

# CONDITIONS OF STABILITY LOSS DURING THE TEST OF HYDRAULIC FORMING OF DRAWPIECES Part I. Derivation of basic equations and relationships

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The paper contains two parts. In the first part, basic relationships were derived and some problems connected with stability loss during hydraulic forming of round metallic drawpieces with liquid pressure were discussed. The aim of the considerations is to test drawability of sheets by estimation of acceptable values of plastic strains and the corresponding heights of spherical shells. The analysis was based on some selected real conditions of stability loss. The influence of the coefficient of material hardening for the drawpiece, the coefficient of normal anisotropy and coefficients of plane anisotropy on acceptable values of plastic strains and heights of the formed drawpieces corresponding to the given condition of stability loss was also tested.

**Key words:** hydraulic forming, stability loss, boundary strains, height of drawpieces, normal and plane anisotropy.

## 1. Introduction

During processes of plastic working, various technological operations connected with sheet-metal forming are very important. In the case of thin sheets, the processes where a required shape of the drawpiece is obtained by biaxial tension under the plane stress state are important, and there are many operations of sheet forming when biaxial uniform tension occurs. Here, we can mention technological processes of stretch drawing or forming of sheets fixed at the periphery. Then, drawpieces are usually formed by means of liquids (synthetic oils), synthetic resins or a rigid punch (Jentet, 1961; Michałowski, 1967; Okamoto *et al.*, 1968).

A test of hydraulic forming of sheets by a liquid (Jovignot's test) is an example of a laboratory test simulating real forming conditions and belonging to the mentioned processes of technological forming.

Figure 1 shows a scheme of Jovignot's test, where a specimen 1 is fixed between the die and the blankholder so that the periphery cannot be displaced. The test is usually realized up to the moment when the drawpiece cracks in the most weakened point of the central part in the surroundings of point M. A more detailed description of experiments and the obtained test results is presented in (Banasiak, 1971). From that paper it appears that almost 95% drawpieces had cracks caused by local reduction of the sheet thickness in the surroundings of point M. Technological test of the sheet disk forming with a liquid belongs to axially-symmetric problems (Kaczanow, 1969; Marciniak, 1961; 1968; Piwnik, 1985; 1991; Sokołowski, 1957; Szczepinski, 1967; Śloderbach and Sawicki 1984; Życzkowski, 1981; Życzkowski and Szuwalski 1982).

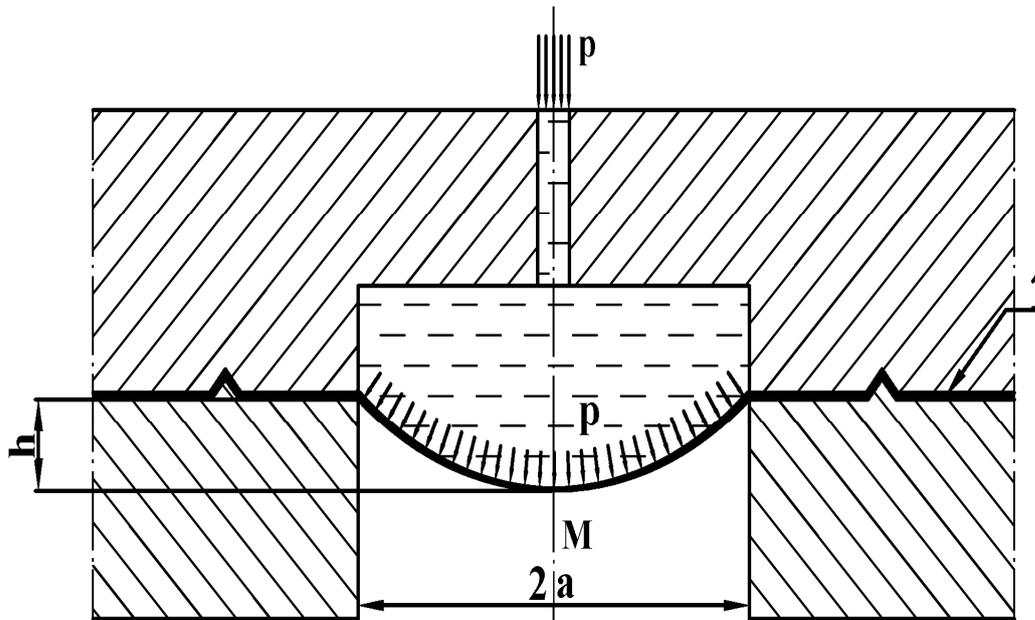


Fig.1. Scheme of forming of a sheet disk fixed at the periphery by liquid pressure (Jovignot's test).

## 2. The aim and basic assumptions

The aim of this chapter is the presentation of a method for determination of the height  $h$  (parameter of sheet drawability) of the formed cap for four practical engineering conditions of stability loss. The other aim is to test the influence of some material parameters on that height (and on drawability of the sheet material), such as the coefficient of hardening  $n$ , coefficient of normal anisotropy  $r$  and coefficients  $r_1$  and  $r_2$  of anisotropy on the sheet plane (plane anisotropy). From most experimental data it appeared that reduction of wall thickness in the cap vertex at the crack moment exceeded 50%. Thus, during forming high plastic strains occur (even  $> 60\%$ ), whereas the maximum elastic strains are of a low order, for example 0.5%. An anisotropic model of a rigid-plastic body with hardening, Levy-Mises laws of plastic flow, and logarithmic measures of principal strains were assumed analysis of the problem.

According to Bressan and Williams (1983), El-Sebaie and Mellor (1972), Hill (1983), Johnson and Mellor (1975), Marciniak (1968; 1971), Marciniak and Kuczynski (1967), Mellor (1969), Moore and Wallace (1964/1965), Swift (1952), the following types of stability loss of the deformed shell were taken into account:

- liquid pressure acting on the formed shell reaches its maximum value,
- the product of the plasticizing stress and the actual wall thickness in its certain point (it is usually the vertex point) reaches the maximum),
- occurrence of local reduction of the drawpiece thickness causing a local change of the ratio of principal stresses under which a local reduction of thickness initiates as one or some chases,
- formation of so-called unstability being a shearing band in the place of thickness reduction under the plane stress state (PSS) for a case of tensile strains when the shear stress  $\tau$  reaches its critical value.

It is assumed that the drawpiece material satisfies the Huber-Mises-Hencky (H-M-H) condition of plasticity generalized for a case of bodies of anisotropic properties by Hill (1986). Levy-Mises associated laws of plastic flow and plastic incompressibility of the drawpiece material are also assumed.

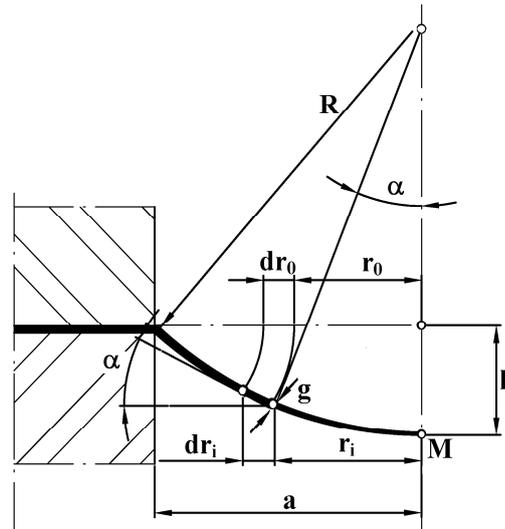


Fig.2. Scheme of forming the spherical shell and the basic geometric dimensions.

The assumed scheme of strains and stresses in the formed disk was the same as in the case of the deformed membraneous shell. According to the membraneous theory, the drawpiece forming from a sheet disk fixed at the periphery and subjected to liquid pressure takes a form of a spherical cap of gradually reducing thickness, see Fig.2.

Logarithmic components of strains (circumferential, longitudinal and those along the thickness of the considered element of the shell) are Marciniak (1966; 1968),

$$\varphi_1 = \ln \frac{r_i}{r_0}, \quad \varphi_2 = \ln \frac{dr_i}{dr_0 \cdot \cos \alpha}, \quad \varphi_3 = \ln \frac{g}{g_0}. \tag{2.1}$$

According to the above assumptions, the drawpiece takes the shape of a spherical cap when  $\varphi_1 = \varphi_2$ . Substituting (1)<sub>1,2</sub> to the condition ( $\varphi_1 = \varphi_2$ ) and after transformations we obtain a differential equation (Marciniak, 1966; 1968).

The condition of plastic incompressibility takes the form (Marciniak, 1971)

$$\varphi_1 + \varphi_2 + \varphi_3 = 0. \tag{2.2}$$

Let us solve the considered differential equation. Let us include the boundary condition (when  $r_i = a$  then  $g = g_0$ ), the condition of plastic incompressibility of the material (2.2), and geometric relationship  $\{a^2 + (R - h)^2 = R^2\}$  resulting from Fig.2. Then, we obtain

$$\varphi_3 = \ln \frac{g}{g_0} = 2 \ln \frac{2R - h}{R + \sqrt{R^2 - r_i^2}}. \tag{2.3}$$

Including suitable geometric relationships resulting for a spherical cap in Fig.2 into expression (2.3), after transformations we have

$$\varphi_3 = \ln \frac{g}{g_0} = 2 \ln \frac{2a^2}{a^2 + h^2 + \sqrt{(a^2 + h^2)^2 - 4h^2 r_i^2}} \quad (2.4)$$

where  $g_0$  – the initial thickness of the drawpiece,  $g$  – the actual thickness of the drawpiece wall corresponding to the actual radius  $r_i$ .

Let us remember that the maximum reduction of thickness occurs in the vertex of the drawpiece along the radius  $r_i = 0$ . The assumption of the membrane scheme of strains causes that  $(\varphi_1 = \varphi_2)$ , thus, in consequence, we obtain  $(\varphi_{(i)} = -\varphi_3)$ . Including the above remarks in Eq.(2.4), after transformations we have (Marciniak, 1971)

$$\varphi_{(i)} = 2 \ln \left[ I + \left( \frac{h}{a} \right)^2 \right], \quad (2.5)$$

and

$$h = a \sqrt{\exp\left(\frac{I}{2}\varphi_{(i)}\right) - I}. \quad (2.6)$$

The above expression determines the dependence of the maximum height of the formed drawpiece on the maximum intensity of strain reached for a given condition of stability loss, see conditions (a-c) given in this chapter of the paper.

### 3. Determination of stability loss states

#### 3.1. The basic equations and relations

1. In the case of sheets we often meet a special case of anisotropy when in the material there are three perpendicular directions: the rolling direction and the direction perpendicular to it, both located on the sheet plane, and the direction normal to the sheet plane. This kind of anisotropy is often called orthotropy.

The theory of plasticity of an anisotropic (orthotropic) body was developed by Hill (1986) and it is cited in many scientific books, monographs and handbooks, it is often applied in engineering practice.

Under the biaxial stress state and on the assumption that directions of the principal stresses  $\sigma_1$  and  $\sigma_2$  on the sheet plane coincide with the rolling direction and the direction perpendicular to it located on the sheet plane, and the direction of the third principal stress ( $\sigma_3 = 0$ ) is perpendicular to the sheet surface, the condition of plasticity, the equivalent strain and the laws of plastic flow take the following forms

$$\sigma_p = \sqrt{\sigma_1^2 + \frac{2\sigma_1\sigma_2}{I+C} + \frac{(I+B)\sigma_2^2}{I+C}},$$

$$\varphi_i = \sqrt{I+C} \sqrt{\frac{(I+B)\varphi_1^2 + 2\varphi_1\varphi_2 + (I+C)\varphi_2^2}{(I+B)(I+C) - I}}, \quad (3.1)$$

$$\frac{\dot{\varphi}_1}{(I+C)\sigma_1 - \sigma_2} = \frac{\dot{\varphi}_2}{(I+B)\sigma_2 - \sigma_1} = \frac{\dot{\varphi}_3}{-C\sigma_1 - B\sigma_2} = \frac{\dot{\varphi}_{(i)}}{(I+C)\sigma_p}.$$

In the above equations, coefficients  $B$  and  $C$  are constants characterizing anisotropy on the material (sheet) plane so as

$$\frac{\dot{\varphi}_1}{\dot{\varphi}_3} = \frac{I}{B} = r_2, \quad \frac{\dot{\varphi}_2}{\dot{\varphi}_3} = \frac{I}{C} = r_1. \quad (3.2)$$

2. So-called normal anisotropy is a special case of anisotropy. It results in the same properties of the sheet in all directions occurring on the sheet plane, but the properties of the sheet in the direction perpendicular to its surface are different. Thus, we obtain ( $r_1 = r_2 = r$ ), so

$$B = C = B \frac{I}{r} \quad \text{or} \quad r = \frac{I}{B} = \frac{I}{C}. \quad (3.3)$$

Isotropy is the simplest kind of the material properties. In such a case, plastic properties of the sheet do not depend on any direction and they are the same in all directions. Then

$$B = C = r = I. \quad (3.4)$$

For an analysis of stability loss during a test of hydraulic forming of drawpieces the following power form of equation was assumed for the hardening curve for the drawpiece material

$$\sigma_p = D(\varphi_0 + \varphi_{(i)})^n. \quad (3.5)$$

It is the Swift equation (Gabryszewski and Gronostajski, 1991).

### 3.2. Boundary strains from the conditions of stability loss

In Johnson and Mellor (1975), Marciniak (1971) it has been shown that in the plastic forming processes under the plane stress state we can distinguish three characteristic kinds of stability loss and determine boundary strains for those states.

a) Stability loss takes place when the drawpiece deformation process proceeds with no synchronous increase of liquid pressure. As it was previously said, physically it means that pressure reaches its maximum value, in the mathematical form it is

$$\frac{dp}{d\varphi_{(i)}} = 0. \quad (3.6)$$

In Marciniak (1971), only a case of isotropic materials was considered. In this paper, we use the same procedure for the determination of the equivalent strain corresponding to the state (3.6) for anisotropic materials:

- for a sheet material of properties of plane anisotropy

$$\varphi_{(i)} = \frac{n}{3} \sqrt{\frac{(I+C)(B+C+4)}{(I+B)(I+C)-I}} - \varphi_0, \quad (3.7)$$

- for a sheet material of properties of normal anisotropy

$$\varphi_{(i)} = \frac{n\sqrt{2(I+r)}}{3}, \quad (3.8)$$

- for a sheet material of isotropic properties according to Marciniak (1971)

$$\varphi_{(i)} = \frac{2}{3}n - \varphi_0. \quad (3.9)$$

b) Stability loss takes place at the moment when a certain part of the drawpiece (usually the vertex part) begins to deform more than the others. Then in the surroundings of that part, a spherical shape of the drawpiece becomes a little disturbed (Marciniak, 1971). This phenomenon takes place when the plastic strength in the drawpiece wall, expressed by the product  $(\sigma_p \cdot g)$ , reaches, as it was previously said, the extreme value, and during further deformation of the drawpiece it begins to reduce. Mathematically, the following condition corresponds to that state

$$\frac{d(\sigma_p \cdot g)}{d\varphi_{(i)}} = 0. \quad (3.10)$$

According to the procedure shown in Marciniak (1971), including the constitutive relationship (3.5) we obtain the following formulas

- for a sheet material of properties of plane anisotropy

$$\varphi_{(i)} = \frac{n}{2} \sqrt{\frac{(I+C)(B+C+4)}{(I+B)(I+C)-I}} - \varphi_0, \quad (3.11)$$

- for a sheet material of properties of normal anisotropy

$$\varphi_{(i)} = n \sqrt{\frac{I+r}{2}} - \varphi_0, \quad (3.12)$$

- for a sheet material of isotropic properties according to Marciniak (1971)

$$\varphi_{(i)} = n - \varphi_0. \quad (3.13)$$

c) The third form of stability loss is caused by non-uniform strain distribution resulting from local reduction of the material thickness (formation of one or more chases) in the vertex part of the drawpiece during continuation of the forming process. Deepening of the chase can be accompanied by further deformation of the areas in the direct neighbourhood of the chase with gradually decreasing rate. This deformation stops when the stress state in the chase reaches the point  $S$  on the ellipse of the condition of plasticity, Fig.3. Then, according to the theory of associated laws of plastic flow formulated by Levy-Mises, we obtain

$$\frac{\dot{\varphi}_2}{\dot{\varphi}_1} = 0. \quad (3.14)$$

The stress state corresponding to the point  $S$  on the ellipse of plasticity, Fig.3, and the mathematical condition (3.14) are as follows:

- for a sheet material of properties of plane anisotropy

$$\sigma_1 = \frac{I+C}{\sqrt{(I+B)(I+C)-I}} \sigma_p, \quad \sigma_2 = \frac{I}{\sqrt{(I+B)(I+C)-I}} \sigma_p, \quad \frac{\sigma_2}{\sigma_1} = \frac{I}{I+C}, \quad (3.15)$$

- for a sheet material of properties of normal anisotropy

$$\sigma_1 = \frac{I+r}{\sqrt{I+2r}} \sigma_p, \quad \sigma_2 = \frac{I}{\sqrt{I+2r}} \sigma_p, \quad \frac{\sigma_2}{\sigma_1} = \frac{r}{I+r}, \quad (3.16)$$

- for a sheet material of isotropic properties according to Marciniak (1971)

$$\sigma_1 = \frac{2}{\sqrt{3}} \sigma_p, \quad \sigma_2 = \frac{I}{\sqrt{3}} \sigma_p, \quad \frac{\sigma_2}{\sigma_1} = \frac{I}{2}. \quad (3.17)$$

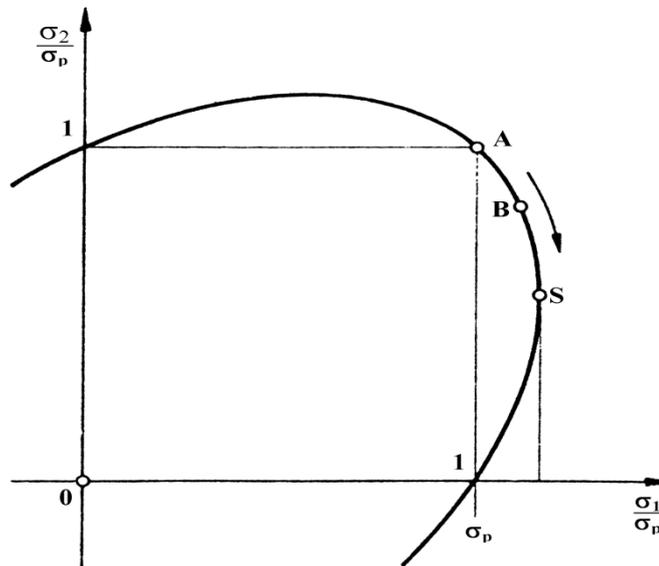


Fig.3. Transient stress states in the formed drawpiece.

In order to determine values of the boundary strains for that form of stability loss, we should use a general condition of instability in forming of metal sheets under the plane stress state (PSS), presented in Moore and Wallace (1964/1965). The condition has the following form

$$\frac{d\sigma_p}{d\varphi_{(i)}} \cdot \frac{I}{\sigma_p} = \frac{I}{z} \quad (3.18)$$

where  $z$  – so-called critical abscissa, see Fig.4, of the following form

$$z = \sqrt{I+C} \frac{\sqrt{[(I+C)\gamma^2 - 2\gamma + (I+B)]^3}}{(I+C)^2 \gamma^3 - (I+2C)\gamma^2 - (I+2B)\gamma + (I+B)^2} \quad (3.19)$$

where  $\gamma$  - is the coefficient of the ratio of principal stresses for a given condition of stability loss.

$$\text{Thus, } \gamma = \frac{\sigma_2}{\sigma_1}.$$

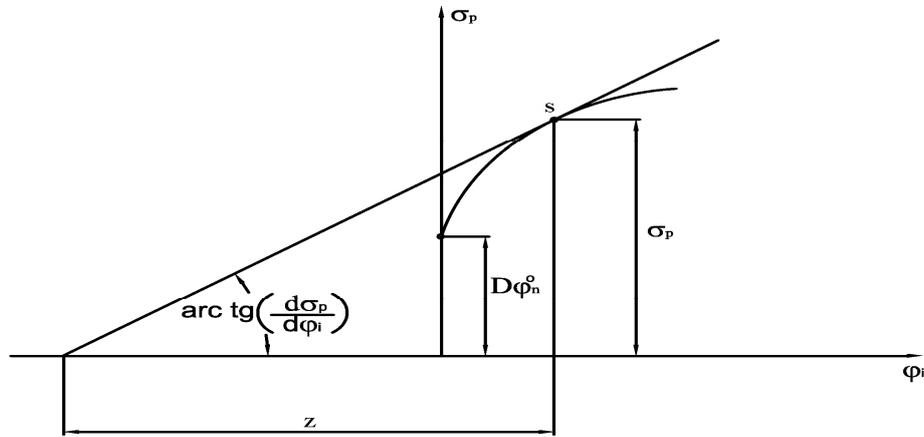


Fig.4. Scheme of the hardening curve with the point of contact s and the subtangent abscissa.

Substituting expression (3.5) to the left side of Eq.(3.20), after suitable operations we obtain the following expression

$$\frac{n}{\varphi_{(i)} + \varphi_0} = \frac{l}{z}. \tag{3.20}$$

Let us substitute the expressions for the coefficient  $\gamma$  for the ratio of principal stresses (3.15)<sub>3</sub> - (3.17)<sub>3</sub> to expression (3.19), and include Eq.(3.20). Now, we can derive suitable formulas for critical (boundary) values of the equivalent strain  $\varphi_{(i)}$  corresponding to a form of stability loss, namely

- for a sheet material of properties of plane anisotropy

$$\varphi_{(i)} = \frac{n(I+C)}{\sqrt{(I+B)(I+C)}-I} - \varphi_0, \tag{3.21}$$

- for a sheet material of properties of normal anisotropy

$$\varphi_{(i)} = \frac{n(I+r)}{\sqrt{I+2r}} - \varphi_0, \tag{3.22}$$

- for a sheet material of isotropic properties

$$\varphi_{(i)} = \frac{2}{\sqrt{3}}n - \varphi_0. \tag{3.23}$$

d) The fourth form of stability loss can occur as a result of local shearing initiation along the wall thickness in the direction of the maximum shear stresses (formation of so-called instability as a shearing band) in the place of reduction of thickness under the plane stress state (PSS) for a case of tensile stresses, see (Bressan and Williams, 1983; Hill, 1986; Johnson and Mellor, 1975; Kaczanow, 1969; Maeciniak, 1971, Sokołowski, 1957; Szczepiński, 1967). When the principal stresses are tensile (have the same signs), then the planes of the maximum shear stresses run through the sheet surface at the angle of  $45^\circ$  along one straight line (Hill, 1986; Kaczanow, 1969; Szczepinski, 1067).

Thus, in this condition the shearing stress  $\tau$  reaches the critical value. The condition directly precedes cracking of the formed drawpiece, see the experimental data in Tab.1.

The condition of stability loss derived in Bressan and Williams (1983), Johnson and Mellor (1975) concerns only the material of isotropic properties. Thus

$$\varphi_{(i)} = \frac{4}{11}(2n + 1) - \varphi_0. \quad (3.24)$$

Finally, we can state that in the end of the 20th century and in recent years many interesting papers has been published, for example (Ostrowska-Maciejewska *et al.*, 2013; Pęcherski, 1996; 1998). In these papers the authors considered the influence of anisotropy induced by plastic strains, and presented macroscopic effects resulting from formation of microscopic shear bands during big plastic deformations in metallic materials. The papers present also mathematical foundations of boundary states for anisotropic materials.

#### 4. Summary and conclusions

1. This chapter presents the results defining a height of the formed spherical cap made from the sheet disk fixed at the edge during a test of hydraulic forming for four chosen conditions of stability loss met in engineering practice. These conditions were determined according to other papers, for example (El-Sebaie and Mellor, 1972; Hill, 1986; Marciniak, 1971; Marciniak and Kuczynski, 1967; Moore and Wallace, 1964/1965; Szczepiński, 1967). The formed disk takes a form of a spherical cap, deformations of which have been specified according to the membrane theory of shells (Marciniak, 1966; 1968; 1971). During such a process, the wall thickness varies depending on the actual radius  $r_i$  and advance of the process (in the considered case on the cap height  $h$ ). The maximum reduction of the wall thickness can be seen in the vertex (central) part of the formed shell.
2. The drawpiece height is also influenced by material parameters of the sheet disk, the hardening coefficient  $n$  and the coefficient  $\varphi_0$ . In all the cases, an increase of the coefficient  $n$  causes an increase of the boundary equivalent strains  $\varphi_{(i)}$  and, in consequence, an increase of the cap height  $h$ . On the other hand, in all the cases under consideration an increase of the preliminary strain  $\varphi_0$  causes a decrease of  $\varphi_{(i)}$  and the cap height  $h$ .
3. In the end of the 20th century and recently many interesting papers have been published, see (Ostrowska-Maciejewska *et al.*, 2013, Pęcherski, 1996; 1998). They address the influence of anisotropy induced by plastic strains on plastic properties of sheets.

#### Nomenclature

- $a$  – external radius of the drawpiece
- $B$  and  $C$  – coefficients of anisotropy on the sheet plane (plane anisotropy)
- $D$  – material constant for power curve of hardening (Swift equation), see (Swift, 1952)
- $g$  – actual thickness of the drawpiece wall corresponding to the actual radius  $r_i$ ,
- $g_0$  – initial thickness of the sheet
- $h$  – height of the drawpiece
- $n$  – coefficient of the hardening curve

- $p$  – liquid pressure during the test of hydraulic forming  
 $R$  – radius of the spherical cap describing a shape of the drawpiece cap  
 $r$  – coefficient of normal anisotropy  
 $r_i$  – actual radius of the drawpiece  
 $r_0$  – actual radius of sheet disk before forming  
 $z$  – value of the abscissa corresponding to the stability loss (Moore and Wallace, 1968)  
 $\alpha$  – angle between the radius  $R$  in the place corresponding to the actual radius and the axis of drawpiece symmetry  
 $\varphi_1, \varphi_2, \varphi_3$  – principal components of logarithmic plastic strains-circumferential, longitudinal and along the thickness in the direction normal to the sheet surface, respectively  
 $\dot{\varphi}_1, \dot{\varphi}_2, \dot{\varphi}_3$  – rates of principal logarithmic plastic strains-circumferential, longitudinal and along the thickness in the direction normal to the sheet surface, respectively  
 $\varphi_{(i)}$  – logarithmic plastic equivalent strain (strain intensity),  $\varphi_{(i)} = \sqrt{\frac{2}{3}(\varphi_1^2 + \varphi_2^2 + \varphi_3^2)}$   
 $\varphi_0$  – logarithmic preliminary strain (material constant)  
 $\frac{\dot{\varphi}_1}{\dot{\varphi}_3} = \frac{I}{B} = r_2$ ,  
 $\frac{\dot{\varphi}_2}{\dot{\varphi}_3} = \frac{I}{C} = r_1$  – coefficients representing properties of plane anisotropy  
 $\gamma$  – ratio of principal stresses ( $\gamma = \sigma_2 / \sigma_1$ )  
 $\sigma_1, \sigma_2, \sigma_3$  – components of principal stresses  
 $\sigma_p$  – stress equal to the plastic strain  
 $\tau$  – shear stresses under the plane stress state  
 $\tau_{cr}$  – critical shear stress under the plane stress state (PSS)

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# CONDITIONS OF STABILITY LOSS DURING THE TEST OF HYDRAULIC FORMING OF DRAWPIECES Part II. Discussion and analysis of the results

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This part of the paper contains the calculation results presented in the form of suitable diagrams and tables. The calculations were made according to the relationships obtained in Part I devoted to a laboratory test of hydraulic forming for some chosen engineering conditions of stability loss. The influence of the coefficient of material hardening, the coefficient of normal anisotropy and the coefficients of plane anisotropy on acceptable plastic strains and heights of the formed drawpieces corresponding to a given condition of stability loss was tested, too.

**Key words:** hydraulic forming, stability loss, boundary strains, height of the drawpiece, normal and plane anisotropy.

## 1. Introduction

As it was said in the introduction to Part I, a test of hydraulic forming of sheets with a liquid, so-called Jovignot's test is a laboratory test simulating real conditions of forming and belonging to technological processes mentioned above. The test is usually applied for investigations on drawability, strength and plastic properties of sheets, and a replacement of a rigid punch by liquid pressure eliminates the influence of friction between the punch and the sheet surface. The stress and strain states in the formed drawpiece are usually different from those occurring in industrial forming operations (except for forming with the use of liquid, rubber or another synthetic resin), but the test is still useful and it is often applied in various versions. Some authors think (Banasiak, 1971; Jentet, 1961; Michałowski, 1967; Okamoto *et al.*, 1968) that the data coming from the test of shallow drawing by a liquid are more useful for sheet drawability estimation than those coming from a test using a rigid punch. Special cases are usually mentioned, namely forming of sheets for bodies of cars and rail-coaches, sheets for aircrafts (Michałowski, 1967), forming of many kinds of dishes and similar goods, bending of tubes and metallic panels for shaped welded tubes (with an intermittent weld), bending of sheets for any housings of machines and devices applied in industry and at home, bending of shells for cleaded energetic pipelines and others.

The test of sheet forming is also often applied in aircraft industry (Michałowski, 1967) and power industry. A modified version of Jovignot's test is applied for investigations of sheet properties under dynamic forming or during technological cladding processes. It is especially important under high strain rates (explosive method) because a lack of friction effects allows a more precise determination of sheet properties influencing their usability for technological forming processes.

In practice, Jovignot's test is often carried out in order to measure the height  $h$  of the drawpiece as a measure of sheet drawability, and register the pressure corresponding to the moment directly before the crack. In more than 95% cases, it is the pressure behind the point  $p_{max}$ , where the working pressure reaches its maximum value (it is one of the points of stability loss of the deformed shell). The crack formation breaks the test and causes disconnection of the pressure pump forcing the liquid.

At present, during tests of such kind it is possible to measure and register continuously actual values of the pressure and the corresponding cap heights. Thus, it is possible to register and measure a value of  $p_{max}$  and the corresponding cap height. In this test, the point  $p_{max}$  is equally important as the maximum point at the curve of uniaxial tension used for the determination of strength of elements of machines and devices.

The assumed scheme of strains and stresses in the formed sheet disk was the same as in the deformed membraneous shell, It is a certain simplification as compared with a real distribution of strains and stresses in the metallic spherical cap. In consequence, the drawpiece heights  $h$  determined in the paper can be lower than the real height. Thus, it can be a certain lower estimation quickly obtained from simple analytic expressions.

As it was said, this part of the paper presents the influence of the coefficient of hardening and preliminary plastic strain, and the influence of normal anisotropy and plane anisotropy (on the sheet surface) on the critical value of the equivalent plastic strain  $\varphi_{(i)}$ , corresponding to the above conditions of stability loss (a-d). Under the above assumptions we may know only the critical equivalent logarithmic plastic strain  $\varphi_{(i)}$  - it is enough for the determination of the critical value of the drawpiece height  $h$ .

According to the membraneous theory, the drawpiece was made from a sheet disk fixed at the periphery and it was subjected to liquid pressure. Under the assumptions contained in Part I of this paper, it becomes a spherical cap of gradually reducing thickness (from the periphery to the centre). Under the assumed constant volume of the material, an increase of the drawpiece area must be compensated by a reduction of thickness; the maximum reduction of thickness occurs in the vertex part.

## 2. Discussion of the obtained results

According to Part I of this paper, the behaviour of instability states of the formed drawpiece in Jovignot's test can be shown in the graph of the system pressure – equivalent strain ( $p - \varphi_{(i)}$ ), see Fig.1.

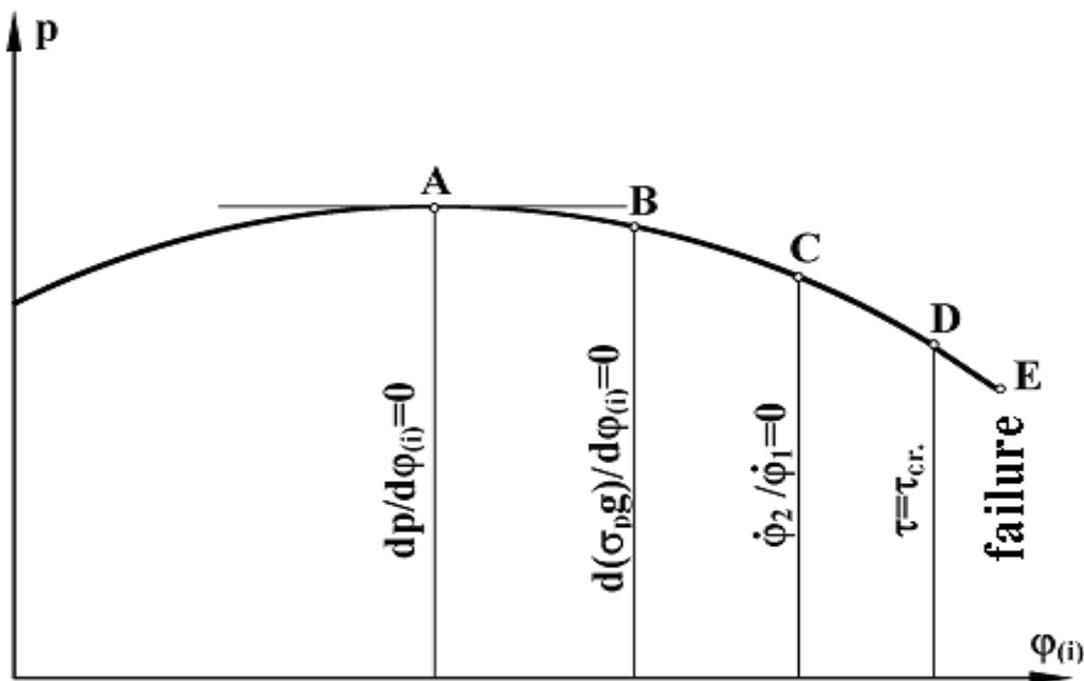


Fig.1. Scheme of the process of deformation of the drawpiece formed by a liquid with characteristic points determining the given conditions of stability loss and the crack point.

From Fig.1 it appears that for a given material the first (a), second (b), third (c) and fourth (d) form of stability loss follow one by one during the process of drawpiece forming. When the point representing the third form of stability loss as initiation of a chase is exceeded, local initiation of the plane state of deformation (PSD) takes place, and areas outside the chase pass into the rigid state. After exceeding the

fourth form of stability loss (shearing) most drawpieces are subjected to cracking soon. At the next stage of the forming process, the drawpiece is subjected to macrocracking (usually in the vertex part), and in Jovignot's test the drawpiece forming process is finished.

The obtained calculation results are presented in Table 1 and in suitable diagrams being nomograms. Figure 2 shows the results for the drawpiece made of an isotropic material. Curve 1 was obtained from Eq.(3.9), part I, from the condition for the maximum of liquid pressure ( $dp/d\varphi_{(i)} = 0$ ), and curve 2 – from Eq.(3.13), part I. Curve 3 comes from Eq.(3.23), part I for the condition ( $\dot{\varphi}_2/\dot{\varphi}_1 = 0$ ) – initiation of the plane state of deformation (PSD) – formation and development of the chase up to the crack. Curve 4 was obtained from Eq.(3.24), Part I resulting from the condition for reaching the critical values ( $\tau = \tau_{cr}$ ) by the shear stress (Bressan and Williams, 1983) under plane state of stresses (PSS) for a case when two principal stresses are tensile (El-Sebaie, 1972; Gabryszewski and Gronostajski, 1991; Hill, 1986; 1983; Kaczanow, 1969; Szczepinski, 1967; Życzkowski, 1981).

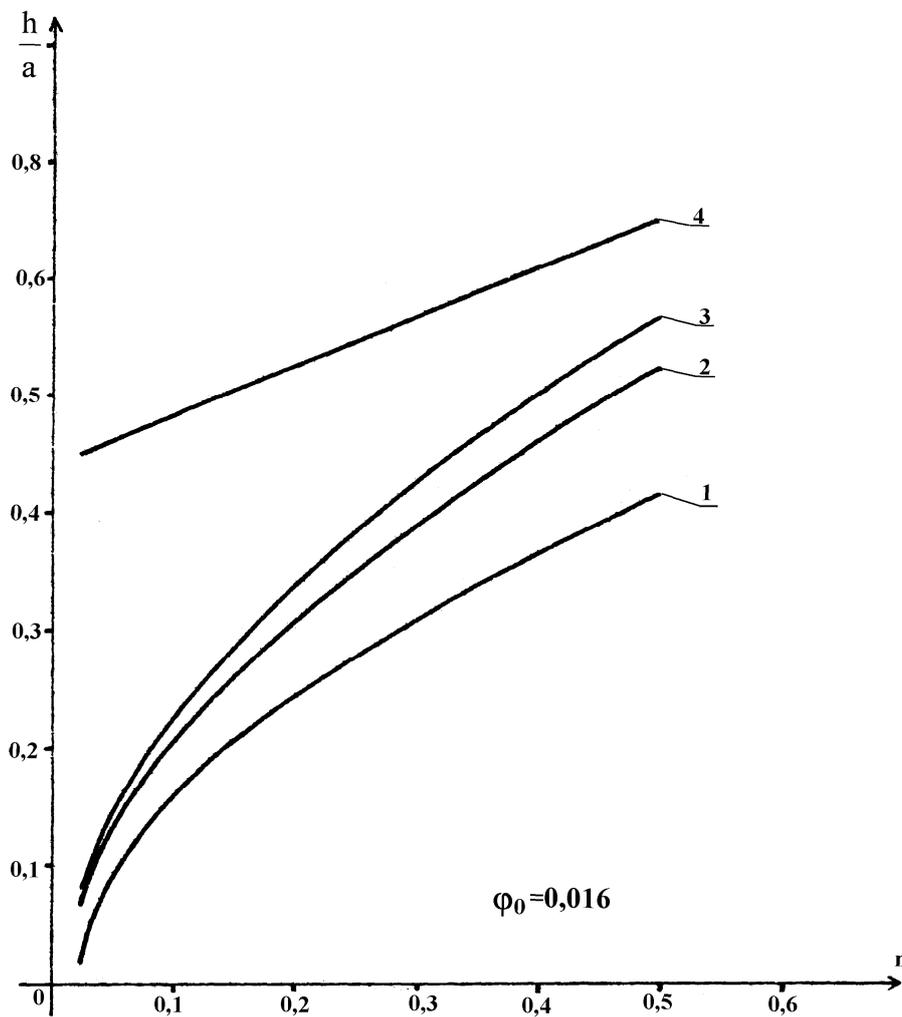


Fig.2. Diagrams of height of the formed drawpiece versus the coefficient of hardening for four cases of stability loss.

Table 1. Comparison of the experimental (Bressan and Williams, 1983) and theoretical results.

Material	$n$	$\varphi_0$	$\frac{g}{g_0}$			
			Experiment		Theory	
			Instability	Cracking	Eq.(3.24)	Eq.(3.23)
Soft copper	0.30	0.010	0.555	0.50	0.71	0.51
Semi-hard copper	0.30	0.070	0.590	0.58	0.75	0.54
Soft bronze	0.48	0.000	-----	0.50	0.57	0.43
Semi-hard bronze	0.48	0.078	-----	0.51	0.62	0.46
Soft aluminium	0.25	0.000	0.565	0.48	0.74	0.52
Killed steel	0.20	0.000	0.570	0.48	0.79	0.53
Stainless steel	0.50	0.010	0.520	0.50	0.56	0.42

Table 1 contains a comparison of the experimental data obtained in Bressan and Williams (1983) and the calculation results for the third and fourth condition of stability loss [see Eqs (3.23) and (3.24) part I] for an isotropic material.

Figure 3 presents the results obtained for a sheet of properties of normal anisotropy for three conditions of stability loss depending on a value of the coefficient of hardening  $n$  and some chosen values ( $r = 0.5; 1; 2; 3; 4$  and  $5$ ) of the coefficient of normal anisotropy  $r$ . The assumed material constant was  $\varphi_0 = 0.016$  (Marciniak, 1971). From the figure it appears that the height of the formed drawpiece  $h$  increases together with the next condition of stability loss and with an increase of the coefficient of hardening  $n$  and normal anisotropy  $r$ . It is easy to predict it because the coefficient of hardening causes an increase of the acceptable stress, and an increase of the coefficient of normal anisotropy  $r$  elongates the ellipse of plasticity in plane state of stresses (PSS), see (Gabryszewski and Gronostajski, 1991; Marciniak, 1971; Moore and Wallace, 1964/1965) in the case of positive principal stresses and, in consequence, an increase in values of acceptable stresses in the drawpiece forming process by a liquid. An increase of  $r$  also causes that reduction of the drawpiece wall is less, so the resistance against wall reduction increases. Thus, materials of greater coefficients  $r$  have better drawability properties (Gabryszewski and Gronostajski, 1991; Marciniak, 1968; 1971; Moore and Wallace, 1964/1965).

Figures 4-6 present calculation results for the sheet of anisotropic properties on the plane. The assumed values of pairs of coefficients ( $r_1$  and  $r_2$ ), representing properties of plane anisotropy were defined according to the data from literature (Banasiak, 1971; Marciniak, 1971). From the diagrams it appears that the influence of plane anisotropy on the height of the formed drawpiece  $h$  is ambiguous. Basing on the obtained calculation results, we can generally state that anisotropy on the sheet plane does not influence drawability (height of the formed drawpiece) in a distinct way. The graphs were presented depending on the hardening parameter  $n$  for three successive (a-c) conditions of stability loss. The constant  $\varphi_0 = 0.016$  was assumed for calculations.

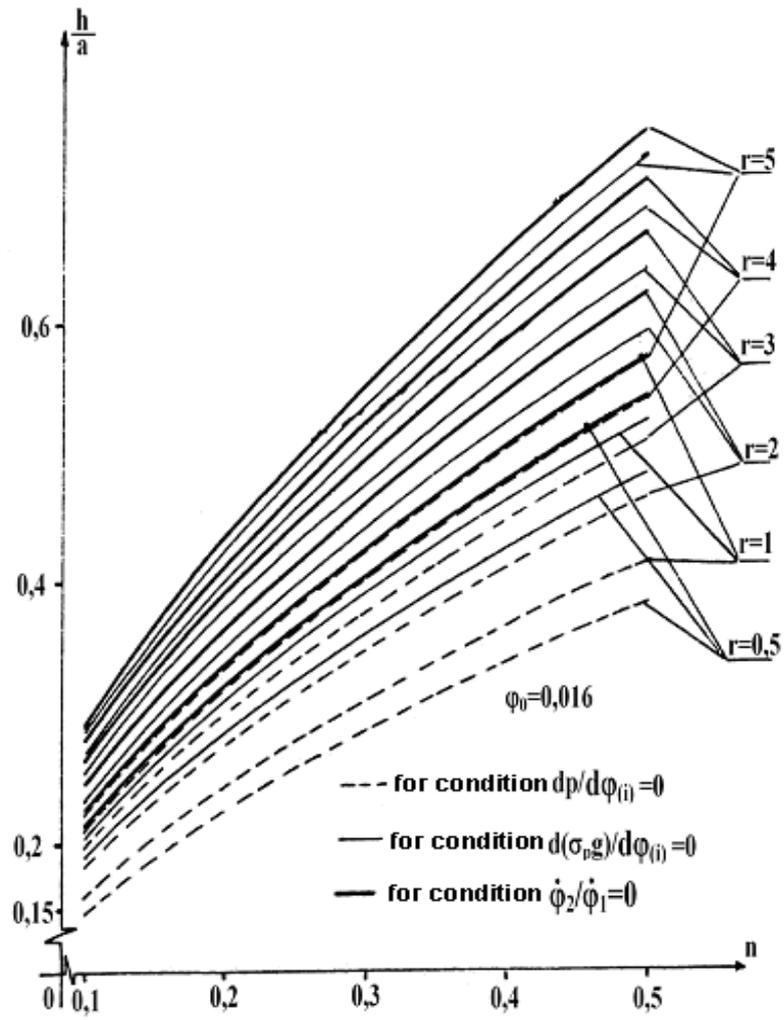


Fig.3. Graphs of height of the formed drawpiece depending on the parameter of hardening  $n$  and the coefficient of normal anisotropy  $r$  for three conditions of stability loss.

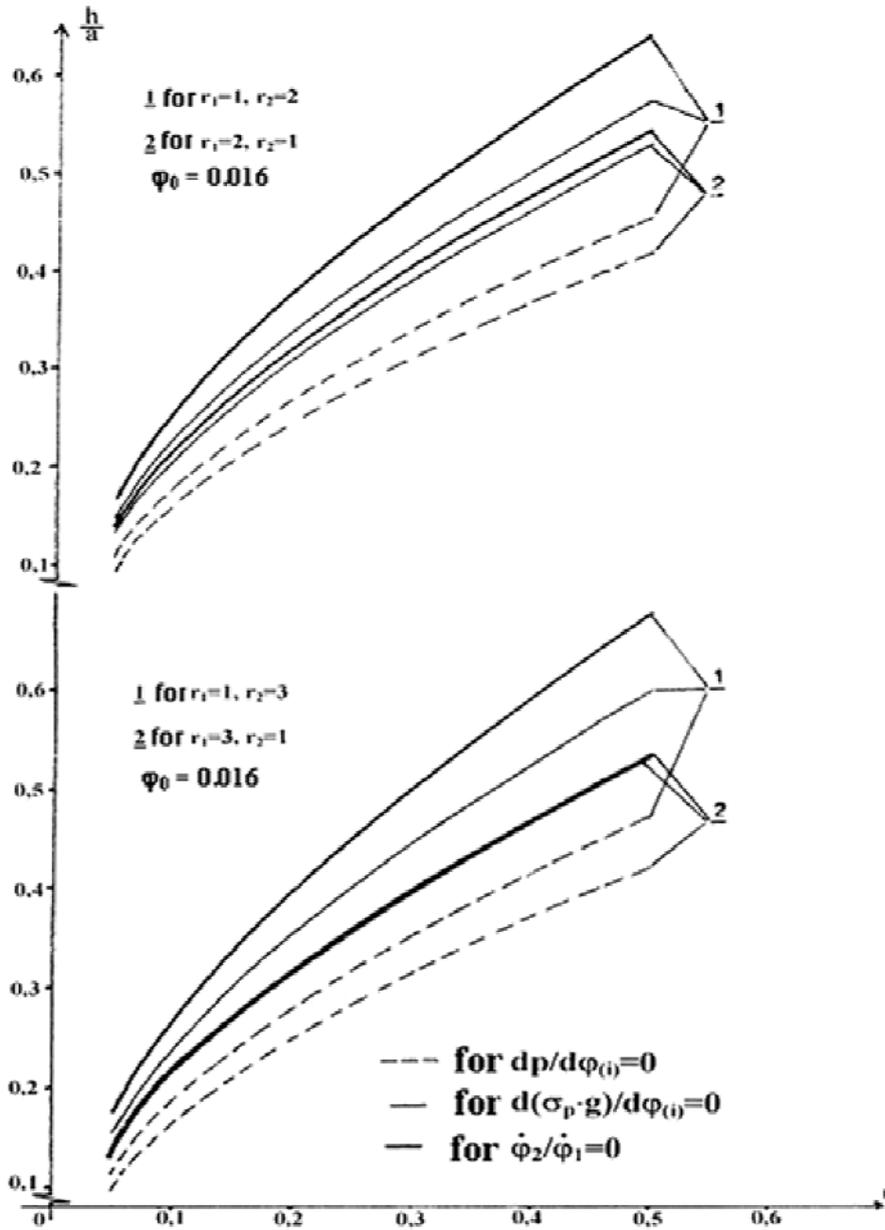


Fig.4. Graphs of spherical cap height depending on the coefficient of hardening  $n$  and some selected pairs of coefficients of plane anisotropy  $r_1$  and  $r_2$  for three conditions of stability loss.

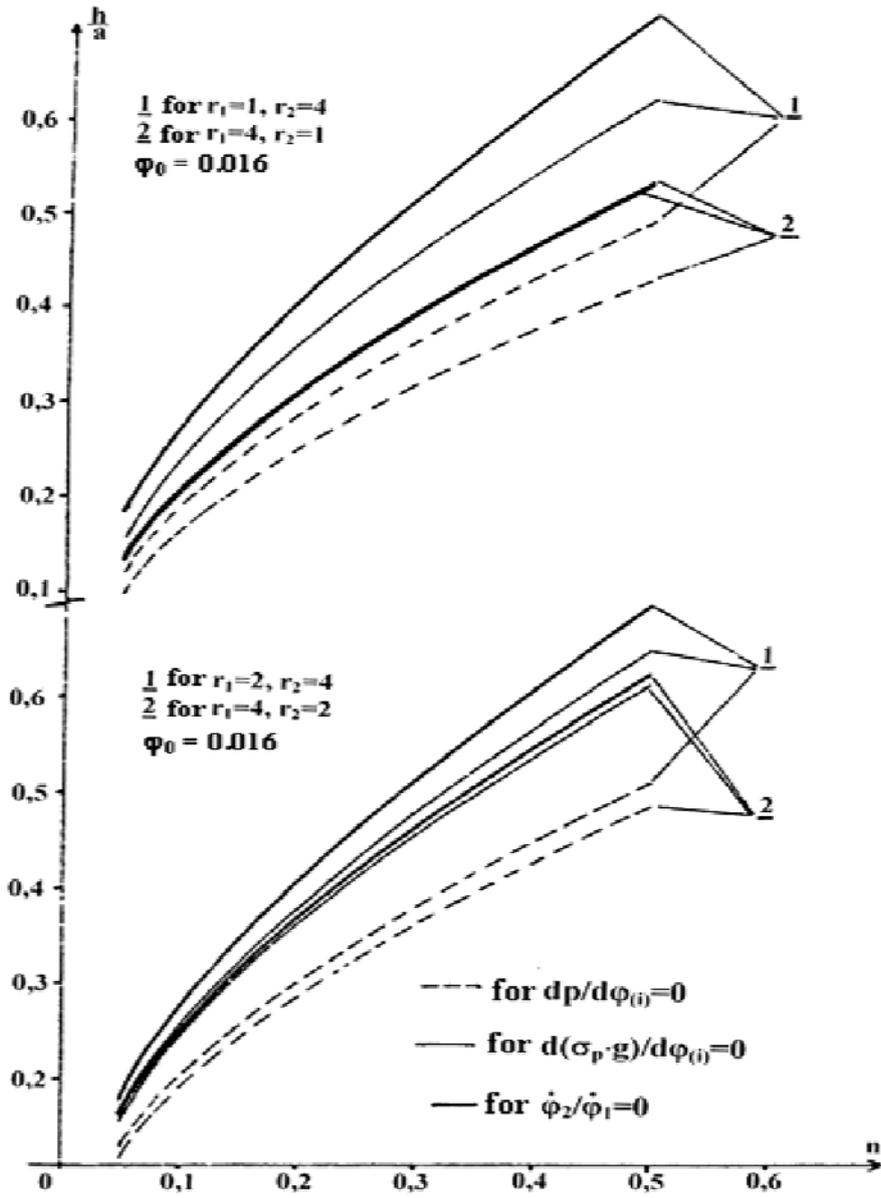


Fig.5. Graphs of spherical cap height depending on the coefficient of hardening  $n$  and some selected pairs of coefficients of plane anisotropy  $r_1$  and  $r_2$  for three conditions of stability loss.

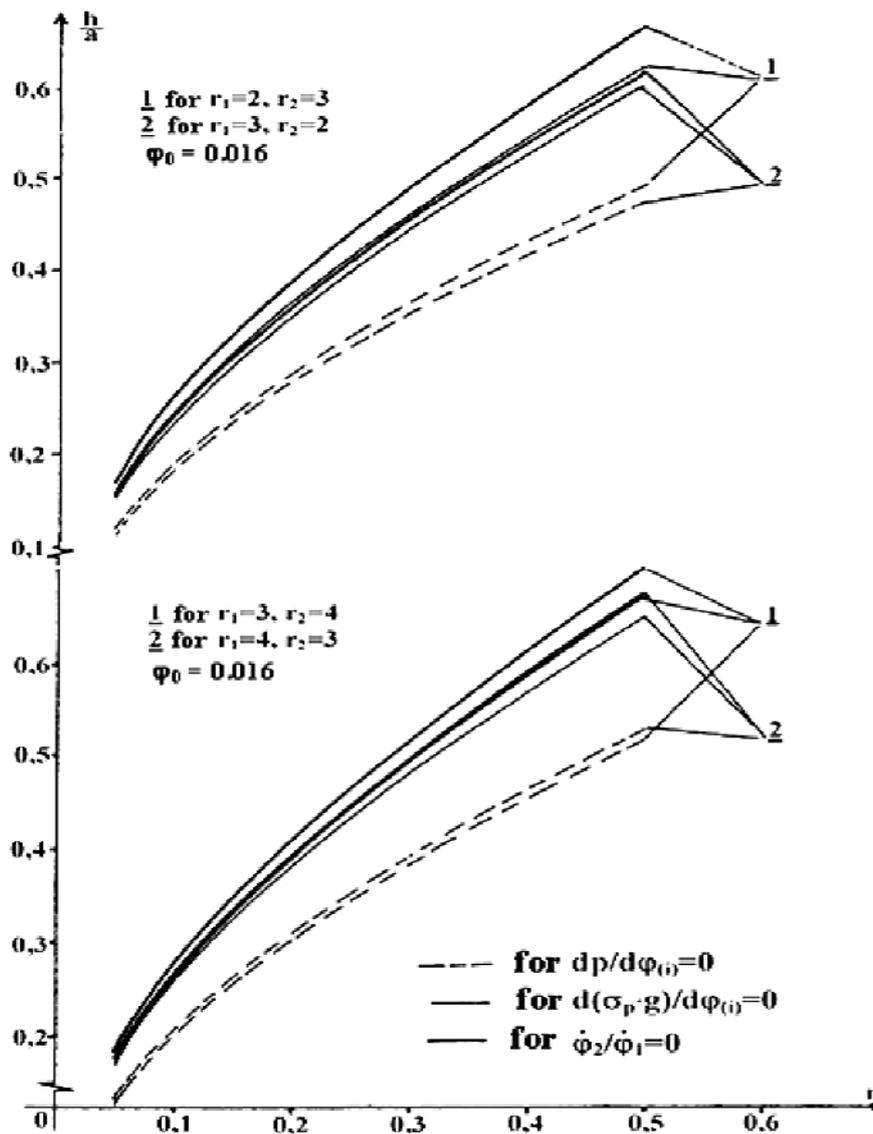


Fig.6. Graphs of spherical cap height depending on the coefficient of hardening  $n$  and some selected pairs of coefficients of plane anisotropy  $r_1$  and  $r_2$  for three conditions of stability loss.

### 3. Final remarks and conclusions

1. In Fig.1 it can be seen that in the case of a given material the first-a, second-b, third-c and fourth-d form of stability loss in the process of drawpiece forming follow one by one. When the point representing the third form of stability loss is exceeded and chase is initiated, local initiation of the plane state of deformations (PSD) begins, and areas outside the chase pass into the rigid state. After exceeding the fourth form of stability loss (shearing), most drawpieces are subjected to cracking after a moment. At the next stage of forming, the drawpiece is subjected to macrocracking (usually at the vertex part of the drawpiece), and during Jovignot's test the drawpiece forming process is finished.

2. From Fig.2 it appears that the maximum shell height  $h$  and maximum equivalent strain  $\varphi_{(i)}$  are obtained from the fourth condition of stability loss (instability in form of shearing) under plane state of

stresses (PSS), when both stresses are tensile (they have the same positive signs), and the minimum one – from the condition for the maximum of liquid pressure  $p_{\max}$ .

3. Figure 3 shows the influence of normal anisotropy  $r$  on the height  $h$  of the formed spherical cap for three (a-c) conditions of stability loss, because in the literature the fourth condition relates only to isotropic materials. From the diagrams it appears that an increase of the coefficient of anisotropy  $r$  for the (H-M-H) condition of plasticity including the influence of normal anisotropy causes elongation of the ellipse of plasticity (Banasiak, 1971; Marciniak, 1971; Moore and Wallace, 1964/1965; Szczepinski, 1967; Śloderbach and Sawicki, 1984), and in consequence an increase of acceptable values of stresses and strains.

4. Figures 4-6 presents the influence of coefficients of anisotropy on the drawpiece plane ( $r_1$  and  $r_2$ ) on the drawpiece height  $h$  for three (a-c) conditions of stability loss. It is difficult to determine the influence of coefficients of plane anisotropy on the cap height  $h$  and generally speaking we can state that it does not exist. It results from the fact that the (H-M-H) ellipsis of plasticity is subjected not only to elongation under the change of ( $r_1 = 1/C$ ) but also to rotation around the beginning of the coordinate system under the change of ( $r_2 = 1/B$ ). A more detailed analysis of that phenomenon can be found in Banasiak (197), Marciniak and Kuczynski (1967). Generally speaking, an increase of the coefficients  $r_1$  and  $r_2$  causes a decrease of differences between the results for the second-b and third-c forms of stability loss, see Figs 9-10. An increase of the coefficient  $r_2$  under a constant and low value of the coefficient  $r_1$  causes that differences in the cap height  $h$  for three different conditions of stability are big, see Figs 8-9. The above calculation results seem to prove a thesis formulated by Prof. M. Zakrzewski from the Wrocław University of Technology. In the 1990s he found that in the sheet forming process plane anisotropy did not influence drawability (acceptable strains, drawpiece height, forming depth). Its influence is manifested by formation of „ears”, see (Banasiak, 1971; Gabryszewski and Gronostajski, 1991; Marciniak, 1961; Ostrowska-Maciejewska *et al.*, 2013) at free peripheries of the formed drawpieces. In Jovignot’s test the „ears” do not appear because a periphery of the sheet disk is fixed in the matrix.

5. The derived expressions for the drawpiece height can give lower calculation results than the experimental results. It can be caused by the assumed spherical shape of the formed shell resulting from the membrane theory of strain distribution ( $\varphi_1 = \varphi_2$ ) whereas in fact that shape is more similar to paraboloid of revolution (Marciniak, 1971). It can be an effect of non-uniformity of distribution of strains and their components in the vertex part, especially at the end stage of forming, where ( $\varphi_1 \neq \varphi_2$ ).

6. Let us note that the applied first three conditions of stability loss (a-c), except for the fourth one (which, according to Tab.1, sometimes gives thickness reduction less than in the case of experiments) determine states preceding macrocrack of the drawpiece. In many cases, they are lower (safe) estimations of actual height of the formed drawpiece in Jovignot’s test.

7. As it was mentioned in Part I of this paper, many interesting papers were published at the end of the 20th century, for example (Ostrowska –Maciejewska *et al.* 2013; Pęcherski, 1996; 1998) where the authors analysed the influence of anisotropy induced by plastic strains on plastic properties of drawability of metallic materials. These papers present macroscopic effects resulting from the formation of microscopic shear bands during big plastic deformations of metallic materials. They also present mathematical foundations of boundary states for anisotropic metallic materials.

## Nomenclature

- $a$  – external radius of the drawpiece
- $B$  and  $C$  – coefficients of anisotropy on the sheet plane (plane anisotropy)
- $D$  – material constant for power curve of hardening (Swift equation), see (Swift, 1952)
- $g$  – actual thickness of the drawpiece wall corresponding to the actual radius  $r_i$
- $g_0$  – initial thickness of the sheet
- $h$  – height of the drawpiece
- $n$  – coefficient of the hardening curve
- $p$  – liquid pressure during the test of hydraulic forming
- $r$  – coefficient of normal anisotropy

$r_1 = 1 / C$ , – coefficients representing properties of plane anisotropy  
 $r_2 = 1 / B$

$\varphi_1, \varphi_2, \varphi_3$  – principal components of logarithmic plastic strains – circumferential, longitudinal and along the thickness in the direction normal to the sheet surface, respectively

$\dot{\varphi}_1, \dot{\varphi}_2, \dot{\varphi}_3$  – rates of principal logarithmic plastic strains - circumferential, longitudinal and along the thickness in the direction normal to the sheet surface, respectively

$\varphi_{(i)}$  – logarithmic plastic equivalent strain (strain intensity)

$$\varphi_{(i)} = \sqrt{\frac{2}{3}(\varphi_1^2 + \varphi_2^2 + \varphi_3^2)}$$

$\varphi_0$  – logarithmic preliminary strain (material constant)

$\gamma$  – ratio of principal stresses ( $\gamma = \sigma_2 / \sigma_1$ )

$\sigma_1, \sigma_2, \sigma_3$  – components of principal stresses [Pa]

$\sigma_p$  – stress equal to the plastic strain [Pa]

$\tau$  – shear stresses under the plane stress state [Pa]

$\tau_{cr}$  – critical shear stress under the plane stress state (PSS) in [Pa]

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