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Brief note

APPLICATION OF GRAPHS IN THE ANALYSIS OF VIBRATING 3-DIMENSIONAL SYSTEMS

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Vibrating 3-dimensional mechanical systems are described by hybrid graphs. By dividing the hybrid graph into a tree and associated co-tree it is possible to describe in terms of matrices: across-flow-variables, stiffness and flexibility. The application of the formalism of graphs makes it possible to analyze mechanical systems in terms of dynamic characteristic and trajectory motion of determinate points.

Key words: abstract graph, dynamic flexibility, transfer functions.

1. Introduction

The foundations for the contemporary analysis can be traced back to the early research in the discipline of linear graph theory applied to engineering problems.[1-5]. Koenig and Blacwell [1] established unambiguously the link between linear graphs and physical systems. Among the continuum of mechanical system models we can distinguish labeling graphs, which can be used for formulating and formalizing mechanical tasks. Mechanical coupled systems are effectively modelled using hybrid graphs which combine characteristics of polar and flow graphs and are superior in terms of algorithmic approach. A hybrid graph used to describe a model of a mechanical system will depend on the selected algebraic method and applying matrixes will be the most convenient method to determine dynamic characteristics of the system.

2. A hybrid graph as a model of a vibrating 3-dimensional mechanical system

Using the symbols as described in [6] we will present a method of mapping a mechanical system with n degrees of freedom with linear conjugates into a hybrid graph $\tilde{X}_{01}^{S}$.

This system is presented as a dynamic structure (Fig.1) [6, 7]

$$\boldsymbol{S}=[{}_1\boldsymbol{S},\ {}_2\boldsymbol{S},\ {}_3\boldsymbol{S}], \tag{2.1}$$

where:
${}_1\boldsymbol{S}$- is a set of across variables, e.g., set of displacements
${}_2\boldsymbol{S}$-is a set of through variables, e.g., set of forces
${}_3\boldsymbol{S}$-is a set of conjugates, i.e., set of relations between across and flow variables.

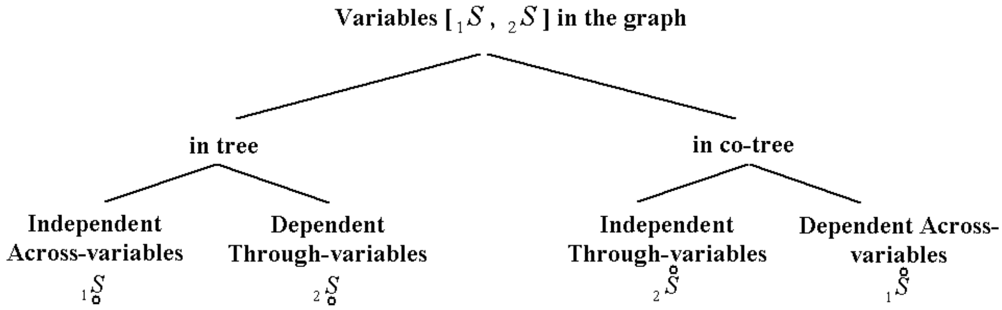

Fig.1. The variables of the graph - in the tree and co-tree.

Relationships between across variables $_1\boldsymbol{S}$ and flow variables $_2\boldsymbol{S}$ form a system of equations called coefficient polar equation in the form of Eq.(2.2) (Fig.2)

$$ {}_2\overset{\circ}{\boldsymbol{S}} = \overset{\circ}{W}(p)\,{}_1\overset{\circ}{\boldsymbol{S}}, \qquad \text{or} \qquad {}_1\underset{\circ}{\boldsymbol{S}} = {}_1\underset{\circ}{W}(p)_2\underset{\circ}{\boldsymbol{S}} \tag{2.2} $$

where:

$\overset{\circ}{W}(p)$ and $\underset{\circ}{W}(p)$ and are diagonal matrices of coefficient terminal equations.

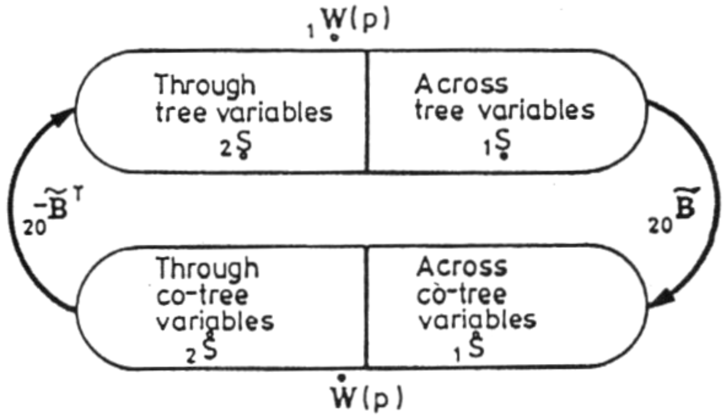

Fig.2. Relationship between across ($_1S$) and through ($_2S$) variables

A fundamental step in the modeling of vibrating 3-dimensional systems using graphs is assigning an abstract graph to the system

$$ X=[{}_1X,\ {}_2X,\ {}_3X], \tag{2.3} $$

where:

$_1X$-is a set of the graph's vertexes $_1X_{j1} \in {}_1X, j_1=1, 2, \ldots, n_1$,

$_2X={}_2X \cup {}_2X \cup {}_2X$ is a set of the graph's edges,

$_3X \subset {}_1X \times {}_2X \times {}_1X$ is a collection of relations meeting certain defined conditions [1].

The next step is clear, unambiguous mapping $f= \{_if\}$, $i=1, 2$ of variables describing the system into elements of an abstract graph, i.e.

$$ F: [{}_1\boldsymbol{S}, {}_2\boldsymbol{S}, {}_3\boldsymbol{S}] \rightarrow [{}_1X, {}_2X, {}_3X], \tag{2.4} $$

such that

$$ {}_{1}f:\left[{}_{2^{\circ}}X \cup {}_{2}^{\circ}X\right] \rightarrow \left[{}_{1}\boldsymbol{S}\right], \tag{2.5}$$

$$ {}_{2}f:[{}_{21}X] \rightarrow [{}_{3}\boldsymbol{S}], \tag{2.6}$$

and orientating edges we obtain the hybrid graph

$$ {}_{01}^{S}\tilde{X} = \left[X, {}_{1}f, {}_{2}f\right], \tag{2.7}$$

as a network model of the mechanical system with linear coupling.

To save the graph we need to represent the graph in an algebraic form. In this case, we will code the graph's structure using:
- matrix of the cuts (cocycles) ${}_{2}\boldsymbol{B} = {}_{0}\boldsymbol{E}_{20}\boldsymbol{B}$,
- matrix of the fundamentals cycles ${}_{3}\boldsymbol{B} = {}_{30}\boldsymbol{B}\,\boldsymbol{E}^{o}$,
- matrix of the across-flow-variables ${}_{1S}\boldsymbol{B}$,

wherein

$$ {}_{1S}\boldsymbol{B} = {}_{20}\boldsymbol{B} = -{}_{30}\boldsymbol{B}^{T}, \tag{2.8}$$

is the matrix dynamical system.

The matrix of across-flow-variables can be calculated based on the constructed sequence of independent hyper-cycles $\tilde{H}_{i}^{S}$, which may be determined from a graph ${}_{01}^{S}\tilde{X}$ or directly constructed from the model of the system. In the first case, a hybrid graph will be presented in the form equivalent to its sequence of sub-graphs, which are autonomous hyper-cycles or independent conjugates contours, namely

$$ {}_{01}^{S}\tilde{X} = {}_{i}U\left(\tilde{H}_{i}^{S}\right), \qquad i = 1,2,\ldots n_{2}^{\circ}, \tag{2.9}$$

where n_{2}° is the cyclomatic number.

Using generalized rules for autonomous hyper-cocycles

$$ {}_{2}\boldsymbol{B}\,{}_{2}\boldsymbol{S} = \boldsymbol{0}, \tag{2.10}$$

and hyper-cycles

$$ {}_{2}\boldsymbol{B}\,{}_{1}\boldsymbol{S} = \boldsymbol{0}. \tag{2.11}$$

We can determine the transfer function (dynamic characteristics) or response of the system to a specific excitation by transforming the hybrid graph into a matrix graph.

To determinate specific transfer functions or system response to specific excitation, one should correctly order a set of $[{}_{1}\boldsymbol{S}]$ and $[{}_{2}\boldsymbol{S}]$ into four characteristic subsets ${}_{1\,o}\boldsymbol{S}$, ${}_{1}^{o}\boldsymbol{S}$, ${}_{2\,o}\boldsymbol{S}$, ${}_{2}^{o}\boldsymbol{S}$ follows directly from their classification through variables ${}_{2}\boldsymbol{S}$ and across variables ${}_{1}\boldsymbol{S}$ and form extraction in the graph tree ${}_{o}X$ and co-tree X^{o}.

Subsets of variables ${}_{1\,o}\boldsymbol{S}$, ${}_{1}^{o}\boldsymbol{S}$, ${}_{2\,o}\boldsymbol{S}$, ${}_{2}^{o}\boldsymbol{S}$ generate eight transfer functions, of which , e.g., ${}_{2}\boldsymbol{S}Y_{1}\boldsymbol{S}=Y(p)$ is a matrix of dynamic flexibility of the system.

To define these functions in the form of a matrix, the rules (2.10) and (2.11) should be transformed taking into account across-flow matrices (2.8) and by extracting subsets variables ${}_{1\,0}\boldsymbol{S}_w$, ${}^{0}_{1}\boldsymbol{S}_w$, ${}_{2\,0}\boldsymbol{S}_w$, ${}^{0}_{2}\boldsymbol{S}_w$ which define excitation.

Based on the obtained dependency of variables depending on expressing across variables ${}^{0}_{1}\boldsymbol{S}$ chords by independent across variables of the tree branch ${}^{0}_{1}\boldsymbol{S}$ (generalized coordinates) and depending on capturing across-flow variables ${}_{20}\boldsymbol{S}$ (generalized forces) we can construct a matrix graph of information flow.

Based on the matrix graph of information flow, we determine functions of the mechanical system under analysis and we conduct a numerical experiment using matrix products in numerical machines.

Due to the fact that the matrix graph of information flow has a loop, calculations require its reduction to a single edge with a resultant weight.

By applying methods of fictitious sources of across variables one can obtain in the place of a matrix graph with a loop, a matrix graph in the form of a simple path comprising serially connected edges. For this kind of a graph matrix of dynamic flexibility has the form

$$\boldsymbol{Y}(p) = -\left[{}_{1S}\boldsymbol{BW}(p)\cdot\left({}_{1S}\tilde{\boldsymbol{B}}\right)^{T}\right]^{-1}. \tag{2.12}$$

Using the appropriate program one can define a set of amplitude-frequency-phase characteristics of the system corresponding to the predetermined frequency of the dynamic excitation. This set may be an input to another program, which can be used to calculate the respective amplitudes and angles of phase shifts demanding components of the system response.

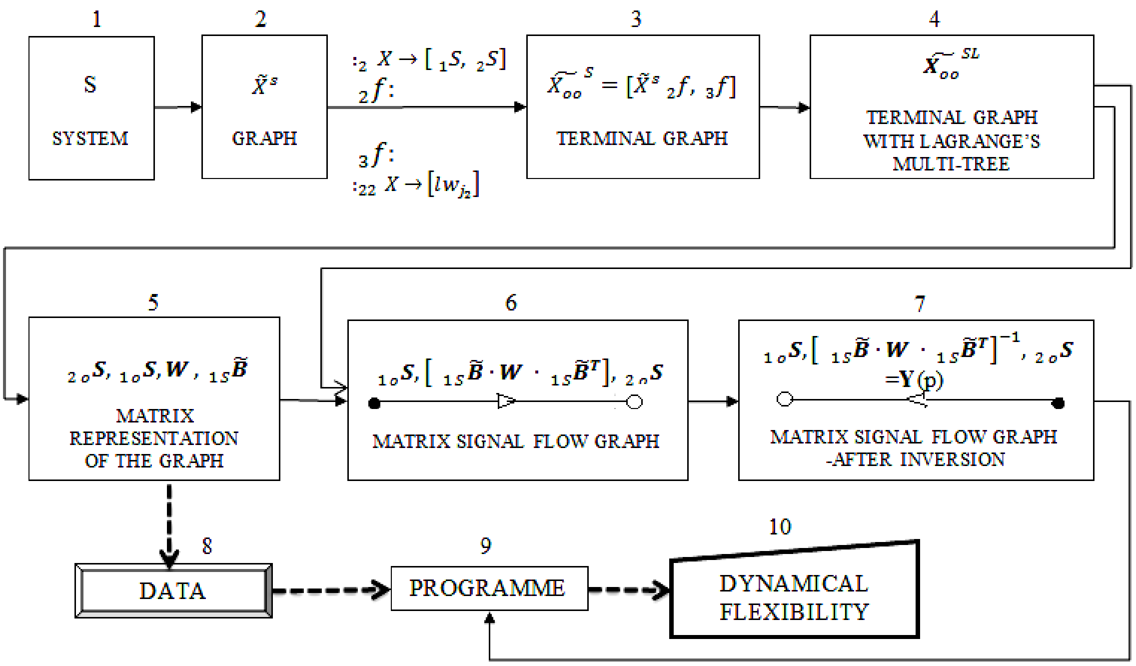

Fig.3. Block diagram procedure of determination of the dynamic flexibility.

In the case of the determination of the dynamic flexibility the method of the graph procedure is shown in the block diagram as follows (Fig.3):

Step 1) Select a model of a mechanical system **S.**

Step 2) This model is transformed to a non- planar graph.

Step 2/3) Define the functions on the graph.
Step 3) Edges are assigned variables, and we obtain a terminal graph as a model of the mechanical system.
Step 4) Select the Lagrange's multi-tree. For such a selection of the graph's tree the matrices describing its dynamical structure take the straight form.
Step 5) Select matrices of variables and terminal equations.
Step 6) Select a matrix signal flow graph in the form of a single branch with resultant weight.
Step 7) Select an inverse matrix signal flow graph.
Step 8) For characteristic parameters using a programme (2.9) we calculate the dynamic flexibility matrix of the mechanical system (2.10).

The procedure method for the mechanical system (*S*) presented as shown in Fig.4.

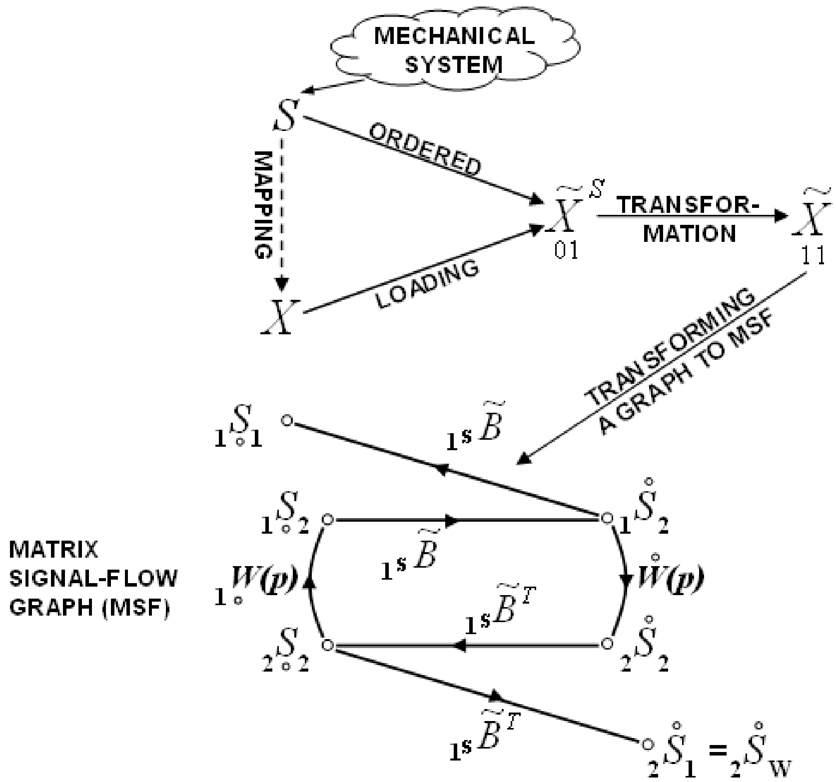

Fig.4. Matrix signal-flow graph (MSF) as a representation of dynamic characteristics of the mechanical system.

3. Conclusions

Hybrid graphs, as formal relation systems, can be used to formulate and formalize tasks in the field of machine vibrations. Hybrid graphs do not require preparing direct mathematical models, i.e., differential equations of motion, to describe and analyze complex mechanical systems with linear coupling. Moreover, by representing coupling of modeled systems by the ordered set of edges in the set of hyper-cycles we can expand the application of graphs to direct modelling of 3-dimentional mechanical systems [7, 8].
In future research these graph methods can be intensively developed [9, 10].

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