

Brief note

RADIATIVE FLOW PAST A PARABOLIC STARTED ISOTHERMAL VERTICAL PLATE WITH UNIFORM MASS DIFFUSION

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A theoretical solution of thermal radiation effects on an unsteady flow past a parabolic starting motion of an infinite isothermal vertical plate with uniform mass diffusion has been studied. The plate temperature as well as the concentration level near the plate are raised uniformly. The dimensionless governing equations are solved using the Laplace-transform technique. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The effects of velocity profiles are studied for different physical parameters such as the thermal radiation parameter, thermal Grashof number, mass Grashof number and Schmidt number. It is observed that the velocity increases with increasing values the thermal Grashof number or mass Grashof number. The trend is just reversed with respect to the thermal radiation parameter.

Key words: parabolic, radiation, isothermal, vertical plate, heat and mass transfer.

1. Introduction

Radiative heat and mass transfer play an important role in manufacturing industries. They are of importance in the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, combustion and furnace design, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as in numerous agricultural, health and military applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the combined effect of thermal radiation and mass diffusion.

Natural convection of a flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method by was studied Gupta *et al.* (1979). Kafousias and Raptis (1981) extended this problem to include mass transfer effects subjected to variable suction or injection. Soundalgekar (1982) studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux were analyzed by Singh and Singh (1983). Free convection effects on flow past an exponentially accelerated vertical plate were studied by Singh and Naveen Kumar (1984). The skin friction for an accelerated vertical plate was

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studied analytically by Hossain and Shayo (1986). Mass transfer effects on an exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion were studied by Jha *et al.* (1991). Agrawal *et al.* (1998) studied free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of the magnetic field. Agrawal *et al.* (1999) further extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of an infinite vertical plate with a transverse magnetic plate. The governing equations are tackled using the Laplace transform technique.

It is proposed to study a flow past an infinite isothermal vertical plate subjected to parabolic motion with uniform mass diffusion in the presence of thermal radiation. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of an exponential and complementary error function.

2. Mathematical analysis

An unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical plate with uniform diffusion, in the presence of thermal radiation is considered. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The x' -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞ . At time $t' > 0$, the plate is started with a velocity $u = u_0 \cdot t'^2$ in its own plane against the gravitational field and the temperature from the plate is raised to T_w and the concentration level near the plate are also raised to C'_w . The plate is infinite, then all the terms in the governing equations will be independent of x' and there is no flow along the y -direction. Then under the usual Boussinesq's approximation the unsteady starting motion is governed by the following equations

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2}, \quad (2.1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \quad (2.2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2}, \quad (2.3)$$

with the following initial and boundary conditions

$$\begin{aligned} u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all } y, t' \leq 0, \\ t' > 0: u = u_0 \cdot t'^2, \quad T = T_w, \quad C' = C'_w \quad \text{at } y = 0, \\ u \rightarrow 0 \quad T \rightarrow T_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y \rightarrow \infty. \end{aligned} \quad (2.4)$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma (T_\infty^4 - T^4). \quad (2.5)$$

It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \quad (2.6)$$

By using Eqs (2.5) and (2.6), Eq.(2.2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y'^2} + 16a^* \sigma T_\infty^3 (T_\infty - T). \quad (2.7)$$

On introducing the following non-dimensional quantities

$$U = u \left(\frac{u_0}{v^2} \right)^{1/3}, \quad t = \left(\frac{u_0^2}{v} \right)^{1/3} t', \quad Y = y \left(\frac{u_0}{v^2} \right)^{1/3}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad \text{Gr} = \frac{g\beta(T - T_\infty)}{(v \cdot u_0)^{1/3}}, \quad \text{Gc} = \frac{g\beta(C' - C'_\infty)}{(v \cdot u_0)^{1/3}}, \quad (2.8)$$

$$R = \frac{16a^* \sigma \cdot T_\infty^3}{k} \left(\frac{v^2}{u_0} \right)^{2/3}, \quad \text{Pr} = \frac{\mu C_p}{k}, \quad \text{Sc} = \frac{v}{D},$$

in Eqs (2.1), (2.3) and (2.7), we get

$$\frac{\partial U}{\partial t} = \text{Gr}\theta + \text{Gc}C + \frac{\partial^2 U}{\partial Y^2}, \quad (2.9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\text{Pr}} \theta, \quad (2.10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{\text{Sc}} \frac{\partial^2 C}{\partial Y^2}. \quad (2.11)$$

The initial and boundary conditions in non-dimensional quantities are

$$U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } Y, t \leq 0,$$

$$t > 0: \quad U = t^2, \quad \theta = 1, \quad C = 1 \quad \text{at } Y = 0, \quad (2.12)$$

$$U \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } Y \rightarrow \infty.$$

The dimensionless governing Eqs (2.6) to (2.8) and the corresponding initial and boundary conditions (2.9) are tackled using the Laplace transform technique

$$\theta = \frac{I}{2} \left[\exp(2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{\operatorname{Pr}} + \sqrt{at}) + \exp(-2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{\operatorname{Pr}} - \sqrt{at}) \right], \quad (2.13)$$

$$C = \operatorname{erfc}(\eta\sqrt{\operatorname{Sc}}), \quad (2.14)$$

$$\begin{aligned} U = & \frac{t^2}{6} \left[(3 + 12\eta^2 + 4\eta^4) \operatorname{erfc}(\eta) - \frac{\eta}{\sqrt{\pi}} (10 + 4\eta^2) \exp(-\eta^2) \right] + \\ & - d \exp(ct) \left[\exp(2\eta\sqrt{ct}) \operatorname{erfc}(\eta\sqrt{\operatorname{Pr}} + \sqrt{ct}) + \exp(-2\eta\sqrt{ct}) \operatorname{erfc}(\eta\sqrt{\operatorname{Pr}} - \sqrt{ct}) \right] + \\ & - e \left[(1 + 2\eta^2) \operatorname{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}} \exp(-\eta^2) \right] + 2d \operatorname{erfc}(\eta) + \\ & - d \left[\exp(2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{\operatorname{Pr}} + \sqrt{bt}) + \exp(-2\eta\sqrt{Rt}) \operatorname{erfc}(\eta\sqrt{\operatorname{Pr}} - \sqrt{bt}) \right] + \\ & + d \exp(ct) \left[\exp(-2\eta\sqrt{\operatorname{Pr}(b+c)t}) \operatorname{erfc}(\eta\sqrt{\operatorname{Pr}} - \sqrt{(b+c)t}) + \right. \\ & \left. + \exp(2\eta\sqrt{\operatorname{Pr}(b+c)t}) \operatorname{erfc}(\eta\sqrt{\operatorname{Pr}} + \sqrt{(b+c)t}) \right] + \\ & + e \left[(1 + 2\eta^2 \operatorname{Sc}) \operatorname{erfc}(\eta\sqrt{\operatorname{Sc}}) - \frac{2\eta\sqrt{\operatorname{Sc}}}{\sqrt{\pi}} \exp(-\eta^2 \operatorname{Sc}) \right] \end{aligned} \quad (2.15)$$

where, $b = \frac{R}{\operatorname{Pr}}, \quad c = \frac{R}{1 - \operatorname{Pr}}, \quad d = \frac{\operatorname{Gr}}{2c(1 - \operatorname{Pr})}, \quad e = \frac{\operatorname{Gct}}{(1 - \operatorname{Sc})} \quad \text{and} \quad \eta = \frac{Y}{2\sqrt{t}}.$

3. Results and discussion

For physical understanding of the problem numerical computations are carried out for different physical parameters Gr, Gc, Sc and t . The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the values of the Prandtl number Pr are chosen such that they represent air ($\operatorname{Pr} = 0.71$). The numerical values of the velocity are computed for different physical parameters like the Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

Figure 1 presents concentration profiles for different values of the Schmidt number ($\operatorname{Sc} = 0.16, 0.6, 2.01$) and time $t = 0.4$. The effect of concentration is important in the concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

The temperature profiles are calculated for different values of the thermal radiation parameter ($R = 0.2, 2, 5$) and are shown in Fig.2 for air ($\operatorname{Pr} = 0.71$). The effect of the thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing the radiation parameter.

The effect of velocity for different values of the radiation parameter ($R = 2, 5, 10$), $\operatorname{Gr} = 5 = \operatorname{Gc}$ and $t = 0.4$ is shown in Fig.3. The trend shows that the velocity increases with decreasing the radiation parameter. It is observed that the velocity decreases in the presence of high thermal radiation.

Figure 4 demonstrates the effects of different thermal Grashof number ($\operatorname{Gr} = 2, 5$) and mass Grashof number ($\operatorname{Gc} = 5, 10$) on the velocity at $t = 0.4$. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

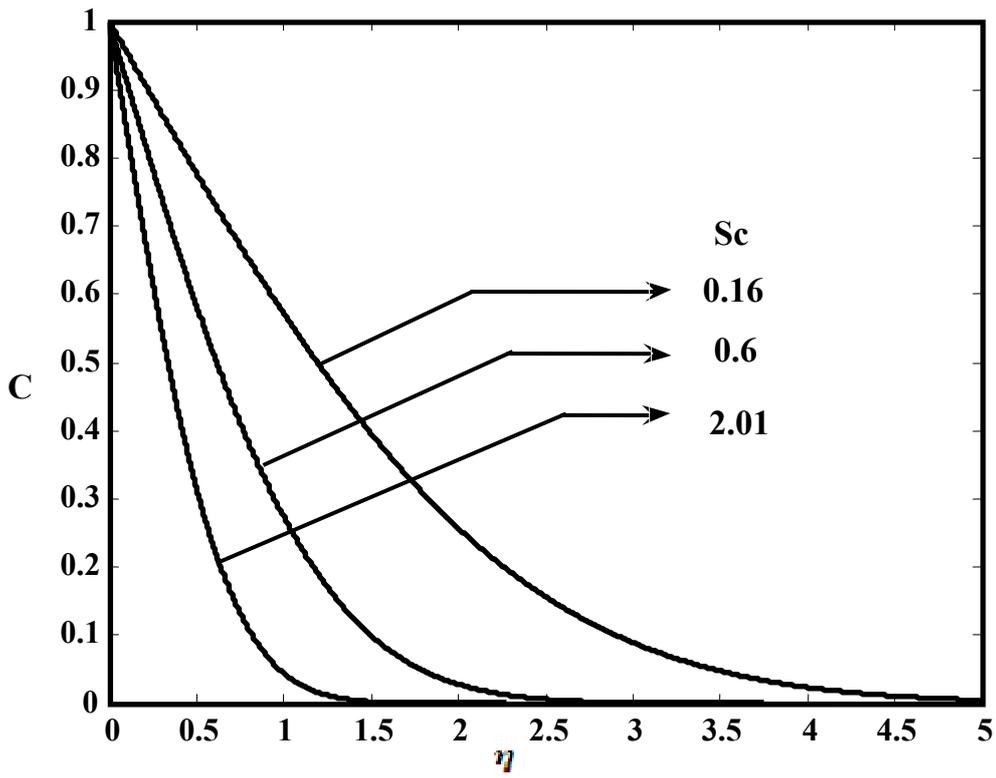


Fig.1. Concentration profiles for different values of Sc .

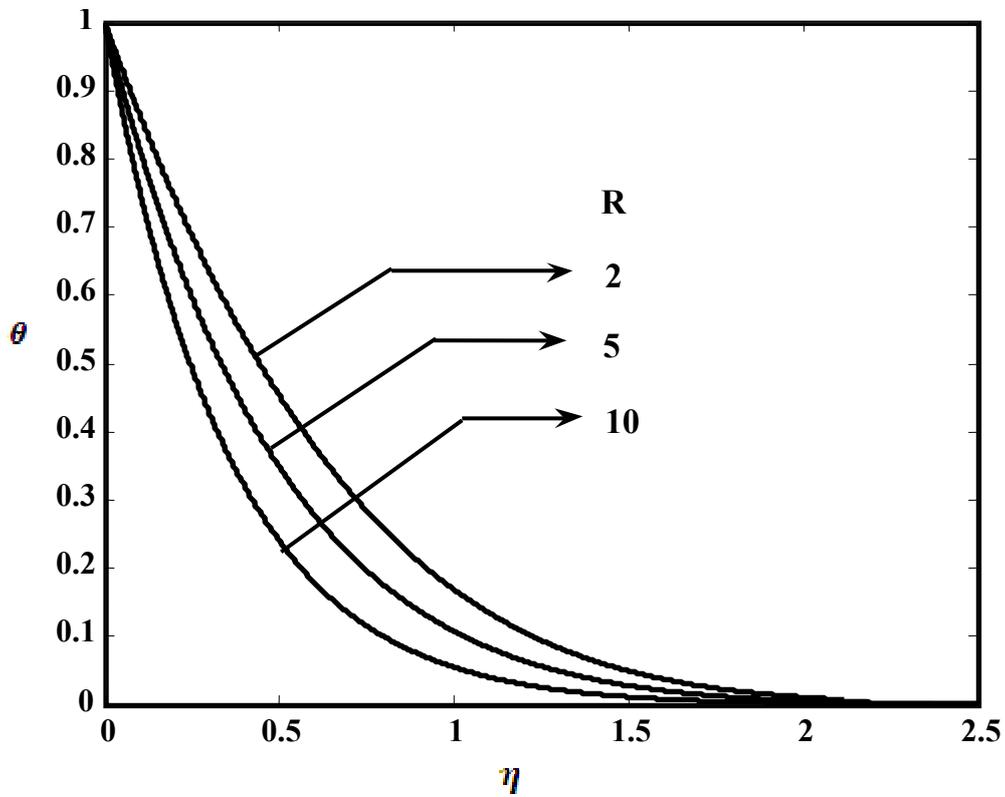


Fig.2. Temperature profiles for different values of R .

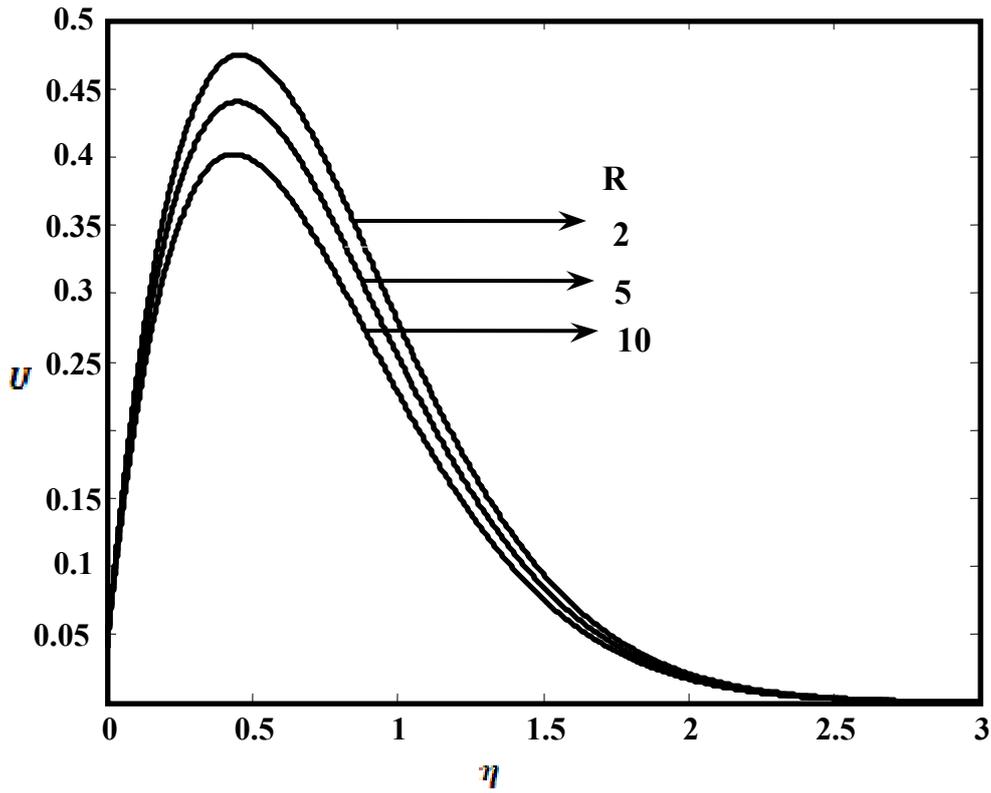


Fig.3. Velocity profiles for different values of R .

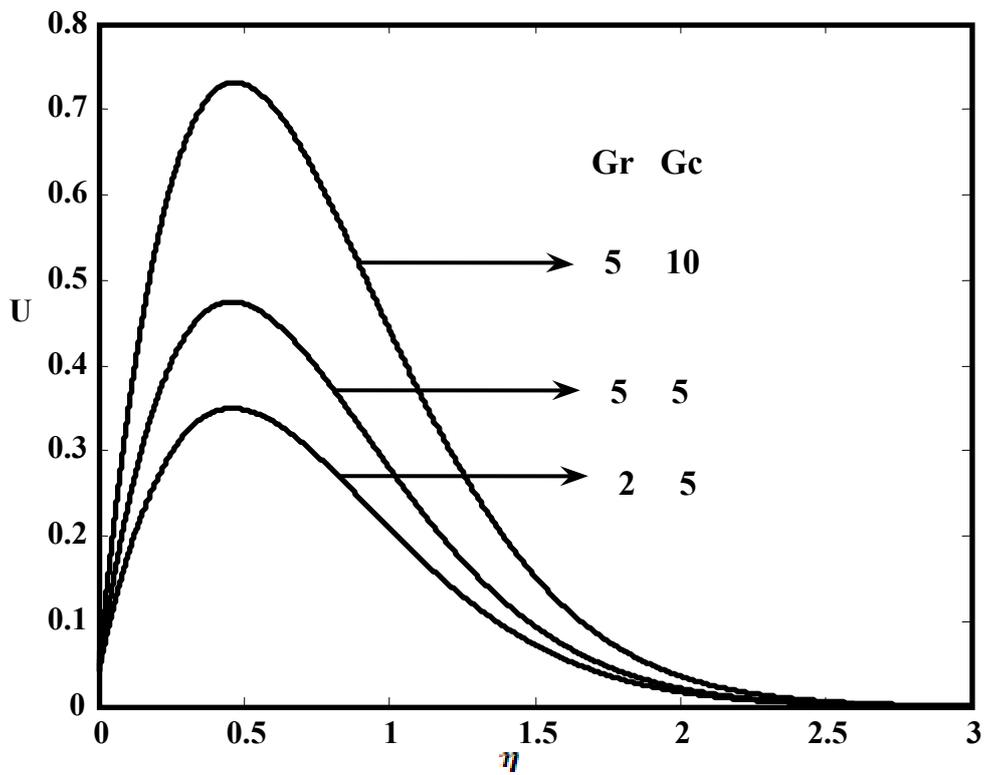


Fig.4. Velocity profiles for different values of Gr and Gc .

4. Conclusion

The theoretical solution of flow past a parabolic starting motion of an infinite vertical plate in the presence of variable temperature and uniform mass diffusion has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effects of different physical parameters such as the thermal radiation parameter, thermal Grashof number and mass Grashof number are studied graphically. The conclusions of the study are as follows:

- (i) The velocity increases with increasing the thermal Grashof number or mass Grashof number, but the trend is just reversed with respect to the thermal radiation parameter.
- (ii) The temperature of the plate increases with decreasing values of the thermal radiation parameter.
- (iii) The plate concentration increases with decreasing values of the Schmidt number.

Nomenclature

- A – constants
 C – dimensionless concentration
 C' – species concentration in the fluid $kg\ m^3$
 C_p – specific heat at constant pressure $J \cdot kg^{-1} \cdot K$
 D – mass diffusion coefficient $m^2 \cdot s^{-1}$
erfc – complementary error function
 Gc – mass Grashof number
 Gr – thermal Grashof number
 g – acceleration due to gravity $m \cdot s^{-2}$
 k – thermal conductivity $W \cdot m^{-1} \cdot K^{-1}$
 Pr – Prandtl number
 Sc – Schmidt number
 T – temperature of the fluid near the plate K
 t' – time s
 u – velocity of the fluid in the x' -direction $m \cdot s^{-1}$
 u – dimensionless velocity
 u_0 – velocity of the plate $m \cdot s^{-1}$
 Y – dimensionless coordinate axis normal to the plate
 y – coordinate axis normal to the plate m
 β – volumetric coefficient of thermal expansion K^{-1}
 β^* – volumetric coefficient of expansion with concentration K^{-1}
 η – similarity parameter
 θ – dimensionless temperature
 μ – coefficient of viscosity $Ra \cdot s$
 ν – kinematic viscosity $m^2 \cdot s^{-1}$
 ρ – density of the fluid $kg \cdot m^{-3}$
 τ – dimensionless skin-friction $kg \cdot m^{-1} \cdot s^2$

Subscripts

- w – conditions at the wall
 ∞ – free stream conditions

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