STUDY OF THREE DIMENSIONAL PROPAGATION OF WAVES IN HOLLOW POROELASTIC CIRCULAR CYLINDERS

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Employing Biot's theory of wave propagation in liquid saturated porous media, waves propagating in a hollow poroelastic circular cylinder of infinite extent are investigated. General frequency equations for propagation of waves are obtained each for a pervious and an impervious surface. Degenerate cases of the general frequency equations of pervious and impervious surfaces, when the longitudinal wavenumber k and angular wavenumber n are zero, are considered. When $k=0$, the plane-strain vibrations and longitudinal shear vibrations are uncoupled and when $k{\neq}0$ these are coupled. It is seen that the frequency equation of longitudinal shear vibrations is independent of the nature of the surface. When the angular (or circumferential) wavenumber is zero, i.e., $n=0$, axially symmetric vibrations and torsional vibrations are uncoupled. For $n{\neq}0$ these vibrations are coupled. The frequency equation of torsional vibrations is independent of the nature of the surface. By ignoring liquid effects, the results of a purely elastic solid are obtained as a special case.

Key words: Biot's theory, poroelastic cylindrical shell, pervious surface, impervious surface, wavenumber, frequency.

1. Introduction

Free vibrations of a solid cylinder of isotropic elastic material were given by Love (1944). Gazis (1959) discussed the propagation of free harmonic waves along a hollow elastic circular cylinder of infinite extent. Employing Biot's (1956) theory, Tajuddin and Sarma (1980) studied the torsional vibrations of poroelastic cylinders. Wisse *et al.* (2002) presented the experimental results of guided wave modes in porous cylinders. Chao et al. (2004) studied the shock-induced borehole waves in porous formations. The model consisting of concentric cylinders having different saturation levels is used to study the propagation of waves in porous cylinders by Berryman and Pride (2005) who obtained two distinct analytical expressions for torsional wave modes. Farhang *et al.* (2007) investigated the wave propagation in transversely isotropic cylinders. Sharma and Sharma (2010) studied the three dimensional free vibrations of transradially thermoelastic spheres. Karpfinger *et al.* (2010) used the method of discretization of cylindrical structures along the radial axis to study the dispersion properties of Stonely waves and attenuation in poroelastic layers. The author (2006; 2007; 2008; 2010a; 2010b; 2011) studied different problems wave propagation in poroelastic cylinders.

In the present analysis, the free vibrations of an infinite hollow poroelastic circular cylinder are studied employing general displacement components in cylindrical polar coordinates, following Biot's (1956) theory. General frequency equations of pervious and impervious surfaces are obtained. Let the infinite poroelastic hollow cylinder be homogeneous and isotropic. Degenerate cases of the general frequency equations of pervious and impervious surfaces, when the longitudinal wavenumber k and angular wavenumber n are zero, are considered. When $k=0$, the plane-strain vibrations and longitudinal shear vibrations are uncoupled and when $k{\neq}0$ these are coupled. When the angular (or circumferential) wavenumber is zero, i.e., $n=0$, axially symmetric vibrations and torsional vibrations are uncoupled. For $n{\neq}0$

these vibrations are coupled. By ignoring liquid effects, the results of a purely elastic solid are obtained as a special case. The plots of non-dimensional frequency as a function of ratio of wall thickness to wavelength are presented for non-axially symmetric vibrations each for a pervious and an impervious surface. Results are discussed for two different poroelastic materials namely, sandstone saturated with kerosene (Fatt, 1959) and sandstone saturated with water (Yew and Jogi, 1976). It is seen that frequency is same for $n=1$ and 2 in the case of a thin poroelastic cylindrical shell each for a pervious and an impervious surface. Frequency is not the same, however, for a thick shell or a poroelastic solid cylinder.

The considered problem is of importance in civil engineering, ceramic industry where the frequency play an important role. The investigation is also applicable to Bio-Mechanics, wherein osseous tissue, bony elements saturated with a fluid are approximated by a hollow poroelastic cylinder.

2.   Governing equations

The equations of motion of a homogeneous, isotropic poroelastic solid (Biot, 1956) in the presence of dissipation b are

$$N\nabla^2\boldsymbol{u}+(A+N)\nabla e+Q\nabla\in=\frac{\partial^2}{\partial t^2}(\rho_{11}\boldsymbol{u}+\rho_{12}\boldsymbol{U})+b\frac{\partial}{\partial t}(\boldsymbol{u}-\boldsymbol{U}),$$

$$\tag{2.1}$$

$$Q\nabla e+R\nabla\in=\frac{\partial^2}{\partial t^2}(\rho_{12}\boldsymbol{u}+\rho_{22}\boldsymbol{U})-b\frac{\partial}{\partial t}(\boldsymbol{u}-\boldsymbol{U}),$$

here ∇^2 is the Laplacian operator, $\boldsymbol{u}=(u,v,w)$ and $\boldsymbol{U}=(U,V,W)$ are displacements of the solid and liquid respectively, e and $\in$ are the dilatations of the solid and liquid, A, N, Q, R are all poroelastic constants and ρ_{ij} $(i,j=1,2)$ are the mass coefficients following Biot (1956).

The stresses σ_{ij}, and the liquid pressure s are

$$\sigma_{ij}=2Ne_{ij}+(Ae+Q\in)\delta_{ij},\qquad(i,j=r,\theta,z),$$

$$\tag{2.2}$$

$$s=Qe+R\in$$

where δ_{ij} is the well-known Kronecker delta function.

3.   Solution of the problem

Let (r,θ,z) be the cylindrical polar coordinates. Consider a homogeneous, isotropic, infinite hollow poroelastic cylinder with inner and outer radii r_1 and r_2, respectively, having thickness h $[=(r_2-r_1)>0]$ whose axis is in the direction of the z-axis.

Let $\boldsymbol{u}=\nabla\Phi_1+\nabla\times\boldsymbol{\Psi}_1,\qquad\boldsymbol{U}=\nabla\Phi_2+\nabla\times\boldsymbol{\Psi}_2.$ (3.1)

In Eq.(3.1), $\boldsymbol{u}$, $\boldsymbol{U}$ are displacements of the solid and liquid, respectively, and Φ_1, Φ_2 are scalar potentials, $\boldsymbol{\Psi}_1$, $\boldsymbol{\Psi}_2$ are vector potentials.

Let $\boldsymbol{\Psi}_1=(h_r,h_\theta,h_z),\qquad\boldsymbol{\Psi}_2=(H_r,H_\theta,H_z).$ (3.2)

Equations (2.1), together with Eqs (3.1) reduce to

$$P\nabla^2\Phi_1 + Q\nabla^2\Phi_2 = \left(\rho_{11}\ddot{\Phi}_1 + \rho_{12}\ddot{\Phi}_2\right) + b\left(\dot{\Phi}_1 - \dot{\Phi}_2\right),$$

$$Q\nabla^2\Phi_1 + R\nabla^2\Phi_2 = \left(\rho_{12}\ddot{\Phi}_1 + \rho_{22}\ddot{\Phi}_2\right) - b\left(\dot{\Phi}_1 - \dot{\Phi}_2\right),$$

$$N\nabla^2\mathbf{\Psi}_1 = \left(\rho_{11}\ddot{\mathbf{\Psi}}_1 + \rho_{12}\ddot{\mathbf{\Psi}}_2\right) + b\left(\dot{\mathbf{\Psi}}_1 - \dot{\mathbf{\Psi}}_2\right), \qquad (3.3)$$

$$0 = \left(\rho_{12}\ddot{\mathbf{\Psi}}_1 + \rho_{22}\ddot{\mathbf{\Psi}}_2\right) + b\left(\dot{\mathbf{\Psi}}_1 - \dot{\mathbf{\Psi}}_2\right)$$

where a dot over a quantity represents differentiation with respect to time *t* and *P=A+2N*.

Let
$$\Phi_1 = f_1(r)\cos(n\theta)e^{i(kz+\omega t)}, \qquad \Phi_2 = f_2(r)\cos(n\theta)e^{i(kz+\omega t)},$$

$$h_r = g_r(r)\sin(n\theta)e^{i(kz+\omega t)}, \quad h_\theta = g_\theta(r)\cos(n\theta)e^{i(kz+\omega t)}, \quad h_z = g_3(r)\sin(n\theta)e^{i(kz+\omega t)},$$

$$\qquad (3.4)$$

$$H_r = G_r(r)\sin(n\theta)e^{i(kz+\omega t)}, \quad H_\theta = G_\theta(r)\cos(n\theta)e^{i(kz+\omega t)}, \quad H_z = G_3(r)\sin(n\theta)e^{i(kz+\omega t)}.$$

In Eqs (3.4), *k* is the axial wavenumber, *n* is the integer number of waves around the circumference or also known as the angular wavenumber, ω is circular frequency. The second and third Eqs of (3.3) with (3.2), when first two Eqs of (3.3) remain the same, are reduced to

$$P\nabla^2\Phi_1 + Q\nabla^2\Phi_2 = \left(\rho_{11}\ddot{\Phi}_1 + \rho_{12}\ddot{\Phi}_2\right) + b\left(\dot{\Phi}_1 - \dot{\Phi}_2\right),$$

$$Q\nabla^2\Phi_1 + R\nabla^2\Phi_2 = \left(\rho_{12}\ddot{\Phi}_1 + \rho_{22}\ddot{\Phi}_2\right) - b\left(\dot{\Phi}_1 - \dot{\Phi}_2\right),$$

$$N\left(\nabla^2 h_r - \frac{h_r}{r^2} - \frac{2}{r^2}\frac{\partial h_\theta}{\partial\theta}\right) = \left(\rho_{11}\ddot{h}_r + \rho_{12}\ddot{H}_r\right) + b\left(\dot{h}_r - \dot{H}_r\right),$$

$$N\left(\nabla^2 h_\theta - \frac{h_\theta}{r^2} + \frac{2}{r^2}\frac{\partial h_r}{\partial\theta}\right) = \left(\rho_{11}\ddot{h}_\theta + \rho_{12}\ddot{H}_\theta\right) + b\left(\dot{h}_\theta - \dot{H}_\theta\right), \qquad (3.5)$$

$$N\nabla^2 h_z = \left(\rho_{11}\ddot{h}_z + \rho_{12}\ddot{H}_z\right) + \left(\dot{h}_z - \dot{H}_z\right),$$

$$0 = \left(\rho_{12}\ddot{h}_r + \rho_{22}\ddot{H}_r\right) - b\left(\dot{h}_r - \dot{H}_r\right),$$

$$0 = \left(\rho_{12}\ddot{h}_\theta + \rho_{22}\ddot{H}_\theta\right) - b\left(\dot{h}_\theta - \dot{H}_\theta\right),$$

$$0 = \left(\rho_{12}\ddot{h}_z + \rho_{22}\ddot{H}_z\right) - b\left(\dot{h}_z - \dot{H}_z\right).$$

Equations (3.5) with the help of Eqs (3.4) reduce to

$$P\Delta f_1 + Q\Delta f_2 = -\omega^2 \left[K_{11} f_1 + K_{12} f_2 \right],$$

$$Q\Delta f_1 + R\Delta f_2 = -\omega^2 \left[K_{12} f_1 + K_{22} f_2 \right],$$

$$N\left[\Delta g_r - \frac{g_r}{r^2} + \frac{2ng_\theta}{r^2} \right] = -\omega^2 \left[K_{11} g_r + K_{12} G_r \right],$$

$$N\left[\Delta g_\theta - \frac{g_\theta}{r^2} + \frac{2ng_r}{r^2} \right] = -\omega^2 \left[K_{11} g_\theta + K_{12} G_\theta \right],$$

$$N\Delta g_3 = -\omega^2 \left[K_{11} g_3 + K_{12} G_3 \right],$$

$$0 = -\omega^2 \left[K_{12} g_r + K_{22} G_r \right],$$

$$0 = -\omega^2 \left[K_{12} g_\theta + K_{22} G_\theta \right],$$

$$0 = -\omega^2 \left[K_{12} g_3 + K_{22} G_3 \right]$$

$$(3.6)$$

where

$$\Delta = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{n^2}{r^2} - k^2,$$

$$K_{11} = \rho_{11} - \frac{ib}{\omega}, \qquad K_{12} = \rho_{12} + \frac{ib}{\omega}, \qquad K_{22} = \rho_{22} - \frac{ib}{\omega}.$$

$$(3.7)$$

The general solutions of Eqs (3.6) can be obtained in terms of the Bessel functions of the first and second kind J and Y or the modified the Bessel functions of first and second kind I and K, depending on whether its arguments $\alpha_1 r$, $\alpha_2 r$, $\alpha_3 r$ are real or imaginary. Equation (3.6) after a long calculation yields

$$f_1(r) = C_1 Z_n(\alpha_1 r) + C_2 W_n(\alpha_1 r) + C_3 Z_n(\alpha_2 r) + C_4 W_n(\alpha_2 r),$$

$$g_3(r) = A_3 Z_n(\alpha_3 r) + B_3 W_n(\alpha_3 r),$$

$$2g_1(r) = g_r - g_\theta = 2A_1 Z_{n+1}(\alpha_3 r) + 2B_1 W_{n+1}(\alpha_3 r),$$

$$2g_2(r) = g_r + g_\theta = 2A_2 Z_{n-1}(\alpha_3 r) + 2B_2 W_{n-1}(\alpha_3 r),$$

$$(3.8)$$

where Z_n denotes Bessel function or modified Bessel function of first kind, i.e., J_n or I_n and W_n denotes the Bessel function or modified Bessel function of second kind, i.e., Y_n or K_n, depending on its arguments

$$\alpha_1^2 = \xi_1^2 - k^2, \qquad \alpha_2^2 = \xi_2^2 - k^2, \qquad \alpha_3^2 = \xi_3^2 - k^2,$$

$$(3.9)$$

are positive or negative and

$$\omega^2 = \xi_i^2 V_i^2 \qquad (i = 1, 2, 3). \tag{3.10}$$

In Eq.(3.10) V_1, V_2 are dilatational wave velocities of first kind and second kind respectively, and V_3 is shear wave velocity.

The gauge invariance property (Gazis, 1959) is used to eliminate two integration constants from Eq.(3.8). Any one of the potential functions g_1, g_2 or g_3 can be set equal to zero, without loss of generality of solution. Choosing $g_2=0$, we obtain

$$g_r = -g_\theta = g_1. \tag{3.11}$$

The displacement vector of the solid $\boldsymbol{u} = (u, v, w)$ with the help of Eqs (3.1), (3.2) is given by

$$u = \frac{\partial \Phi_1}{\partial r} + \frac{1}{r}\frac{\partial h_z}{\partial \theta} - \frac{\partial h_\theta}{\partial z}, \qquad v = \frac{1}{r}\frac{\partial \Phi_1}{\partial \theta} + \frac{\partial h_r}{\partial z} - \frac{\partial h_z}{\partial r}, \qquad w = \frac{\partial \Phi_1}{\partial z} + \frac{\partial h_\theta}{\partial r} + \frac{h_\theta}{r} - \frac{1}{r}\frac{\partial h_r}{\partial \theta}. \tag{3.12}$$

Substituting Eqs (3.4) in Eqs (3.12), the displacement components of the solid are

$$u = \left(f_1' + \frac{n}{r}g_3 - ikg_\theta \right)\cos(n\theta)e^{i(kz+\omega t)}, \qquad v = \left(-\frac{n}{r}f_1 + ikg_r - g_3' \right)\sin(n\theta)e^{i(kz+\omega t)},$$

$$w = \left(ikf_1 + g_\theta' + g_\theta - \frac{n}{r}g_r \right)\cos(n\theta)e^{i(kz+\omega t)}, \tag{3.13}$$

Substituting Eq.(3.11) in Eqs (3.13), the solid displacement is

$$u = \left(f_1' + \frac{n}{r}g_3 + ikg_1 \right)\cos(n\theta)e^{i(kz+\omega t)}, \qquad v = \left(-\frac{n}{r}f_1 + ikg_1 - g_3' \right)\sin(n\theta)e^{i(kz+\omega t)},$$

$$w = \left(ikf_1 - g_1' - \frac{(n+1)}{r}g_1 \right)\cos(n\theta)e^{i(kz+\omega t)} \tag{3.14}$$

where the 'prime' over a quantity denotes differentiation with respect to r.

The proper selection of either the Bessel function or modified Bessel function is shown in Tab.1.

Table 1

Interval	Functions used
$\lvert kV_1 \rvert < \omega$	$J(\alpha_1 r)$, $Y(\alpha_1 r)$, $J(\alpha_2 r)$, $Y(\alpha_2 r)$, $J(\alpha_3 r)$, $Y(\alpha_3 r)$.
$\lvert kV_3 \rvert < \omega < \lvert kV_1 \rvert$	$I(\beta_1 r)$, $K(\beta_1 r)$, $I(\beta_2 r)$, $K(\beta_2 r)$, $J(\alpha_3 r)$, $Y(\alpha_3 r)$.
$\omega < \lvert kV_1 \rvert$	$I(\beta_1 r)$, $K(\beta_1 r)$, $I(\beta_2 r)$, $K(\beta_2 r)$, $I(\beta_3 r)$, $K(\beta_3 r)$.

β_1, β_2, β_3, are the absolute values of α_1, α_2, α_3, respectively, when these values are imaginary, and α_1, α_2, α_3 are defined in Eq.(3.9).

The dilatations of the solid and liquid media are

$$e = \Delta f_1(r)\cos(n\theta)e^{i(kz+\omega t)}, \qquad \in = \Delta f_2(r)\cos(n\theta)e^{i(kz+\omega t)} \tag{3.15}$$

where Δ is defined in Eq.(3.7).

By substituting Eqs (3.14) into strain displacement relations and then using Eq.(2.2), the stresses σ_{ij} and the liquid pressure s are

$$\sigma_{rr} + s = \big[C_1 M_{11}(r) + C_2 M_{12}(r) + C_3 M_{13}(r) + C_4 M_{14}(r) +$$
$$+ A_3 M_{15}(r) + B_3 M_{16}(r) + A_1 M_{17}(r) + B_1 M_{18}(r) \big]\cos(n\theta)e^{i(kz+\omega t)}, \tag{3.16}$$

$$\sigma_{r\theta} = \big[C_1 M_{21}(r) + C_2 M_{22}(r) + C_3 M_{23}(r) + C_4 M_{24}(r) +$$
$$+ A_3 M_{25}(r) + B_3 M_{26}(r) + A_1 M_{27}(r) + B_1 M_{28}(r) \big]\sin(n\theta)e^{i(kz+\omega t)}, \tag{3.17}$$

$$\sigma_{rz} = \big[C_1 M_{31}(r) + C_2 M_{32}(r) + C_3 M_{33}(r) + C_4 M_{34}(r) +$$
$$+ A_3 M_{35}(r) + B_3 M_{36}(r) + A_1 M_{37}(r) + B_1 M_{38}(r) \big]\cos(n\theta)e^{i(kz+\omega t)}, \tag{3.18}$$

$$s = \big[C_1 M_{41}(r) + C_2 M_{42}(r) + C_3 M_{43}(r) + C_4 M_{44}(r) \big]\cos(n\theta)e^{i(kz+\omega t)}, \tag{3.19}$$

$$\frac{\partial s}{\partial r} = \big[C_1 N_{41}(r) + C_2 N_{42}(r) + C_3 N_{43}(r) + C_4 N_{44}(r) \big]\cos(n\theta)e^{i(kz+\omega t)}, \tag{3.20}$$

where the coefficients $M_{ij}(r)$ and $N_{ij}(r)$ are given as

$$M_{11}(r) = \left\{ 2N\left[\frac{n(n-1)}{r^2} - \lambda_1\alpha_1^2 \right] + \right.$$
$$\left. + \left[(Q+R)\delta_1^2 - (A+Q) \right]\left(k^2 + \lambda_1\alpha_1^2 \right) \right\} Z_n(\beta_1 r) + \frac{2\lambda_1 N\beta_1}{r} Z_{n+1}(\beta_1 r),$$

$$M_{12}(r) = \left\{ 2N\left[\frac{n(n-1)}{r^2} - \lambda_1\alpha_1^2 \right] + \right.$$
$$\left. + \left[(Q+R)\delta_1^2 - (A+Q) \right]\left(k^2 + \lambda_1\alpha_1^2 \right) \right\} W_n(\beta_1 r) + \frac{2N\beta_1}{r} Z_{n+1}(\beta_1 r),$$

$$M_{13}(r) = \left\{ 2N\left[\frac{n(n-1)}{r^2} - \lambda_1\alpha_2^2 \right] + \right.$$
$$\left. + \left[(Q+R)\delta_2^2 - (A+Q) \right]\left(k^2 + \lambda_1\alpha_2^2 \right) \right\} Z_n(\beta_2 r) + \frac{2\lambda_1 N\beta_2}{r} Z_{n+1}(\beta_2 r),$$

$$M_{14}(r) = \left\{ 2N \left[\frac{n(n-1)}{r^2} - \lambda_1 \alpha_2^2 \right] + \right.$$

$$\left. + \left[(Q+R)\delta_2^2 - (A+Q) \right] \left(k^2 + \lambda_1 \alpha_2^2 \right) \right\} W_n(\beta_2 r) + \frac{2N\beta_2}{r} Z_{n+1}(\beta_2 r),$$

$$M_{15}(r) = \frac{2Nn(n-1)}{r^2} Z_n(\beta_3 r) - \frac{2\lambda_2 Nn\beta_3}{r} Z_{n+1}(\beta_3 r),$$

$$M_{16}(r) = \frac{2Nn(n-1)}{r^2} W_n(\beta_3 r) - \frac{2Nn\beta_3}{r} Z_{n+1}(\beta_3 r),$$

$$M_{17}(r) = 2Nik\beta_3 Z_n(\beta_3 r) - \frac{2N(n+1)ik}{r} Z_{n+1}(\beta_3 r),$$

$$M_{18}(r) = 2\lambda_2 Nik\beta_3 W_n(\beta_3 r) - \frac{2N(n+1)ik}{r} W_{n+1}(\beta_3 r),$$

$$M_{21}(r) = \frac{2Nn(1-n)}{r^2} Z_n(\beta_1 r) + \frac{2\lambda_1 Nn\beta_1}{r} Z_{n+1}(\beta_1 r),$$

$$M_{22}(r) = \frac{2Nn(1-n)}{r^2} W_n(\beta_1 r) + \frac{2Nn\beta_1}{r} W_{n+1}(\beta_1 r),$$

$$M_{23}(r) = \frac{2Nn(1-n)}{r^2} Z_n(\beta_2 r) + \frac{2\lambda_1 Nn\beta_2}{r} Z_{n+1}(\beta_2 r),$$

$$M_{24}(r) = \frac{2Nn(1-n)}{r^2} W_n(\beta_2 r) + \frac{2Nn\beta_2}{r} W_{n+1}(\beta_2 r),$$

$$M_{25}(r) = \left[\frac{2Nn(1-n)}{r^2} + \lambda_2 N\alpha_3^2 \right] Z_n(\beta_3 r) - \frac{2\lambda_2 N\beta_3}{r} Z_{n+1}(\beta_3 r),$$

$$M_{26}(r) = \left[\frac{2Nn(1-n)}{r^2} + \lambda_2 N\alpha_3^2 \right] W_n(\beta_3 r) - \frac{2N\beta_3}{r} W_{n+1}(\beta_3 r),$$

$$M_{27}(r) = ikN\beta_3 Z_n(\beta_3 r) - \frac{2N(n+1)ik}{r} Z_{n+1}(\beta_3 r),$$

$$M_{28}(r) = \lambda_2 ikN\beta_3 W_n(\beta_3 r) - \frac{2N(n+1)ik}{r} W_{n+1}(\beta_3 r),$$

$$M_{31}(r) = \frac{2Nnik}{r} Z_n(\beta_1 r) - 2\lambda_1 Nik\beta_1 Z_{n+1}(\beta_1 r),$$

$$M_{32}(r) = \frac{2Nnik}{r}W_n(\beta_1 r) - 2Nik\beta_1 W_{n+1}(\beta_1 r),$$

$$M_{33}(r) = \frac{2Nnik}{r}Z_n(\beta_2 r) - 2\lambda_1 Nik\beta_2 Z_{n+1}(\beta_2 r),$$

$$M_{34}(r) = \frac{2Nnik}{r}W_n(\beta_2 r) - 2Nik\beta_2 W_{n+1}(\beta_2 r),$$

$$M_{35}(r) = \frac{Nnik}{r}Z_n(\beta_3 r), \qquad M_{36}(r) = \frac{Nnik}{r}W_n(\beta_3 r),$$

$$M_{37}(r) = \frac{-Nn\beta_3}{r}Z_n(\beta_3 r) - N(k^2 - \lambda_2 \alpha_3^2)Z_{n+1}(\beta_3 r),$$

$$M_{38}(r) = \frac{-\lambda_2 Nn\beta_3}{r}W_n(\beta_3 r) - N(k^2 - \lambda_2 \alpha_3^2)Z_{n+1}(\beta_3 r),$$

$$M_{41}(r) = \left(R\delta_1^2 - Q\right)\left(k^2 + \lambda_1 \alpha_1^2\right)Z_n(\beta_1 r), \qquad M_{42}(r) = \left(R\delta_1^2 - Q\right)\left(k^2 + \lambda_1 \alpha_1^2\right)W_n(\beta_1 r),$$

$$M_{43}(r) = \left(R\delta_2^2 - Q\right)\left(k^2 + \lambda_1 \alpha_2^2\right)Z_n(\beta_2 r), \qquad M_{44}(r) = \left(R\delta_2^2 - Q\right)\left(k^2 + \lambda_1 \alpha_2^2\right)W_n(\beta_2 r),$$

$$N_{41}(r) = \frac{n}{r}\left(R\delta_1^2 - Q\right)\left(k^2 + \lambda_1 \alpha_1^2\right)Z_n(\beta_1 r) - \beta_1\left(R\delta_1^2 - Q\right)\left(\alpha_1^2 + \lambda_1 k^2\right)Z_{n+1}(\beta_1 r),$$

$$N_{42}(r) = \frac{n}{r}\left(R\delta_1^2 - Q\right)\left(k^2 + \lambda_1 \alpha_1^2\right)W_n(\beta_1 r) - \beta_1\left(R\delta_1^2 - Q\right)\left(\lambda_1 \alpha_1^2 + k^2\right)W_{n+1}(\beta_1 r),$$

$$N_{43}(r) = \frac{n}{r}\left(R\delta_2^2 - Q\right)\left(k^2 + \lambda_1 \alpha_2^2\right)Z_n(\beta_2 r) - \beta_2\left(R\delta_2^2 - Q\right)\left(\alpha_2^2 + \lambda_1 k^2\right)Z_{n+1}(\beta_2 r),$$

$$N_{44}(r) = \frac{n}{r}\left(R\delta_2^2 - Q\right)\left(k^2 + \lambda_1 \alpha_2^2\right)W_n(\beta_2 r) - \beta_2\left(R\delta_2^2 - Q\right)\left(\lambda_1 \alpha_2^2 + k^2\right)W_{n+1}(\beta_2 r), \qquad (3.21)$$

In Eqs (3.21),

$$\delta_i^2 = \frac{1}{\left(RK_{12} - QK_{22}\right)}\left[\left(RK_{11} - QK_{12}\right) - V_i^{-2}\left(PR - Q^2\right)\right]. \qquad (i = 1, 2) \qquad (3.22)$$

Also λ_1 and λ_2 are parameters which are introduced in order to account for the differences in the recurrence and differentiation formulas between the different kinds of Bessel functions. The values of these parameters are 1 and -1 when Bessel functions of first kind and second kind and modified Bessel functions of first and second kind are used. It is seen from Table-I that λ_1 and λ_2 vary as follows

$$|kV_1|<\omega: \qquad \lambda_1=1, \qquad\qquad \lambda_2=1,$$

$$|kV_3|<\omega<|kV_1|: \qquad \lambda_1=-1, \qquad\qquad \lambda_2=1, \qquad\qquad (3.23)$$

$$\omega<|kV_1|: \qquad \lambda_1=-1, \qquad\qquad \lambda_2=-1.$$

4. Boundary conditions - frequency equation

The boundary conditions for traction free inner and outer surfaces of the hollow poroelastic cylinder in the case of a pervious surface are

$$\sigma_{rr}+s=0, \qquad \sigma_{r\theta}=0, \qquad \sigma_{rz}=0, \qquad s=0, \quad \text{at} \quad r=r_1 \quad \text{and} \quad r=r_2, \qquad (4.1)$$

while the boundary conditions for free vibrations in the case of an impervious surface are

$$\sigma_{rr}+s=0, \qquad \sigma_{r\theta}=0, \qquad \sigma_{rz}=0, \qquad \frac{\partial s}{\partial r}=0, \quad \text{at} \quad r=r_1 \quad \text{and} \quad r=r_2. \qquad (4.2)$$

Equations (3.16)-(3.19) together with Eq.(4.1) yield eight homogeneous equations for eight arbitrary constants C_1, C_2, C_3, C_4, A_3, B_3, A_1, and B_1. A non-trivial solution can be obtained only when the determinant of the coefficients vanishes. Thus the frequency equation for a pervious surface is

$$\begin{vmatrix}
M_{11}(r_1) & M_{12}(r_1) & M_{13}(r_1) & M_{14}(r_1) & M_{15}(r_1) & M_{16}(r_1) & M_{17}(r_1) & M_{18}(r_1) \\
M_{21}(r_1) & M_{22}(r_1) & M_{23}(r_1) & M_{24}(r_1) & M_{25}(r_1) & M_{26}(r_1) & M_{27}(r_1) & M_{28}(r_1) \\
M_{31}(r_1) & M_{32}(r_1) & M_{33}(r_1) & M_{34}(r_1) & M_{35}(r_1) & M_{36}(r_1) & M_{37}(r_1) & M_{38}(r_1) \\
M_{41}(r_1) & M_{42}(r_1) & M_{43}(r_1) & M_{44}(r_1) & 0 & 0 & 0 & 0 \\
M_{11}(r_2) & M_{12}(r_2) & M_{13}(r_2) & M_{14}(r_2) & M_{15}(r_2) & M_{16}(r_2) & M_{17}(r_2) & M_{18}(r_2) \\
M_{21}(r_2) & M_{22}(r_2) & M_{23}(r_2) & M_{24}(r_2) & M_{25}(r_2) & M_{26}(r_2) & M_{27}(r_2) & M_{28}(r_2) \\
M_{31}(r_2) & M_{32}(r_2) & M_{33}(r_2) & M_{34}(r_2) & M_{35}(r_2) & M_{36}(r_2) & M_{37}(r_2) & M_{38}(r_2) \\
M_{41}(r_2) & M_{42}(r_2) & M_{43}(r_2) & M_{44}(r_2) & 0 & 0 & 0 & 0
\end{vmatrix} = 0. \qquad (4.3)$$

Similarly, with the help of Eqs (3.16)-(3.18) and (3.20) together with Eq.(4.2) we get the frequency equation of an impervious surface

$$\begin{vmatrix}
M_{11}(r_1) & M_{12}(r_1) & M_{13}(r_1) & M_{14}(r_1) & M_{15}(r_1) & M_{16}(r_1) & M_{17}(r_1) & M_{18}(r_1) \\
M_{21}(r_1) & M_{22}(r_1) & M_{23}(r_1) & M_{24}(r_1) & M_{25}(r_1) & M_{26}(r_1) & M_{27}(r_1) & M_{28}(r_1) \\
M_{31}(r_1) & M_{32}(r_1) & M_{33}(r_1) & M_{34}(r_1) & M_{35}(r_1) & M_{36}(r_1) & M_{37}(r_1) & M_{38}(r_1) \\
N_{41}(r_1) & N_{42}(r_1) & N_{43}(r_1) & N_{44}(r_1) & 0 & 0 & 0 & 0 \\
M_{11}(r_2) & M_{12}(r_2) & M_{13}(r_2) & M_{14}(r_2) & M_{15}(r_2) & M_{16}(r_2) & M_{17}(r_2) & M_{18}(r_2) \\
M_{21}(r_2) & M_{22}(r_2) & M_{23}(r_2) & M_{24}(r_2) & M_{25}(r_2) & M_{26}(r_2) & M_{27}(r_2) & M_{28}(r_2) \\
M_{31}(r_2) & M_{32}(r_2) & M_{33}(r_2) & M_{34}(r_2) & M_{35}(r_2) & M_{36}(r_2) & M_{37}(r_2) & M_{38}(r_2) \\
N_{41}(r_2) & N_{42}(r_2) & N_{43}(r_2) & N_{44}(r_2) & 0 & 0 & 0 & 0
\end{vmatrix} = 0. \qquad (4.4)$$

In Eqs (4.3), (4.4) the elements $M_{ij}(r)$ and $N_{ij}(r)$ are defined in Eq.(23).

By ignoring the liquid effects in the frequency equation of a pervious surface (4.3), the results of a purely elastic solid are obtained as a special case considered by Gazis (1959). The frequency equation of an impervious surface has no counterpart in a purely elastic solid. Now we consider the particular cases of the general frequency Eqs (3.3), (3.4) when the axial and angular wavenumbers vanish.

4.1. Motion independent of z

When the axial wavenumber k is taken equal to zero, i.e., by considering zeroth azimuthal mode, the frequency Eq.(4.3) of a pervious surface degenerates into the product of two determinants as

$$D_1 D_2 = 0, \tag{4.5}$$

where

$$D_1 = \begin{vmatrix} M_{11}(r_1) & M_{12}(r_1) & M_{13}(r_1) & M_{14}(r_1) & M_{15}(r_1) & M_{16}(r_1) \\ M_{21}(r_1) & M_{22}(r_1) & M_{23}(r_1) & M_{24}(r_1) & M_{25}(r_1) & M_{26}(r_1) \\ M_{41}(r_1) & M_{42}(r_1) & M_{43}(r_1) & M_{44}(r_1) & 0 & 0 \\ M_{11}(r_2) & M_{12}(r_2) & M_{13}(r_2) & M_{14}(r_2) & M_{15}(r_2) & M_{16}(r_2) \\ M_{21}(r_2) & M_{22}(r_2) & M_{23}(r_2) & M_{24}(r_2) & M_{25}(r_2) & M_{26}(r_2) \\ M_{41}(r_2) & M_{42}(r_2) & M_{43}(r_2) & M_{44}(r_2) & 0 & 0 \end{vmatrix},$$

$$D_2 = \begin{vmatrix} M_{37}(r_1) & M_{38}(r_1) \\ M_{37}(r_2) & M_{38}(r_2) \end{vmatrix}. \tag{4.6}$$

The terms $M_{ij}(r)$ appearing in D_1 and D_2 are given in Eqs (3.21) for $k=0$. It is clear that for $k=0$, α_1^2, α_2^2, α_3^2, are all positive thereby Bessel functions of first and second kind enter the solution.

Arguing on similar lines the frequency equation of an impervious surface (4.4) when $k=0$ reduces to the product of two determinants

$$D_3 D_4 = 0, \tag{4.7}$$

where the determinants D_3, D_4 are

$$D_3 = \begin{vmatrix} M_{11}(r_1) & M_{12}(r_1) & M_{13}(r_1) & M_{14}(r_1) & M_{15}(r_1) & M_{16}(r_1) \\ M_{21}(r_1) & M_{22}(r_1) & M_{23}(r_1) & M_{24}(r_1) & M_{25}(r_1) & M_{26}(r_1) \\ N_{41}(r_1) & N_{42}(r_1) & N_{43}(r_1) & N_{44}(r_1) & 0 & 0 \\ M_{11}(r_2) & M_{12}(r_2) & M_{13}(r_2) & M_{14}(r_2) & M_{15}(r_2) & M_{16}(r_2) \\ M_{21}(r_2) & M_{22}(r_2) & M_{23}(r_2) & M_{24}(r_2) & M_{25}(r_2) & M_{26}(r_2) \\ N_{41}(r_2) & N_{42}(r_2) & N_{43}(r_2) & N_{44}(r_2) & 0 & 0 \end{vmatrix},$$

$$D_4 = \begin{vmatrix} M_{37}(r_1) & M_{38}(r_1) \\ M_{37}(r_2) & M_{38}(r_2) \end{vmatrix}. \tag{4.8}$$

Equation (4.5) is satisfied, if either D_1 or D_2 is equal to zero. The case of $D_1=0$ corresponds to plane-strain vibrations of thick-walled hollow poroelastic cylinders, discussed by Malla Reddy and Tajuddin

(2000), for a pervious surface. Equation (4.7) is satisfied when either $D_3=0$, or $D_4=0$. The case of $D_3=0$, corresponds to the plane-strain vibrations of thick-walled hollow poroelastic cylinders for an impervious surface discussed by Malla Reddy and Tajuddin (2000). From Eqs (4.6) and (4.8) it is clear that D_2 and D_4 are same. The case of $D_2=0$, corresponds to longitudinal shear vibrations which involve only longitudinal displacement w. Also, since $D_2=0$, is the same for a pervious and an impervious surface, hence we find that longitudinal shear vibrations are independent of the nature of the surface. From Eqs (4.5) and (4.7) we find that plane-strain vibrations and longitudinal shear vibrations are uncoupled for a pervious and an impervious surface, respectively, when the motion is independent of the longitudinal coordinate *z*, and these vibrations are coupled for the non-zero longitudinal wavenumber *k*, that is $k \neq 0$. The frequency equation of longitudinal shear vibrations $D_2=0$, when expanded gives

$$J'_n(\alpha_3 r_1)Y'_n(\alpha_3 r_2) - J'_n(\alpha_3 r_2)Y'_n(\alpha_3 r_1) = 0, \tag{4.9}$$

and the amplitude ratio is given as

$$\frac{A_1}{B_1} = -\frac{Y'_n(\alpha_3 r_1)}{J'_n(\alpha_3 r_1)}. \tag{4.10}$$

The frequency equation of longitudinal shear vibrations of hollow poroelastic cylinders (4.9) is discussed by Tajuddin and Ahmed Shah (2010a).

4.2. Motion independent of θ

When the motion is independent of the angular coordinate θ (i.e., *n=0*), the frequency Eq.(4.3) for a pervious surface is reduced to the product of two determinants given by

$$D_5 D_6 = 0 \tag{4.11}$$

where

$$D_5 = \begin{vmatrix} M_{11}(r_1) & M_{12}(r_1) & M_{13}(r_1) & M_{14}(r_1) & M_{17}(r_1) & M_{18}(r_1) \\ M_{31}(r_1) & M_{32}(r_1) & M_{33}(r_1) & M_{34}(r_1) & M_{37}(r_1) & M_{38}(r_1) \\ M_{41}(r_1) & M_{42}(r_1) & M_{43}(r_1) & M_{44}(r_1) & 0 & 0 \\ M_{11}(r_2) & M_{12}(r_2) & M_{13}(r_2) & M_{14}(r_2) & M_{17}(r_2) & M_{18}(r_2) \\ M_{31}(r_2) & M_{32}(r_2) & M_{33}(r_2) & M_{34}(r_2) & M_{37}(r_2) & M_{38}(r_2) \\ M_{41}(r_2) & M_{42}(r_2) & M_{43}(r_2) & M_{44}(r_2) & 0 & 0 \end{vmatrix},$$

$$D_6 = \begin{vmatrix} M_{25}(r_1) & M_{26}(r_1) \\ M_{25}(r_2) & M_{26}(r_2) \end{vmatrix}. \tag{4.12}$$

The terms $M_{ij}(r)$ in D_5 and D_6 are given by Eqs (3.21) for *n=0*.

Similarly, the frequency equation for an impervious surface (4.4), for *n=0* yields into the product of two determinants as

$$D_7 D_8 = 0, \tag{4.13}$$

where the determinants D_7, D_8 are

$$D_7 = \begin{vmatrix} M_{11}(r_1) & M_{12}(r_1) & M_{13}(r_1) & M_{14}(r_1) & M_{17}(r_1) & M_{18}(r_1) \\ M_{31}(r_1) & M_{32}(r_1) & M_{33}(r_1) & M_{34}(r_1) & M_{37}(r_1) & M_{38}(r_1) \\ N_{41}(r_1) & N_{42}(r_1) & N_{43}(r_1) & N_{44}(r_1) & 0 & 0 \\ M_{11}(r_2) & M_{12}(r_2) & M_{13}(r_2) & M_{14}(r_2) & M_{17}(r_2) & M_{18}(r_2) \\ M_{31}(r_2) & M_{32}(r_2) & M_{33}(r_2) & M_{34}(r_2) & M_{37}(r_2) & M_{38}(r_2) \\ N_{41}(r_2) & N_{42}(r_2) & N_{43}(r_2) & N_{44}(r_2) & 0 & 0 \end{vmatrix},$$

$$D_8 = \begin{vmatrix} M_{25}(r_1) & M_{26}(r_1) \\ M_{25}(r_2) & M_{26}(r_2) \end{vmatrix}. \tag{4.14}$$

From Eqs (4.12) and (4.14) it is clear that the determinants D_6 and D_8 are same. Now Eq.(4.11) is satisfied if $D_5=0$ or $D_6=0$. The case of $D_5=0$ gives the frequency equation of axially symmetric vibrations of an infinite hollow poroelastic cylinder for a pervious surface discussed by Ahmed Shah (2008). Similarly, Eq.(4.13) is satisfied when $D_7=0$ or $D_8=0$. The case of $D_7=0$ gives the frequency equation of axially symmetric vibrations of an infinite hollow poroelastic cylinder for an impervious surface discussed by Ahmed Shah (2008). The case of $D_6=0$ or $D_8=0$ when simplified yields the equation

$$J_2(\alpha_3 r_1)Y_2(\alpha_3 r_2) - J_2(\alpha_3 r_2)Y_2(\alpha_3 r_1) = 0, \tag{4.15}$$

which is the frequency of torsional vibrations of infinite hollow poroelastic cylinders studied by Tajuddin and Ahmed Shah (2007) in the presence of dissipation. Moreover, the frequency Eq.(4.15) is independent of the nature of the surface, i.e., pervious or impervious. Also, Eq.(4.15) is the same for pervious and impervious surfaces.

4.3.   Equivoluminal modes

For $n=0$, the stress free boundary conditions at the inner and outer surfaces of the hollow poroelastic cylinder are satisfied if

$$\alpha_3^2 = k^2 > 0. \tag{4.16}$$

The dilatational and equivoluminal potentials f_1 and g_1 are coupled through the boundary conditions. To consider purely equivoluminal modes, we set

$$f_1 = f_2 = 0, \qquad g_3 = 0. \tag{4.17}$$

Substituting Eqs (4.17) into Eq.(3.15), it is seen that the dilatations of solid and liquid media are zero. Hence from Eq.(2.2), the liquid pressure is zero. Therefore the equivoluminal modes are independent of the nature of the surface, i.e., pervious and impervious. Accordingly, no distinction between pervious and impervious surfaces is seen. Therefore from Eqs (4.16) and (4.17), we have

$$g_1'(\alpha_3 r_1) = g_1'(\alpha_3 r_2) = 0, \qquad [\sigma_{rr} + s = 0] \tag{4.18}$$

where

$$g_1(\alpha_3 r) = A_1 J_1(\alpha_3 r) + B_1 Y_1(\alpha_3 r). \tag{4.19}$$

Substituting Eq.(4.19) into Eq.(4.18) and eliminating the constants A_1, B_1, the frequency equation of purely equivoluminal modes is

$$J_1'(\alpha_3 r_1)Y_1'(\alpha_3 r_2) - J_1'(\alpha_3 r_2)Y_1'(\alpha_3 r_1) = 0, \tag{4.20}$$

with the amplitude ratio

$$\frac{A_1}{B_1} = -\frac{Y_1'(\alpha_3 r_1)}{J_1'(\alpha_3 r_1)}, \tag{4.21}$$

and the non-zero displacement and stress for equivoluminal modes are

$$u = ik\left[A_1 J_1(\alpha_3 r) + B_1 Y_1(\alpha_3 r)\right]e^{i(kz+\omega t)}, \quad w = -\alpha_3\left[A_1 J_0(\alpha_3 r) + B_1 Y_0(\alpha_3 r)\right]e^{i(kz+\omega t)},$$

$$\tag{4.22}$$

$$\sigma_{rr} = -2Nik\left[A_1 J_1'(\alpha_3 r) + B_1 Y_1'(\alpha_3 r)\right]e^{i(kz+\omega t)}.$$

Equation (4.20) is same as Eq.(4.9) of longitudinal shear vibrations for a case of *n=1*. The frequency Eq.(4.20) for the two limiting values of h/r_1 is given below:

4.3.(i) For a thin poroelastic cylindrical shell

When $h/r_1 << 1$, that is for a thin poroelastic cylindrical shell, the frequency Eq.(4.20) by using Hankel-Kirchoff asymptotic approximations (Abramowitz and Stegun, 1965) is reduced to

$$\sin(\alpha_3 h) - \frac{7\alpha_3 h}{8\alpha_3^2 r_1 r_2}\cos(\alpha_3 h) + \frac{49}{64\alpha_3^2 r_1 r_2}\sin(\alpha_3 h) \approx 0. \tag{4.23}$$

As $\alpha_3 r_1 \to \infty$, $\alpha_3 r_2 \to \infty$, with the help of Eq.(4.16), Eq.(4.23) is simplified to

$$\omega \approx \sqrt{2}\frac{q\pi V_3}{h}\left[1 + \frac{7}{8q^2\pi^2}\left(\frac{h}{r_1}\right)^2\right]. \qquad (q = 1, 2, 3, ...) \tag{4.24}$$

Equation (4.24) determines the frequency of purely equivoluminal modes of a poroelastic plate of thickness *h*.

4.3.(ii) For a poroelastic non-hollow cylinder

When $r_1/h \to 0$, that is for a poroelastic non-hollow cylinder, the frequency Eq.(4.20) is reduced asymptotically to

$$J_1'(\alpha_3 h) = 0, \tag{4.25}$$

which is the frequency equation of purely equivoluminal modes of a poroelastic solid cylinder of radius *h*.

5. Non-dimensionalisation of frequency equation

For propagating modes in a non-dissipative medium, the wavenumber *k* is real. To analyze the frequency equations of pervious and impervious surfaces, it is convenient to introduce the following non-dimensional parameters

$$a_1 = PH^{-1}, \qquad a_2 = QH^{-1}, \qquad a_3 = RH^{-1}, \qquad a_4 = NH^{-1},$$

$$m_{11} = \rho_{11}\rho^{-1}, \qquad m_{12} = \rho_{12}\rho^{-1}, \qquad m_{22} = \rho_{22}\rho^{-1}, \qquad \Omega = \omega h C_0^{-1}, \tag{5.1}$$

$$\tilde{x} = \left(V_0 V_1^{-1}\right)^2, \qquad \tilde{y} = \left(V_0 V_2^{-1}\right)^2, \qquad \tilde{z} = \left(V_0 V_3^{-1}\right)^2, \qquad \delta = hL^{-1}$$

where Ω is the non-dimensional frequency, $H=P+2Q+R$, $\rho=\rho_{11}+2\rho_{12}+\rho_{22}$, C_0 and V_0 are the reference velocities ($C_0^2=N/\rho$, $V_0^2=H/\rho$), h is the thickness of the hollow poroelastic cylinder and L is the wavelength. Let

$$g = \frac{r_2}{r_1} \qquad \text{so that} \qquad \frac{h}{r_1} = (g-1). \tag{5.2}$$

6.   Numerical results and discussion

Two types of poroelastic materials are used to carry out the computational work, one is sandstone saturated with kerosene say Material-I, (Fatt, 1959), the other one is sandstone saturated with water, Material-II (Yew and Jogi, 1976), whose non-dimensional physical parameters are given in Tab.2

Table 2

Material/ Parameter	a_1	a_2	a_3	a_4	m_{11}	m_{12}	m_{22}	$\tilde{x}$	$\tilde{y}$	$\tilde{z}$
I	0.843	0.065	0.028	0.234	0.901	-0.001	0.101	0.999	4.763	3.851
II	0.960	0.006	0.028	0.412	0.877	0	0.123	0.913	4.347	2.129

For a given poroelastic material, the frequency Eqs (4.3) and (4.4) of pervious and impervious surfaces when non-dimensionalised by using the Eqs (5.1) and (5.2) present the relation between the non-dimensional frequency Ω and ratio of thickness to wavelength (δ) for given dimensions of the hollow poroelastic cylinder. The ratio of the outer radius to that of the inner radius of the hollow poroelastic cylinder, that is, g takes the values *1.034*, *3* and ∞, which represent the thin poroelastic shell, thick poroelastic shell and poroelastic non-hollow cylinder, respectively. The number of waves around the circumference, or angular wavenumber n take values 1 and 2. The value $n=1$ corresponds to flexural vibrations and $n=2$ corresponds to typical non-axially symmetric vibrations.

The non-dimensional frequency Ω as a function of the ratio of thickness to wavelength δ is computed for the referred material. The frequencies of first five modes are shown in the figures. The frequencies of the thin poroelastic shell of material-I are presented in Fig.1. for $n=1$ and $n=2$. Frequencies are same for $n=1$ and $n=2$, each for a pervious surface and an impervious surface in the case of a thin poroelastic shell. From Fig.1 it is clear that for material-I, the frequency of a pervious surface and an impervious surface is same for the first three modes in *0.1≤δ≤0.7*, while for the fourth and fifth mode the frequency is same in *0.1<δ≤0.5* and *0.8<δ≤1*. In *0<δ≤0.1*, the frequency of an impervious surface is more than of a pervious surface for the second, third, fourth and fifth mode while the frequency of an impervious surface is less than the frequency of a pervious surface in *0.7≤δ≤1* for the first mode. The frequencies of the thick poroelastic shell of material-I with $n=1$ is presented in Fig.2. The frequency of an impervious surface is less than that of a pervious surface for the second, third, fourth and fifth mode in *0<δ<0.1* and *0.0<δ≤1*, while for the first mode the frequency of an impervious surface is less than that of a pervious surface in *0.3<δ<0.5*. The frequency of the

poroelastic non-hollow cylinder of material-I with *n=1* is presented in Fig.3. The frequency of an impervious surface is more than that of a pervious surface for the third, fourth and fifth mode, while the first and second mode have frequency less than that of a pervious surface in *0.1<δ≤0.3*.
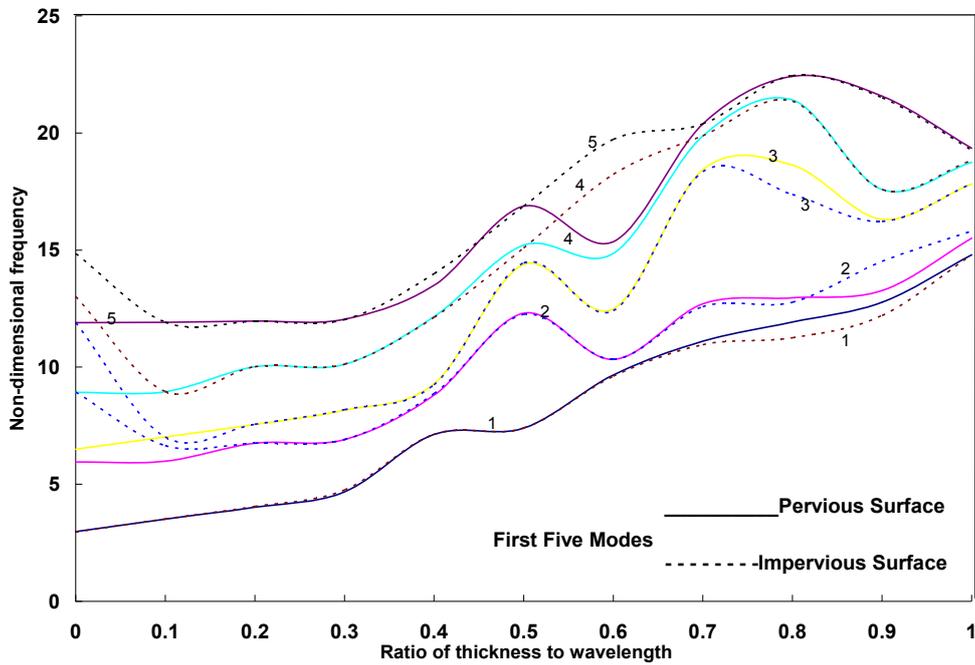
Fig.1. Frequency as a function of wavelength (Mat – I, thin-shell, *n=1, 2*).
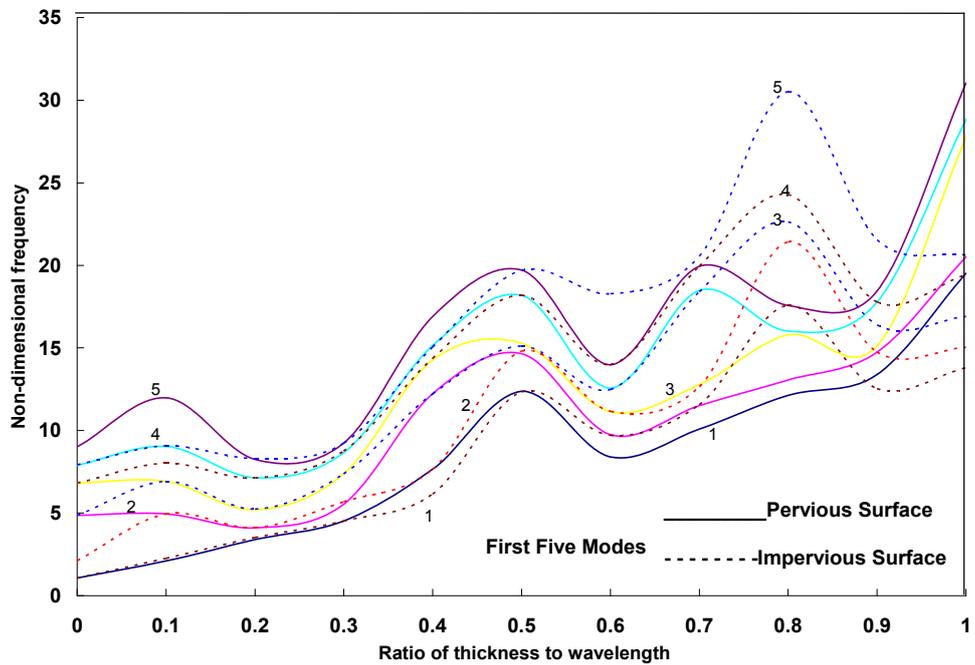
Fig.2. Frequency as a function of wavelength (Mat – I, thick-shell, *n=1*).
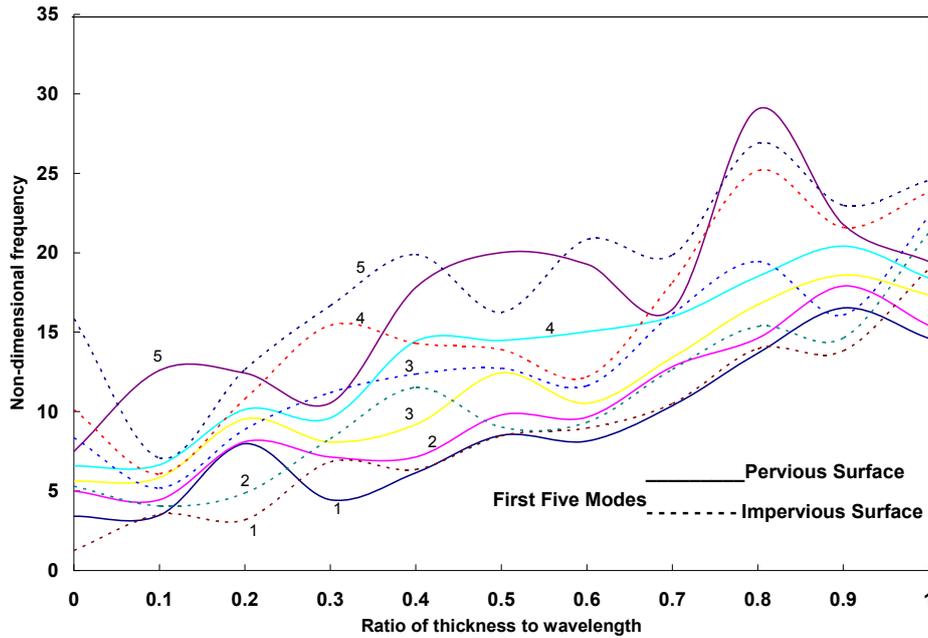
Fig.3. Frequency as a function of wavelength (Mat – I, non-hollow cylinder, *n=1*).

The frequencies of the thick poroelastic shell of material-I when *n=2* are presented in Fig.4. The frequency of a pervious and an impervious surface is same in *0.3<δ<0.8* for the first five modes. In *0<δ<0.3*, the frequency of an impervious surface is less than that of a pervious surface for the first, fourth and fifth mode. In *0.8<δ<1*, the frequency of an impervious surface is less than that of a pervious surface for the first five modes. The frequency of the poroelastic solid cylinder of material-I with *n=2* is shown in Fig.5. The frequency of an impervious surface in the case of the third, fourth and fifth mode is less than that of corresponding modes of a pervious surface in *0.5<δ≤1*. The modes near the origin, when the wavelength is large are presented for material-I in Fig.6. From Fig.6 it is observed that the frequency of an impervious surface is more than that of a pervious surface for *n=1* and *n=2*. Also, these frequencies are constant.
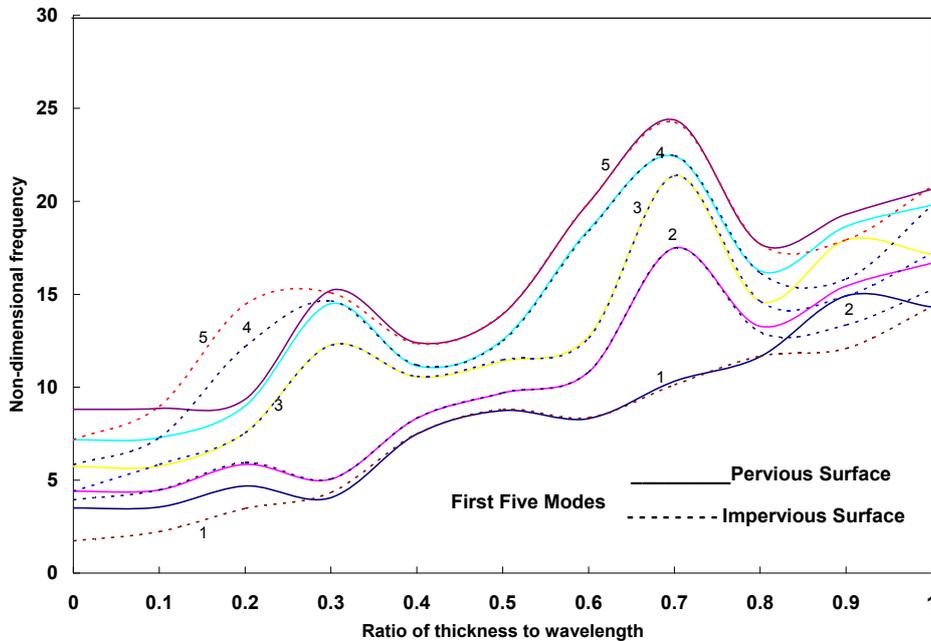
Fig.4. Frequency as a function of wavelength (Mat – I, thich-shell, *n=2*).

Fig.5. Frequency as a function of wavelength (Mat – I, non-hollow cylinder, *n=2*).
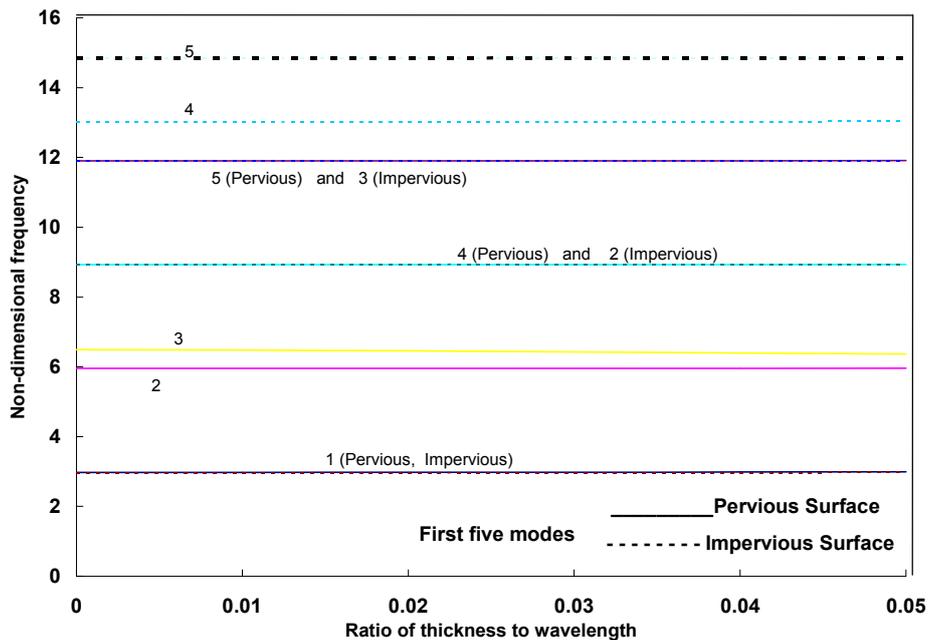
Fig.6. Frequency as a function of wavelength (Mat – I, thin-shell, large wavelength, *n=1, 2*).

The frequencies of the thin poroelastic shell of material-II are shown in Fig.7 for *n=1* and *n=2*. The frequencies are same for *n=1* and *n=2* in the case of a thin poroelastic shell. From Fig.7 it is clear that in *0.3≤δ≤0.5* the frequency of an impervious surface is less than that of a pervious surface for the first five modes while in *0≤δ<0.3* and *0.5≤δ≤1*, the frequency of a pervious and an impervious surfaces are same for all the first five modes. The frequency versus the ratio of thickness to wavelength for a thick poroelastic shell

with *n=1* is presented in Fig.8 in the case of material-II. It is seen that the frequency of an impervious surface for all the first five modes is higher than that of a pervious surface in *0.7<δ<0.9*. The frequency of an impervious surface is less than that of a pervious surface in *0.2<δ<0.4*, and same in the case of the first three modes in *0.4<δ≤1*. The fourth and fifth mode have same frequency for a pervious and an impervious surface in *0.6≤δ≤1*. In *0.4<δ<0.6*, the frequency of an impervious surface is more for the fourth and fifth mode than the corresponding modes of a pervious surface. It is observed from Fig.9 that for the poroelastic solid cylinder of material-II, the frequency of the first five modes of an impervious surface is less than corresponding modes of a pervious surface in *0.8<δ<0.9*. The frequency of the second, third, fourth and fifth mode of an impervious surface is less than that of the corresponding modes of a pervious surface in *0.1<δ<0.3*, while the frequency of an impervious surface for the first mode is less than that of a pervious surface in *0.65≤δ≤0.75*. The frequency of the thick poroelastic shell of material-II is shown in Fig.10 for *n=2*. It is observed that in *0<δ<0.3*, the frequency of an impervious surface is more than that of a pervious surface for the first five modes, just an opposite phenomenon to material-I. The frequency of a pervious surface and an impervious surface is same in *0.3<δ<0.6* and *0.8<δ<1* for all the first five modes. In *0.6<δ<0.8*, the frequency of an impervious surface is more than that of a pervious surface.

The variation of frequency of the poroelastic solid cylinder of material-II when *n=2* is shown in Fig.11. The frequency of an impervious surface is more than that of a pervious surface for the first five modes in *0≤δ≤0.3*, and *0.5<δ≤0.7*. The frequency of an impervious surface of the first five modes is less than that of corresponding modes of a pervious surface in *0.3<δ≤0.5* and *0.7<δ≤1*. The modes near the origin for material-II with *n=1* and *n=2* are presented in Fig.12. From Fig.12 it is seen that the frequency of a pervious and an impervious surface is same for material-II when *n=1* and *n=2*. The frequency of material-I is higher than that of material-II for pervious and impervious surfaces.
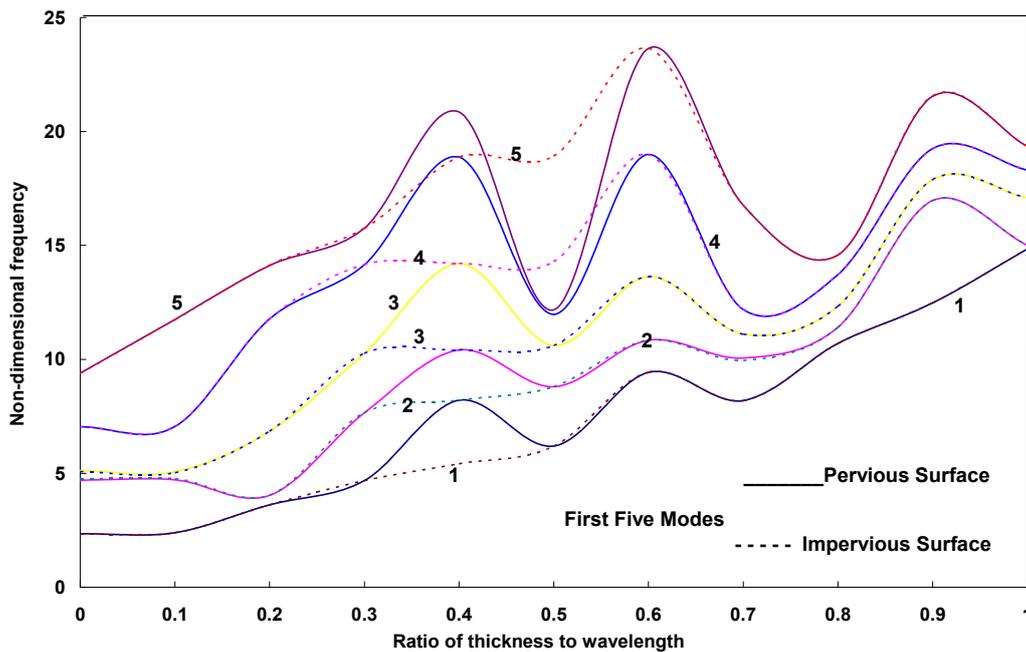
Fig.7. Frequency as a function of wavelength (Mat – II, thin-shell, *n=1, 2*).
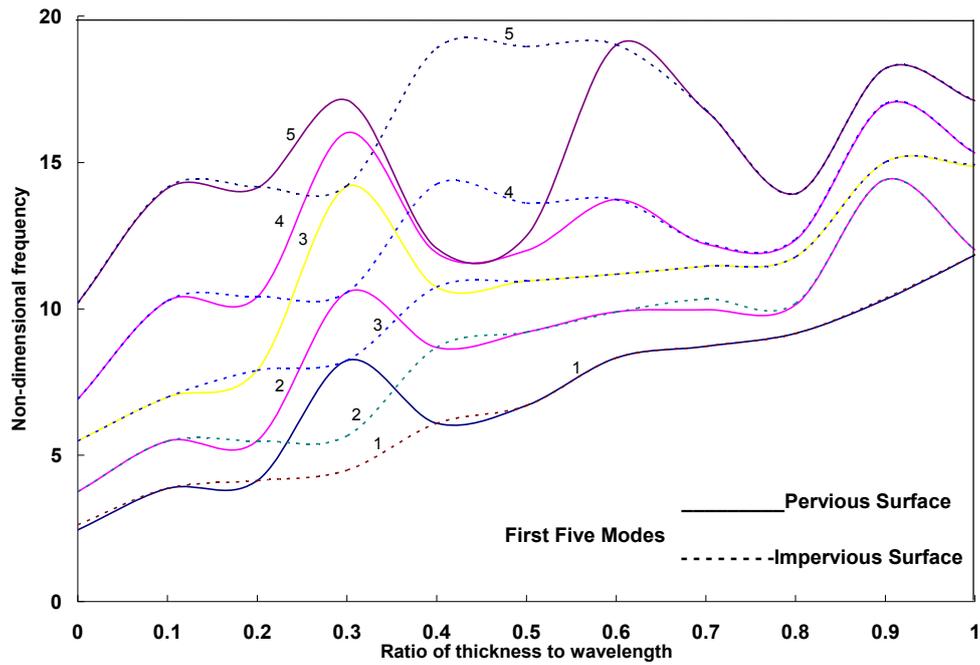
Fig.8. Frequency as a function of wavelength (Mat – II, thick-shell, *n=1*).
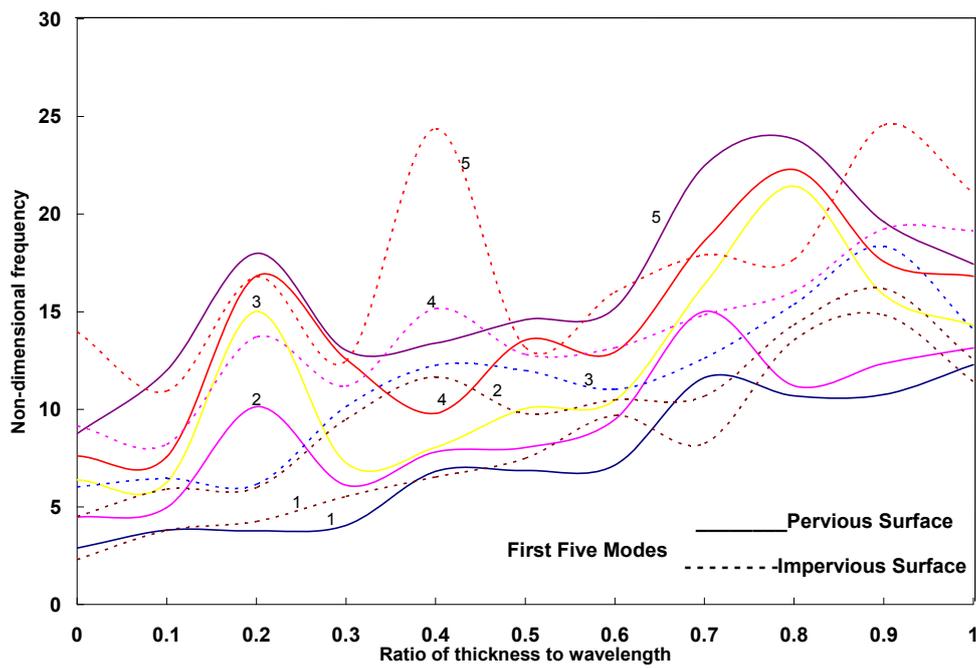
Fig.9. Frequency as a function of wavelength (Mat – II, non-hollow cylinder, *n=1*).
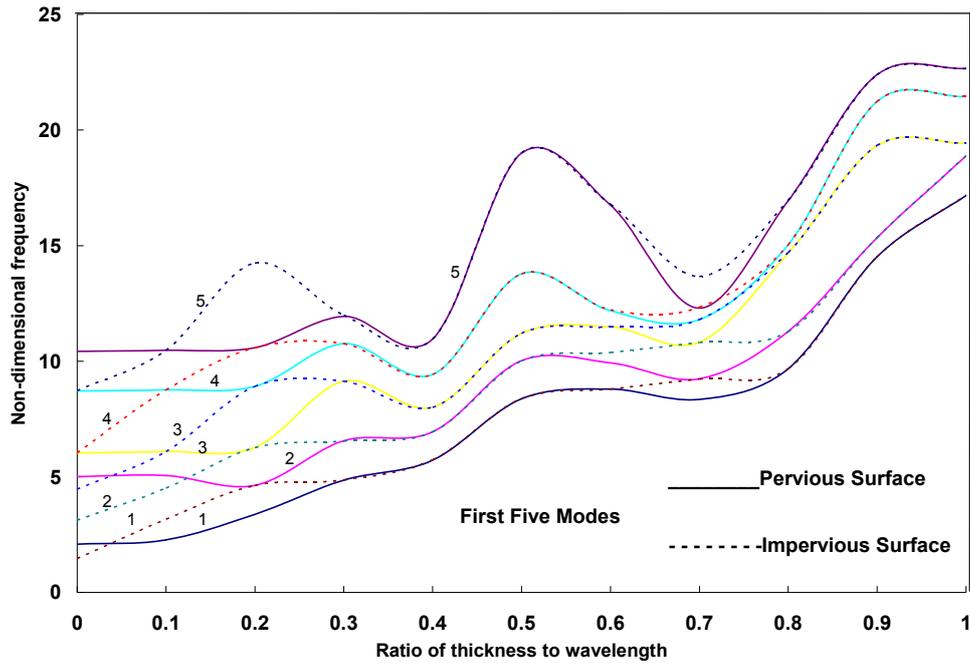
Fig.10. Frequency as a function of wavelength (Mat – II, thick-shell, *n=2*).
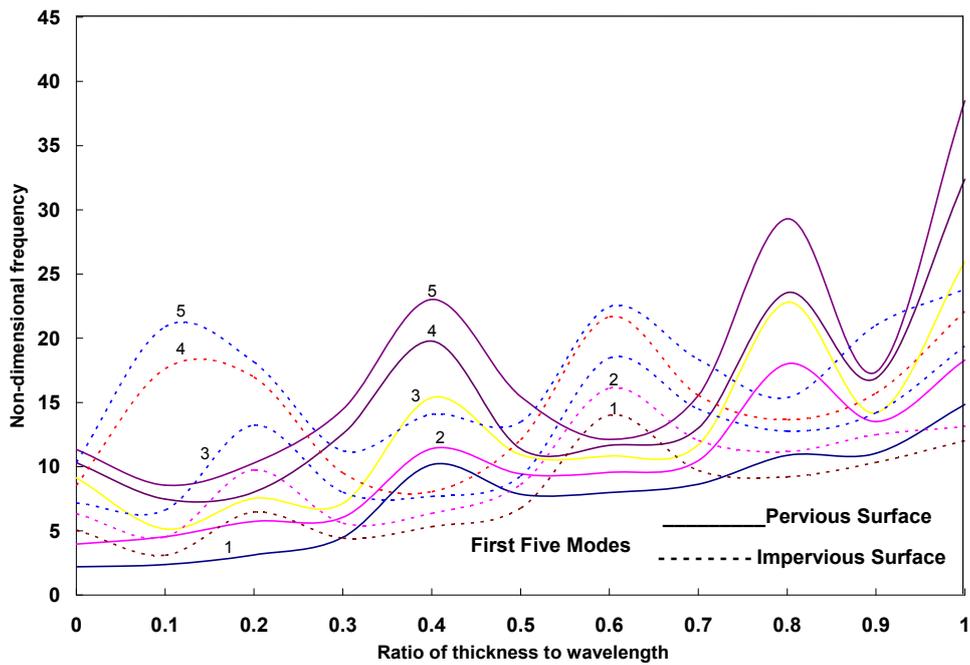
Fig.11. Frequency as a function of wavelength (Mat – II, non-hollow cylinder, *n=2*).
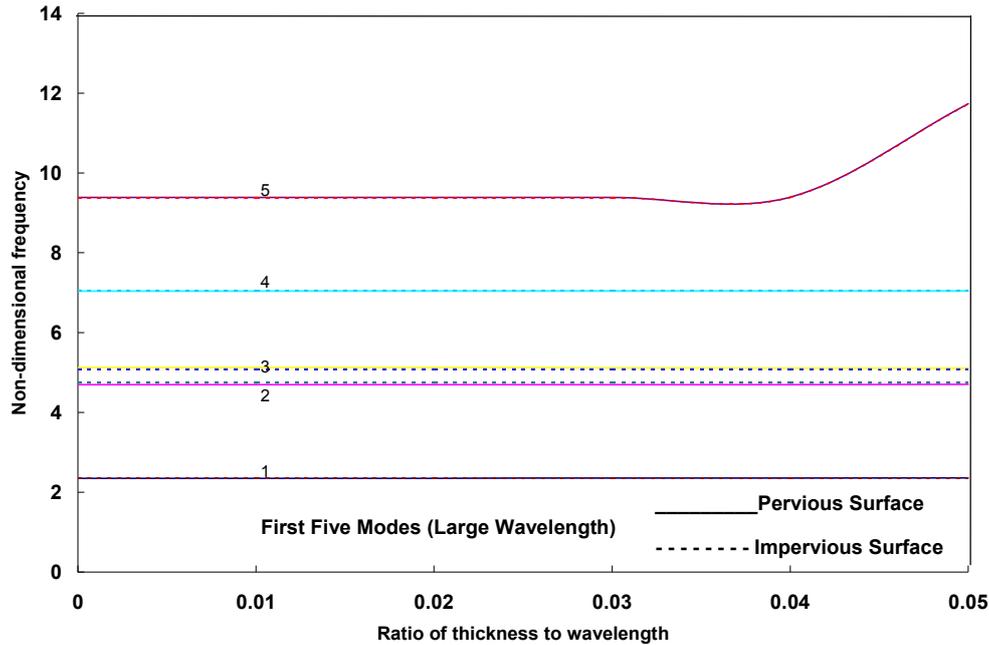
Fig.12. Frequency as a function of wavelength (Mat – II, thin-shell, large wavelength, *n=1, 2*).

7.   Concluding remarks

The investigation of the propagation of wave in hollow poroelastic circular cylinders of infinite extent has lead to the following conclusions:

(i)   When *n≠0* and *k=0,* non-axially symmetric plane-strain modes are generated and uncoupled family of longitudinal shear vibrations are derived from the potential function $g_l(r)$ alone as a special case.

(ii)   When *k≠0* and *n=0* axially symmetric longitudinal modes are generated.

(iii)   When both the number of waves around the circumference n and longitudinal wavenumber *k* are zero, the motion is axially symmetric and of infinite wavelength. The potential functions $f_l(r)$, $g_l(r)$ and $g_3(r)$ generate three types of uncoupled vibrations that are plane-strain extensional, plane-strain shear and longitudinal shear vibrations.

(iv)   For *n≠0* and *k≠0* the potential functions $f_l(r)$, $g_l(r)$ and $g_3(r)$ are coupled through the boundary conditions and generate the non-axially symmetric vibrations.

(v)   The frequency is same for flexural and non-axially symmetric vibrations both for a pervious and impervious surface, in the case of a thin poroelastic cylindrical shell. This is not true for a thick poroelastic shell or a poroelastic non-hollow cylinder.

(vi)   In general, the frequency of an impervious surface is higher than that of a pervious surface.

(vii)   When the wavelength is large, the propagation of waves in thin poroelastic cylindrical shells is non-dispersive.

Nomenclature

A, N, Q, R – poroelastic constants
b – dissipation
e – dilatation of solid
h – thickness of the hollow poroelastic cylinder
I_n, K_n – modified Bessel functions of first and second kind of order *n*
J_n, Y_n – Bessel functions of first and second kind of order *n*

$$k$$ – axial wavenumber

$$n$$ – angular wavenumber

$$(r, \theta, z)$$ – cylindrical polar coordinates

$$r_2, r_1$$ – outer and inner radii of the hollow poroelastic cylinder

$$s$$ – liquid pressure

$$\boldsymbol{U} = (U, V, W)$$ – displacement of liquid

$$\boldsymbol{u} = (u, v, w)$$ – displacement of solid

$$V_1, V_2, V_3$$ – dilatational wave velocities of first and second kind, shear wave velocity

$$\delta$$ – non-dimensional wave number

$$\in$$ – dilatation of liquid

$$\rho_{ij}$$ – stresses

$$\rho_{11}, \rho_{12}, \rho_{22}$$ – mass coefficients

$$\Phi_1, \Phi_2, \Psi_1, \Psi_2$$ – potential functions

$$\Omega$$ – non-dimensional frequency

$$\omega$$ – circular frequency

$$\nabla^2$$ – Laplacian operator

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