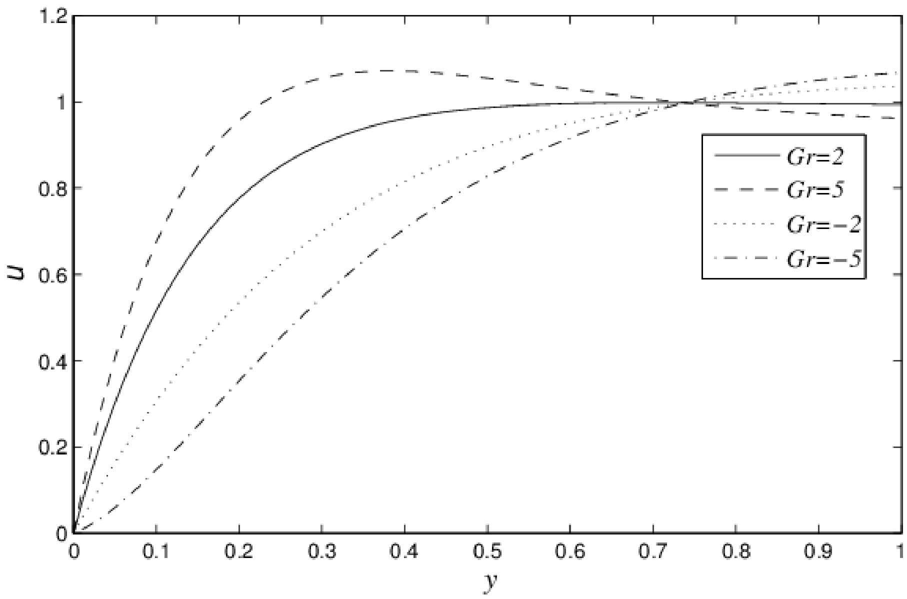

Fig.4. Primary velocity u for Gr=*5.0*, Re=*5.0*, Pr=*0.71*, $\varepsilon = 0.05$, *z=0.0*.

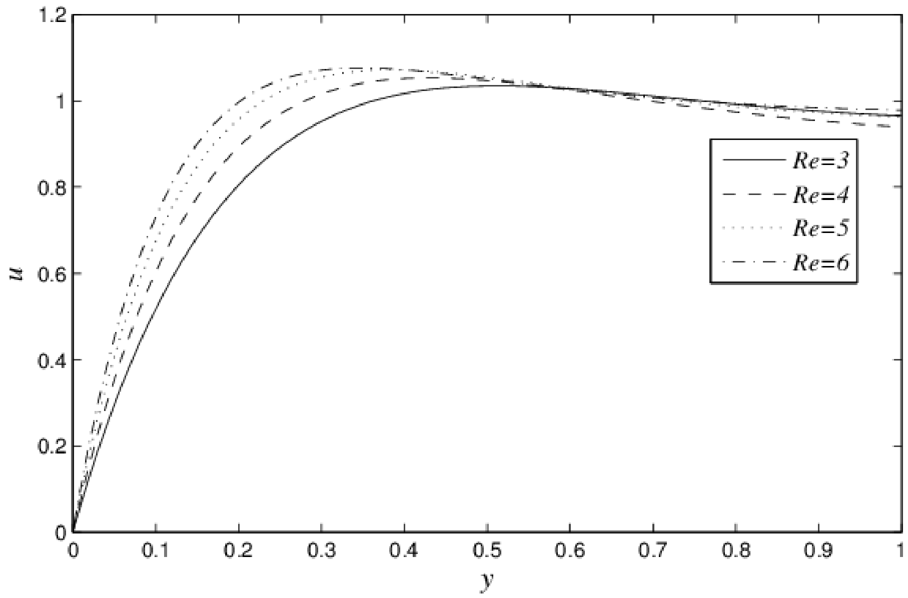

Fig.5. Primary velocity u for Gr=*5.0*, Re=*5.0*, Pr=*0.71*, $\varepsilon = 0.05$, *z=0.0*.

Knowing the velocity field it is interesting to know the shear stress at the plate. The shear stress at the plate $y^* = 0$ due to the primary flow is given by

$$\tau_x^* = \mu\left(\frac{\partial u^*}{\partial y^*}\right)_{y^*=0} = \frac{\mu U_0}{d}\left(\frac{\partial u}{\partial y}\right)_{y=0}. \tag{4.1}$$

In a non-dimensional form the shear stress at the plate $y = 0$ can be written as

$$\tau_x = \frac{\tau_x^* d}{\mu U_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0},$$

$$= u_0'(0) + \varepsilon u_1'(0),$$

$$\begin{aligned}&= -\left(A_1 m_1 - A_2 m_2 - A_4 \mathrm{Re}\right) + \varepsilon\left[-Am_3 - Bm_4 - C\lambda_1 - D\lambda_2 + \right.\\&-C_1\left(m_1 + m_3\right) - C_2\left(m_1 + m_4\right) + C_3\left(\pi - m_1\right) - C_4\left(\pi + m_1\right) +\\&-C_5\left(m_2 + m_3\right) - C_6\left(m_2 + m_4\right) + C_7\left(\pi - m_2\right) - C_8\left(\pi + m_2\right) - C_9\left(m_3 + \mathrm{Re}\right) +\\&\left.-C_{10}\left(m_4 + \mathrm{Re}\right) + C_{11}\left(\pi - \mathrm{Re}\right) + C_{11}\left(\pi - \mathrm{Re}\right) - C_{12}\left(\pi + \mathrm{Re}\right)\right]\cos\pi z.\end{aligned} \tag{4.2}$$

The shear stress due to the primary flow in terms of τ_x is given in Tabs 1 and 2 for different values of the slip parameter, radiation parameter, Reynolds number and Grashoff number and for Gr=5.0, $\varepsilon = 0.05$, $z = 0.0$.

Table.1. Shear stress component due to the main flow for $\mathrm{Gr} = 2.0$, $\mathrm{Pr} = 0.71$, $\mathrm{Re} = 2.0$, $z = 0.0$.

h			τ_x		
	$F = 2.0$	$F = 3.0$	$F = 4.0$	$F = 5.0$	
2	3.50	3.57	3.66	3.75	
3	3.44	3.52	3.62	3.73	
4	3.39	3.48	3.59	3.71	
5	3.34	3.44	3.56	3.69	

Table.2. Shear stress component due to the main flow for $F = 2.0$, $\mathrm{Pr} = 0.71$, $h = 0.5$, $z = 0.0$.

Gr			τ_x		
	$\mathrm{Re} = 2.0$	$\mathrm{Re} = 2.5$	$\mathrm{Re} = 3.0$	$\mathrm{Re} = 3.5$	
2	3.66	4.21	4.69	5.34	
3	4.10	4.75	5.37	6.06	
4	4.53	5.29	6.05	6.18	

The shear stress due to the primary flow increases with an increase in the Reynolds number, Grashoff number as well as the radiation parameter but decreases with an increase in the slip parameter.

Figure 6 represents the variations of θ for different values of the radiation parameter. The temperature profile decreases with the increase of the radiation parameter.

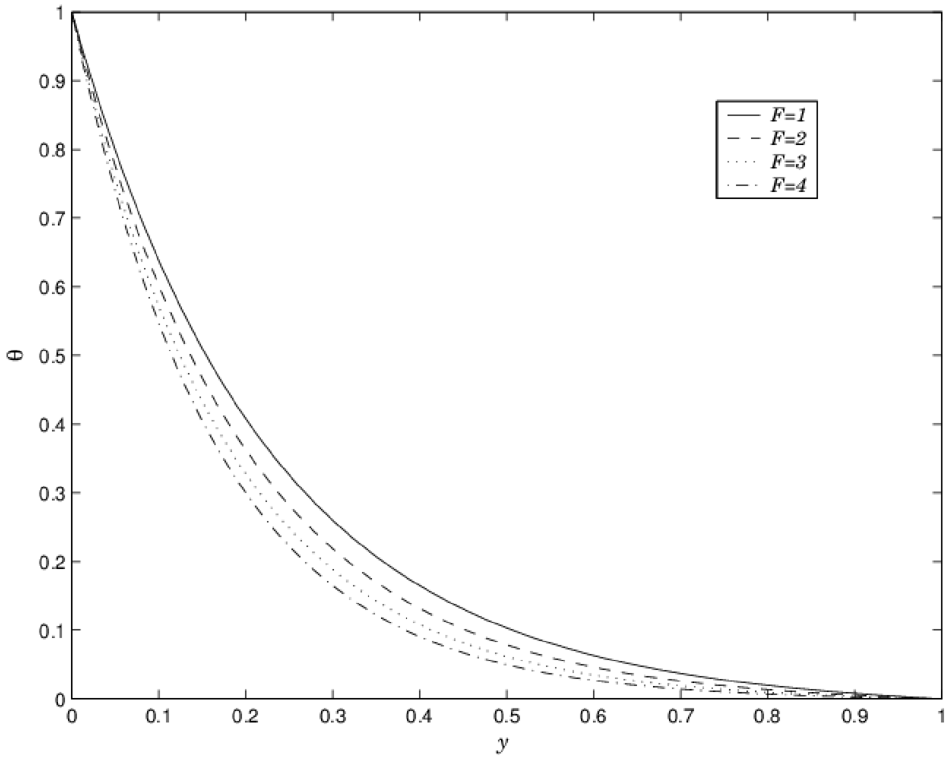

Fig.6. Temperature profile θ for $\mathrm{Gr}=5.0$, $\mathrm{Re}=5.0$, $\mathrm{Pr}=0.71$, $\varepsilon=0.05$, $z=0.0$.

The Nusselt number is the great measure of heat transfer from the plates to the fluid flowing up between the plates due to its practical importance. On definition of the Nusselt number depends on the rate of heat transfer from the plate to the fluid. The heat transfer coefficient from the plate to the fluid may be calculated as

$$q=-k\left(\frac{\partial T^*}{\partial y^*}\right)_{y^*=0}=-\frac{k\left(T_w-T_0\right)}{d}\left(\frac{\partial\theta}{\partial y}\right)_{y=0}. \tag{4.3}$$

In a non-dimensional form the heat transfer coefficient at the plate $y=0$ is given by

$$\begin{aligned}\mathrm{Nu}_1&=\frac{qd}{k\left(T_w-T_0\right)}=-\left(\frac{\partial\theta}{\partial y}\right)_{y=0}=-\theta_0'(0)-\varepsilon\theta_1'(0),\\&=\frac{1}{\left(e^{-m_2}-e^{-m_1}\right)}\left[-m_1e^{-m_2}+m_2e^{-m_1}\right]+\varepsilon\left[B_1\lambda_1+B_2\lambda_2+K_1B_3\left(m_1+m_3\right)+\right.\\&+K_1B_4\left(m_1+m_4\right)-K_1B_5\left(\pi-m_1\right)+K_1B_6\left(\pi+m_1\right)+K_2B_7\left(m_2+m_3\right)+\\&\left.+K_2B_8\left(m_2+m_4\right)-K_2B_9\left(\pi-m_2\right)+K_2B_{10}\left(\pi+m_2\right)\right]\cos\pi z,\end{aligned} \tag{4.4}$$

and the heat transfer coefficient at the plate $y=1$ is given by

$$\mathrm{Nu}_2 = \frac{qd}{k\left(T_w - T_0\right)} = -\left(\frac{\partial\theta}{\partial y}\right)_{y=1} = -\theta_0'\left(1\right) - \varepsilon\theta_1'\left(1\right),$$

$$\begin{aligned}
&= \frac{\left(m_2 - m_1\right)e^{-\left(m+m_2\right)}}{\left(e^{-m_2} - e^{-m_1}\right)} + \varepsilon\Big[B_1\lambda_1 e^{-\lambda_1} + B_2\lambda_2 e^{-\lambda_2} + K_1B_3\left(m_1 + m_3\right)e^{-\left(m_1+m_3\right)} + \\
&+K_1B_4\left(m_1 + m_4\right)e^{-\left(m_1+m_4\right)} - K_1B_5\left(\pi - m_1\right)e^{\left(\pi-m_1\right)} + K_1B_6\left(\pi + m_1\right)e^{-\left(\pi+m_1\right)} + \\
&+K_2B_7\left(m_2 + m_3\right)e^{-\left(m_2+m_3\right)} + K_2B_8\left(m_2 + m_4\right)e^{-\left(m_2+m_4\right)} + \\
&-K_2B_9\left(\pi - m_2\right)e^{\left(\pi-m_2\right)} + K_2B_{10}\left(\pi + m_2\right)e^{-\left(\pi+m_2\right)}\Big]\cos\pi z.
\end{aligned} \tag{4.5}$$

We have plotted the rate of heat transfer in terms of the Nusselt number for different values of the radiation parameter, Prandtl number and Reynolds number and for $\mathrm{Gr} = 5.0$, $\varepsilon = 0.05$, $z = 0.0$. In order to be realistic, the rate of heat transfer of air $\left(\mathrm{Pr} = 0.71\right)$ is plotted in Figs 7 and 8 and for water $\left(\mathrm{Pr} = 7.0\right)$ in Figs 9 and 10. It is found that the rate of heat transfer is much lower in the case of water than in air at the plate $y = 1$ and a reverse effect is seen at the plate $y = 0$. Also, it is seen that the heat transfer is more pronounced at the plate $y = 0$. The rate of heat transfer at the plate $y = 0$ increases whereas that at the plate $y = 1$ decreases with an increase in the radiation parameter in case of both water and air.

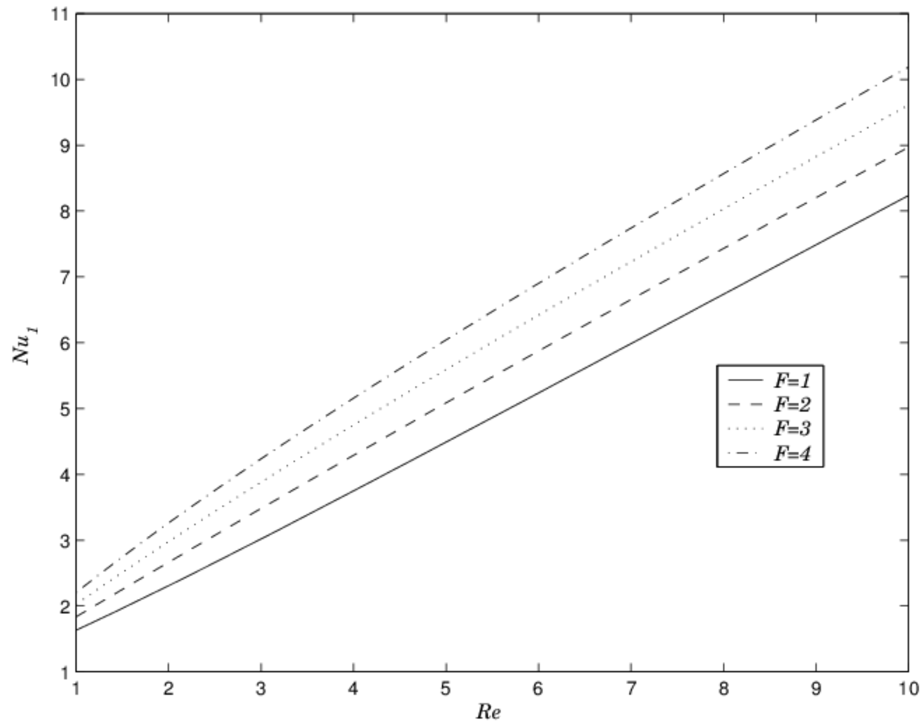

Fig.7. Rate of heat transfer Nu_1 for $\mathrm{Pr} = 0.71$, $\mathrm{Gr} = 5.0$, $\varepsilon = 0.05$, $z = 0.0$.

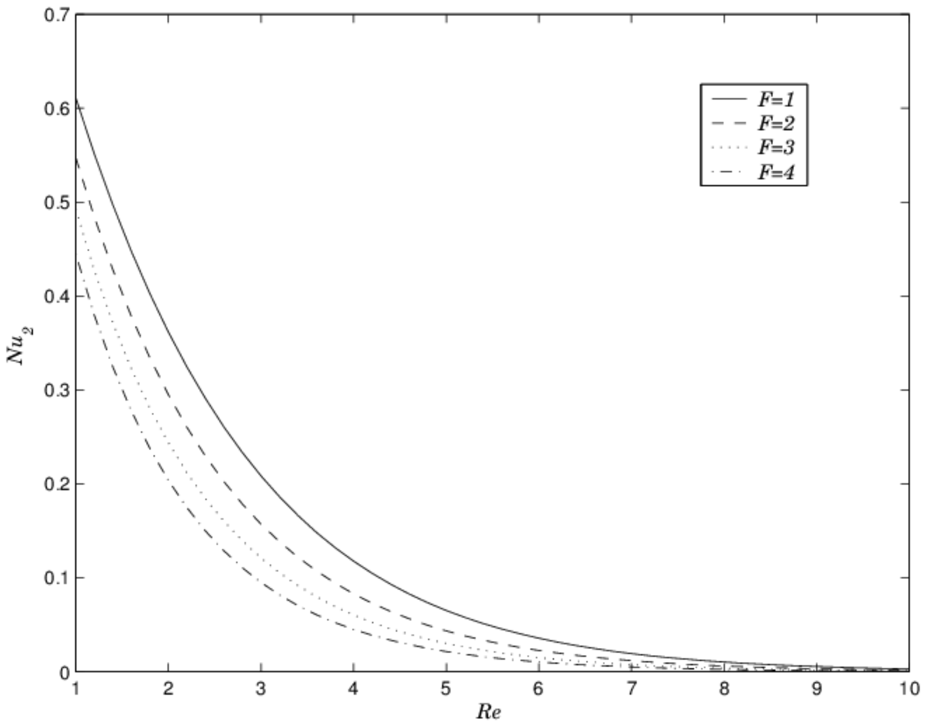

Fig.8. Rate of heat transfer Nu_2 for $\mathrm{Pr} = 0.71$, $\mathrm{Gr} = 5.0$, $\varepsilon = 0.05$, $z = 0.0$.

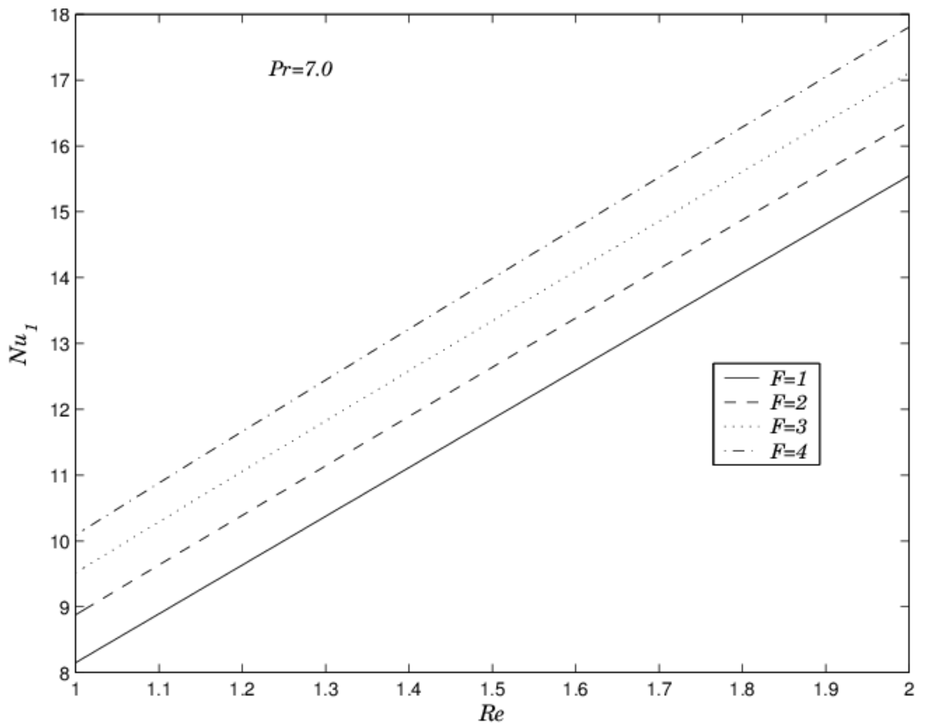

Fig.9. Rate of heat transfer Nu_1 for $\mathrm{Pr} = 7.0$, $\mathrm{Gr} = 5.0$, $\varepsilon = 0.05$, $z = 0.0$.

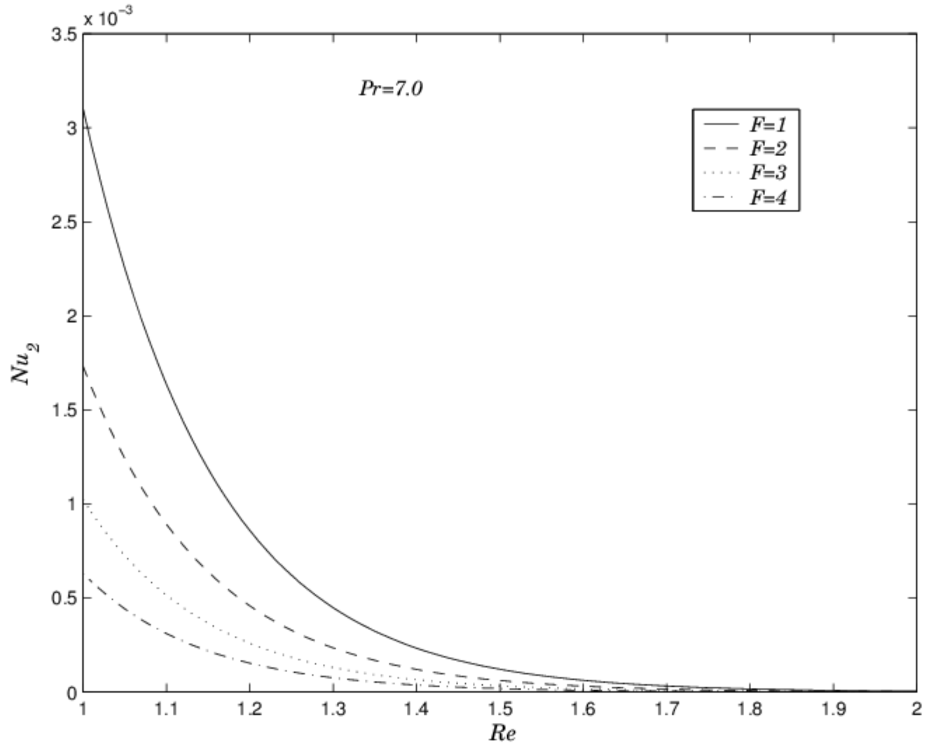

Fig.10. Rate of heat transfer Nu_2 for $Pr = 7.0$, $Gr = 5.0$, $\varepsilon = 0.05$, $z = 0.0$.

It is interesting to note that with an increase in the Reynolds number, the Nusselt number at the plate $y = 0$ rapidly increases whereas that at the plate $y = 1$ tends to zero.

Conclusion

The effect of slip on the steady flow of a viscous incompressible fluid between two vertical porous plates is studied in the presence of radiation. It is found that the slip effect reduces the primary velocity. It is also found that the rate of heat transfer at the left plate (plate with periodic suction) increases whereas that at the right plate(plate with constant injection) decreases with an increase in the radiation parameter *F*, in case of both air and water. It is seen that the heat transfer is more pronounced at the left plate. The rate of heat transfer is much lower in the case of water than that of air at the right plate and the reverse effect is seen at the left plate. Our problem is a non-trivial extension of Guria and Jana [15] in slip flow regime.

Acknowledgement

One of the the authors thanks the UGC of India for financial support for carrying out the research via a Minor Research Project in the year 2012-13 of No. F. PSW-86/12-13(ERO).

Nomenclature

$A_i, i = 1, \cdots 8$ – constants
A, B, C, D – constants
$B_i, i = 1, \cdots 10$ – constants
$C_i, i = 1, \cdots 12$ – constants

C_p – specific heat at constant pressure
d – channel width
F – radiation parameter
Gr – Grashoff number
g – gravitational acceleration
h – slip parameter
K, K_1, K_2 – constants
k – thermal conductivity
$m_i, i = 1, \cdots 4$ – constants
$\mathrm{Nu}_1, \mathrm{Nu}_2$ – Nusselt number at the plates $y = 0$ and $y = 1$
Pr – Prandtl number
p – dimensionless pressure
p^* – pressure
q – local heat transfer at the plate
Re – Reynolds number
$r_i, i = 1, \cdots 4$ – constants
T^* – temperature of the fluid
T_w – plate temperature $\left(y^* = 0\right)$
T_0 – plate temperature $\left(y^* = d\right)$
u, v, w – dimensionless velocity components in the x, y, z -axes respectively
u^*, v^*, w^* – velocity components in the x, y, z -axes respectively
V_0 – constant suction velocity
x, y, z – dimensionless Cartesian coordinate system
x^*, y^*, z^* – Cartesian coordinates system
β – coefficient of thermal expansion
ε – amplitude of suction velocity
θ – non-dimensional temperature
λ_1, λ_2 – constants
μ – viscosity
μ_1, μ_2 – constants
ν – kinematic viscosity
ρ – density
τ_x, τ_z – shear stress due to primary and secondary flows

Dimensional variables are indicated by dropping asterisk and are defined in Eq.(2.8).

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Received: July 14, 2015

Revised: March 22, 2016