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UNSTEADY THREE-DIMENSIONAL MHD FLOW DUE TO IMPULSIVE MOTION WITH HEAT AND MASS TRANSFER PAST A STRETCHING SHEET IN A SATURATED POROUS MEDIUM

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The development of velocity, temperature and concentration fields of an incompressible viscous electrically conducting fluid, caused by impulsive stretching of the surface in two lateral directions and by suddenly increasing the surface temperature from that of the surrounding fluid in a saturated porous medium is studied. The partial differential equations governing the unsteady laminar boundary layer flow are solved analytically. For some particular cases, closed form solutions are obtained, and for large values of the independent variable asymptotic solutions are found. The surface shear stress in x and y directions and the surface heat transfer and surface mass transfer increase with the magnetic parameter and with permeability parameter and the stretching ratio, and there is a smooth transition from the short-time solution to the long-time solution.

Key words: unsteadiness, laminar flow, three dimensional, shear stress, heat and mass transfer, short-time and long-time solution.

1. Introduction

The flow with heat and mass transfer problem in the boundary layer induced by a continuously moving or stretching surface is important in many manufacturing processes. In industry, polymer sheets and filaments are manufactured by continuous extrusion of the polymer from a die to a wind up roller which is located at a finite distance away. The thin polymer sheet constitutes a continuously moving surface with a non-uniform velocity through an ambient fluid. Crane (1970) investigated the flow due to a stretching surface in an otherwise ambient fluid. Since then several authors (Gupta and Gupta, 1977; Chakrabarti and Gupta, 1979; Carraghar and Crane, 1982; Dutta *et al*., 1985; Jeng *et al*., 1986; Dutta, 1989; Andersson, 1995; Chaim, 1996; Vajravelu and Hadjinicolaou, 1997) have studied various aspects of this problem such as the effects of surface mass transfer, magnetic field, arbitrary stretching velocity, variable thermal conductivity or wall temperature (heat flux).

Veena *et al*. analysed heat transfer in a fluid past a linearly stretching sheet with varying thermal conductivity and studied internal heat generation. Pravin *et al*. considered a non-Newtonian Magnetohydrodynamic flow over a stretching sheet with heat and mass transfer.

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Veena *et al* investigated the heat transfer flow in a visco-elastic fluid past a stretching sheet with viscous dissipation and internal heat generation. Subhash and Veena studied the oscillatory motion of a visco-elastic fluid past a stretching sheet. Further Pravin *et al*. studied the unsteady motion of an electrically conducting visco-elastic fluid over a stretching sheet in a saturated porous medium with suction/blowing.

But all the above studies dealt with two-dimensional flows which are different from that of the Blasius flow over a semi-infinite plate due to the entrainment of the fluid. Thus Wang (1984) considered the three dimensional flow caused by a stretching flat surface in two lateral directions in an otherwise ambient fluid.

Takhar *et al*. (2001) studied the unsteady three dimensional MHD flow due to the impulsive motion of a strestching surface and obtained several closed form solutions. They showed that the surface shear stresses in x and y directions and the surface heat transfer increase with the magnetic field and the stretching ratio. In some cases, the flow field could be unsteady due to a sudden stretching of the flat sheet when the surface is impulsively stretched with certain velocity, the inviscid flow is developed instantaneously. The flow problem caused by the impulsive motion of the flat surface or the wedge has been investigated by many authors (Hall, 1969; Dennis, 1972; Watkins, 1975; Smith, 1967; Nanbu, 1971; Williams and Rhyne, 1980; Eringen and Maugin, 1990; Abromowitz and Stegun, 1972).

In this present paper, we considered an unsteady laminar viscous boundary layer flow of an electrically conducting fluid induced by the impulsive stretching of a flat surface in two lateral directions in an otherwise quiescent fluid embedded in a porous medium with heat and mass transfer.

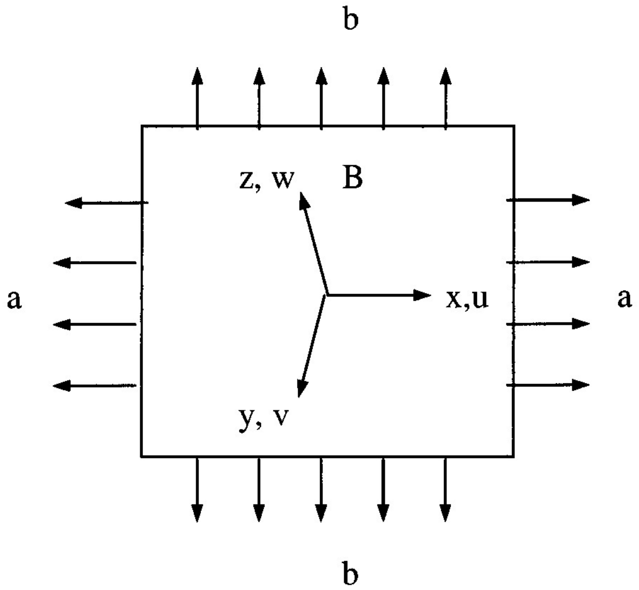

Fig.1. Physical model and co-ordinate system.

The problem is formulated in such a manner that for a small time it reduces to that of the Rayleigh type of flow and for large time it reduces to that of the Wang type flow. The steady state results without the magnetic field and porous medium are compared with those of Wang (1984) and are found to be in excellent agreement.

2. Mathematical formulation

We consider an unsteady laminar incompressible flow of an electrically conducting fluid over a flat surface in two lateral directions in an otherwise quiescent fluid in a porous medium with heat and mass transfer. At the same time, the wall temperature is raised from T_∞ to $T_w\left(T_w > T_\infty\right)$ and wall concentration is raised from C_∞ to $C_w\left(C_w > C_\infty\right)$. The magnetic field is applied in the *z*-direction. It is assumed that the magnetic Reynolds number is small, i.e., $Rm=\mu_0\,\sigma\,V\,L << 1$, where μ_0 is the magnetic permeability, σ is the electrical conductivity and *V*, *L* are the characteristic velocity and length, respectively. Under these conditions we can neglect the effect of the induced magnetic field in comparison to the applied magnetic field. The electrical current flowing in the fluid gives rise to an induced magnetic field if the fluid were an electrical insulator, but here we have taken the fluid to be electrically conducting and embedded in a porous medium. Hence, the applied magnetic field B_0 gives rise to magnetic forces - $F_x = \sigma B_0^2 u/\rho$ and $F_y = \sigma B_0^2 v/\rho$ and applied porous media gives rise to $G_x = \frac{\nu}{k'}u$ and $G_y = \frac{\nu}{k'}v$ in *x* and *y* directions, respectively. The effects of viscous dissipation, Ohmic heating and Hall current are neglected. The wall and ambient temperatures and concentrations are considered to be constant. Under the above assumptions, the boundary layer equations governing the unsteady laminar flow due to an impulsive motion are given by Wang (1984), Takhar *et al*. (2001) as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0\,, \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \nu\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B_0^2 u}{\rho} - \frac{\nu}{k'}u\,, \tag{2.2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial y^2} - \sigma\frac{B_0^2 v}{\rho} - \frac{\nu}{k'}v\,, \tag{2.3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial z^2}\,, \tag{2.4}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D\frac{\partial^2 C}{\partial z^2}\,. \tag{2.5}$$

The initial conditions are given by

$$\begin{gathered} u(x,y,z,t) = v(x,y,z,t) = \mathrm{w}(x,y,z,t) = 0, \\ T(x,y,z,t) = T_\infty \quad \text{for} \quad t < 0, \\ C(x,y,z,t) = C_\infty \quad \text{for} \quad C < 0. \end{gathered} \tag{2.6}$$

The boundary conditions for $t \geq 0$ are given by

$$\begin{gathered} u(x,y,0,t) = u_w = a_0 x; \qquad v(x,y,0,t) = v_w = b_0 y, \\ w(x,y,0,t) = 0; \qquad T(x,y,0,t) = T_w, \qquad C(x,y,0,t) = C_w, \\ u(x,y,\infty,t) = v(x,y,\infty,t) = 0, \\ T(x,y,\infty,t) = T_\infty, \qquad C(x,y,\infty,t) = C_\infty. \end{gathered} \tag{2.7}$$

Here x, y and z are the longitudinal, transverse and normal directions, respectively: t is the time, T is the temperature, C is the concentration, B is the magnetic field applied, k' is the porous parameter in the x-direction, u_w and v_w are the surface velocities in x and y directions, respectively, the subscripts w and ∞; denote conditions at the wall and in the ambient fluid, respectively.

We make use of the scales $R=\dfrac{z}{\sqrt{\nu t}}$, $t^*=\dfrac{u_w t}{x}$ or $\eta=\sqrt{\dfrac{u_w}{\nu x}}\,z$, $t^*=\dfrac{u_w t}{x}$ which are valid for small and large times, respectively. Hence, one has to find such scales where both small and large time solutions fit in properly. Defining

$$\begin{gathered}
\eta=\sqrt{\frac{a_0}{\nu\xi}}\,z\,, \qquad \xi=1-\exp(-t^*), \qquad t^*=a_0 t, \qquad a>0\,, \\
u(x,y,z,t)=a_0 x f'(\xi,\eta), \qquad \nu(x,y,z,t)=a_0 y S'(\xi,\eta), \\
w(x,y,z,t)=-\sqrt{a_0\nu}\,\sqrt{\xi}\,(f+s), \qquad u_w=a_0 x, \\
v_w=b_0 y, \qquad b_0\ge 0, \qquad T(x,y,z,t)=T_\infty+(T_w-T_\infty)\,g(\xi,\eta), \\
C(x,y,z,t)=C_\infty+(C_w-C_\infty)\,h(\xi,\eta), \qquad Mn=\frac{\sigma B_0^2}{\rho a_0}, \\
k_2=\frac{\nu}{k' a_0}, \qquad C_0=\frac{b_0}{a_0}\ge 0, \qquad \mathrm{Pr}=\frac{\nu}{\alpha}, \qquad \mathrm{Sc}=\frac{\nu}{D},
\end{gathered} \tag{2.8}$$

and substituting relations (2.8) in Eqs (2.1) to (2.5) we find that Eq.(2.1) is identically satisfied and Eqs (2.2)-(2.5) reduce to

$$\begin{gathered}
f_{\eta\eta\eta}(\eta)+\frac{1}{2}\eta(1-\xi)f_{\eta\eta}(\eta)+\xi(f+S)f_{\eta\eta}(\eta)-\xi f_\eta^2+ \\
-(Mn+k_2)\xi f_\eta(\eta)=\xi(1-\xi)\frac{\partial f_\eta}{\partial\xi},
\end{gathered} \tag{2.9}$$

$$\begin{gathered}
S_{\eta\eta\eta}(\eta)+\frac{1}{2}\eta(1-\xi)S_{\eta\eta}(\eta)+\xi(f+S)S_{\eta\eta}(\eta)-\xi S_\eta^2+ \\
-(Mn+k_2)\xi S_\eta(\eta)=\xi(1-\xi)\frac{\partial S_\eta}{\partial\xi},
\end{gathered} \tag{2.10}$$

$$\mathrm{Pr}^{-1}g_{\eta\eta}(\eta)+\frac{1}{2}\eta(1-\xi)g_\eta(\eta)+\xi(f+S)g_\eta(\eta)=\xi(1-\xi)\frac{\partial g}{\partial\xi}, \tag{2.11}$$

$$\mathrm{Sc}^{-1}h_{\eta\eta}(\eta)+\frac{1}{2}\eta(1-\xi)h_\eta(\eta)+\xi(f+S)h_\eta(\eta)=\xi(1-\xi)\frac{\partial h}{\partial\xi}. \tag{2.12}$$

The corresponding boundary conditions are given by

$$\begin{gathered}
f(\xi,0)=0\,, \qquad f_\eta(\xi,0)=1, \qquad S(\xi,0)=0, \qquad S_\eta(\xi,0)=C_0, \\
g(\xi,0)=1, \qquad h_\eta(\xi,0)=1,
\end{gathered}$$

$$f_\eta(\xi,\infty)=S_\eta(\xi,\infty)=g(\xi,\infty)=h(\xi,\infty)=0. \tag{2.13}$$

Here η is the dimensionless transformed similarity variable, t^* and ξ are dimensionless times, f_η and S_η are the dimensionless velocity components along the x and y directions, respectively: g is the dimensionless temperature and h is the dimensionless concentration, C_0 is the ratio of the surface velocity gradients along the y and x directions: Pr is the Prandtl number, Sc is the Schmidt number and the suffix denotes a partial derivative with respect to η.

It is justified that Eqs (2.9)-(2.12) are parabolic partial differential equations, but for $\xi=0$ ($t^*=0$) and $\xi=1$ ($t^*\to\infty$) they reduce to ordinary differential equations. For $\xi=0$, Eqs (2.9)-(2.12) reduce to

$$f_{\eta\eta\eta}(\eta)+\frac{1}{2}\eta\, f_{\eta\eta}(\eta)=0, \tag{2.14}$$

$$S_{\eta\eta\eta}(\eta)+\frac{1}{2}\eta\, S_{\eta\eta}(\eta)=0, \tag{2.15}$$

$$\frac{1}{\Pr}g_{\eta\eta}(\eta)+\frac{1}{2}\eta g_\eta(\eta)=0, \tag{2.16}$$

$$\frac{1}{\mathrm{Sc}}h_{\eta\eta}(\eta)+\frac{1}{2}\eta h_\eta(\eta)=0. \tag{2.17}$$

For $\xi=1$, Eqs (2.9)-(2.12) are reduced to

$$f_{\eta\eta\eta}(\eta)+(f+S)f_{\eta\eta}(\eta)-f_\eta^2-(Mn+k_2)f_\eta(\eta)=0, \tag{2.18}$$

$$S_{\eta\eta\eta}(\eta)+(f+S)S_{\eta\eta}(\eta)-S_\eta^2-(Mn+k_2)S_\eta(\eta)=0, \tag{2.19}$$

$$\frac{1}{\Pr}g_{\eta\eta}(\eta)+(f+S)g_\eta(\eta)=0, \tag{2.20}$$

$$\frac{1}{\mathrm{Sc}}h_{\eta\eta}(\eta)+(f+S)h_\eta(\eta)=0. \tag{2.21}$$

The boundary conditions for Eqs (2.14)-(2.17) and (2.18)-(2.21) become

$$\begin{gathered} f(0)=1,\quad f_\eta(0)=1,\quad S(0)=0,\quad S_\eta(0)=C_0,\quad g(0)=1,\quad h(0)=1, \\ f_\eta(\infty)=S_\eta(\infty)=g(\infty)=h(\infty)=0. \end{gathered} \tag{2.22}$$

3. Analytical solution

Equations (2.14)-(2.17) are linear equations and the solutions to those equations satisfying the boundary conditions (2.22) are obtained as

$$f=\frac{S}{C_0}=\eta\ \mathrm{erfc}\frac{\eta}{2}+\frac{1}{\sqrt{\pi}}\left[1-\exp\left(-\frac{\eta^2}{4}\right)\right],$$

$$h=\mathrm{erfc}\frac{\sqrt{\mathrm{Sc}}\eta}{2},\qquad g=\mathrm{erfc}\frac{\sqrt{\mathrm{Pr}}\eta}{2},\qquad f_\eta=\frac{S_\eta}{C_0}=\mathrm{erfc}\frac{\eta}{2},$$

$$f_{\eta\eta}=\frac{S_{\eta\eta}}{C_0}=-\frac{1}{\sqrt{\pi}}\exp\left(-\frac{\eta^2}{4}\right);\qquad g_\eta=-\frac{\sqrt{\mathrm{Pr}}}{\sqrt{\pi}}\exp\left(-\frac{\mathrm{Pr}\,\eta^2}{4}\right),\tag{3.1}$$

$$h_\eta=-\frac{\sqrt{\mathrm{Sc}}}{\sqrt{\pi}}\exp\left(-\frac{\mathrm{Sc}\eta^2}{4}\right)$$

where $\mathrm{erfc}(\eta)=1-\mathrm{erf}(\eta)$,

$$\mathrm{erf}(\eta)=\frac{2}{\sqrt{\pi}}\int_0^\eta \exp\left(-x^2\right)dx\ .$$

Equations (2.18)-(2.21) for $C_0=0$ (two-dimensional case) admit closed form solutions.
For $C_0=0$ and $S=0$ Eq.(2.19) is not required.
For $C_0=0$, the solution of Eq.(2.18) under the boundary conditions (2.22) is given by

$$f(\eta)=\frac{\left[1-\exp(-\gamma\eta)\right]}{\gamma},\qquad \gamma=\sqrt{1+Mn+k_2}\ .\tag{3.2}$$

The solutions of Eqs (2.20) and (2.21) using the solution (3.2) in terms of Kummer's function F (Abromowitz and Stegun, 1972) are given by
where

$$g(\eta)=e^{-\frac{\mathrm{Pr}}{\gamma^2}\gamma\eta}\frac{\mathrm{F}\left[\frac{\mathrm{Pr}}{\gamma^2},1+\frac{\mathrm{Pr}}{\gamma^2},-\frac{\mathrm{Pr}}{\gamma^2}e^{-\gamma\eta}\right]}{\mathrm{F}\left[\frac{\mathrm{Pr}}{\gamma^2},1+\frac{\mathrm{Pr}}{\gamma^2},-\frac{\mathrm{Pr}}{\gamma^2}\right]},\tag{3.3}$$

$$h(\eta)=e^{-\frac{\mathrm{Sc}}{\gamma^2}\eta}\frac{\mathrm{F}\left[\frac{\mathrm{Sc}}{\gamma^2},1+\frac{\mathrm{Sc}}{\gamma^2},-\frac{\mathrm{Sc}}{\gamma^2}e^{-\gamma\eta}\right]}{\mathrm{F}\left[\frac{\mathrm{Sc}}{\gamma^2},1+\frac{\mathrm{Sc}}{\gamma^2},-\frac{\mathrm{Sc}}{\gamma^2}\right]}\tag{3.4}$$

where $F(a,b,z)=1+\sum_{i=1}^{\infty}\frac{(a)_i z^i}{(b)_i L^i}$,

$$(a)_i=a(a+1)(a+2)\ \ \ldots\ldots\ldots\ \ (a+i-1),$$

$$(b)_i=b(b+1)(b+2)\ \ \ldots\ldots\ldots\ \ (b+i-1).$$

The dimensionless temperature gradient and concentration gradient at the wall are expressed as

$$g_\eta(0)=\frac{\Pr}{\gamma}\left[1-\frac{\Pr}{\gamma^2+\Pr}\frac{F\left[\frac{\Pr}{\gamma^2}+1,2+\frac{\Pr}{\gamma^2},-\frac{\Pr}{\gamma^2}\right]}{F\left[\frac{\Pr}{\gamma^2},1+\frac{\Pr}{\gamma^2},-\frac{\Pr}{\gamma^2}\right]}\right], \tag{3.5}$$

$$h_\eta(0)=\frac{Sc}{\gamma}\left[1-\frac{Sc}{\gamma^2+Sc}\frac{F\left[\frac{Sc}{\gamma^2}+1,2+\frac{Sc}{\gamma^2},-\frac{Sc}{\gamma^2}\right]}{F\left[\frac{Sc}{\gamma^2},1+\frac{Sc}{\gamma^2},-\frac{Sc}{\gamma^2}\right]}\right] \tag{3.6}$$

where F is Kummer's function. Also, for $C_0=0$, $\gamma=1$, ($Mn=0$, $k_2=0$) and Pr=Sc=1, the solutions g and h can be expressed in a simple form, and given by

$$g=\frac{e}{e-1}\left[1-\exp\left(-e^{-\eta}\right)\right],$$

$$h=\frac{e}{e-1}\left[1-\exp\left(-e^{-\eta}\right)\right]. \tag{3.7}$$

4. Asymptotic solution

For large η when $\xi=1$, $f_\eta\to 0$, $S_\eta\to 0$, $g\to 0$, $h\to 0$. Hence, the asymptotic behaviour of f_η and S_η, g and h is verified as

$$\lim_{\eta\to\infty}\left(\delta_1-f\right)\to 0; \qquad \lim_{\eta\to\infty}\left(\delta_2-S\right)\to 0. \tag{4.1}$$

Hence, for large $\eta(\eta\to\infty)$, we get

$$f=\delta_1+F, \qquad S=\delta_2+S, \qquad g=G, \qquad h=H, \qquad \delta_3=\delta_1+\delta_2$$

where F, S, G and H are small, linearizing Eqs (2.18)-(2.21) we get

$$F_{\eta\eta\eta}(\eta)+\delta_3 F_{\eta\eta}(\eta)-(Mn+k_2)F_\eta(\eta)=0, \tag{4.2}$$

$$S_{\eta\eta}(\eta)+\delta_3 S_{\eta\eta}(\eta)-(Mn+k_2)S_\eta(\eta)=0, \tag{4.3}$$

$$G_{\eta\eta}(\eta)+\Pr\,\delta_3 G_\eta(\eta)=0, \tag{4.4}$$

$$H_{\eta\eta}(\eta)+Sc\,\delta_3 H_\eta(\eta)=0. \tag{4.5}$$

The boundary conditions as $\eta\to\infty$ are given by

$$F_\eta=S_\eta=G=H=0. \tag{4.6}$$

Equations (4.3)-(4.5) are linear differential equations with constant co-efficients. Thus their solutions satisfying the boundary conditions (4.6) are obtained as

$$F_\eta = S_\eta = A_1 \exp(-\delta_4 \eta), \tag{4.7}$$

$$G = A_2 \exp\left[-\left(\sqrt{\Pr}\,\delta_3\right)\eta\right], \tag{4.8}$$

$$H = A_3 \exp\left[-\left(\sqrt{\mathrm{Sc}}\,\delta_3\right)\eta\right] \tag{4.9}$$

where $\delta_4 = \left[\delta_3 + \left(\delta_3^2 + 4(Mn + k_2)\right)^{1/2}\right]\Big/2$. Here δ_1, δ_2, δ_3, δ_4 are constants and A_1, A_2 and A_3 are some arbitrary constants. It follows from Eqs (4.7)-(4.9) that F_η, S_η, G and H (hence, f_η, S_η, g, h) tend to zero in an exponential manner as $\eta \to \infty$, if $\delta_3 > 0$.

5. Results and discussion

Figures 1 and 2 show the variation of the surface shear stresses in x and y directions $\left[-f_{\eta\eta}(\xi,0)\,;\,-S_{\eta\eta}(\xi,0)\right]$ for different values of permeability parameter k_2 and $Mn = 1.0$ when $C_1=0.5$ and Pr=0.7. From the figures we observe that at the start of the motion ($\xi=0$), the surface shear stresses are independent of k_2.The surface shear stresses increase with the permeability k_2 due to the enhanced Lorentz force which imparts additional momentum into the boundary layer. This reduces the boundary layer thickness which, inturn, increases the surface shear stress.

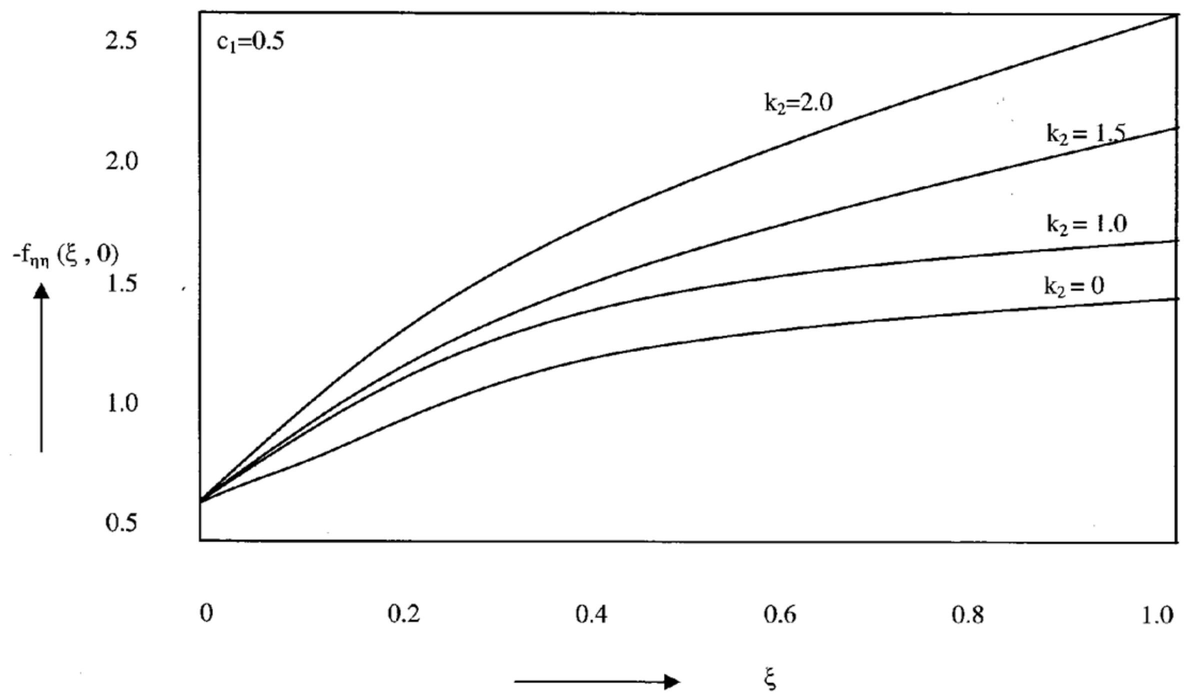

Fig.1. Effect of the permeability parameter k_2 on the surface shear stress in the x-direction for $0 \le \xi \le 1$ and for $Mn=1.0$.

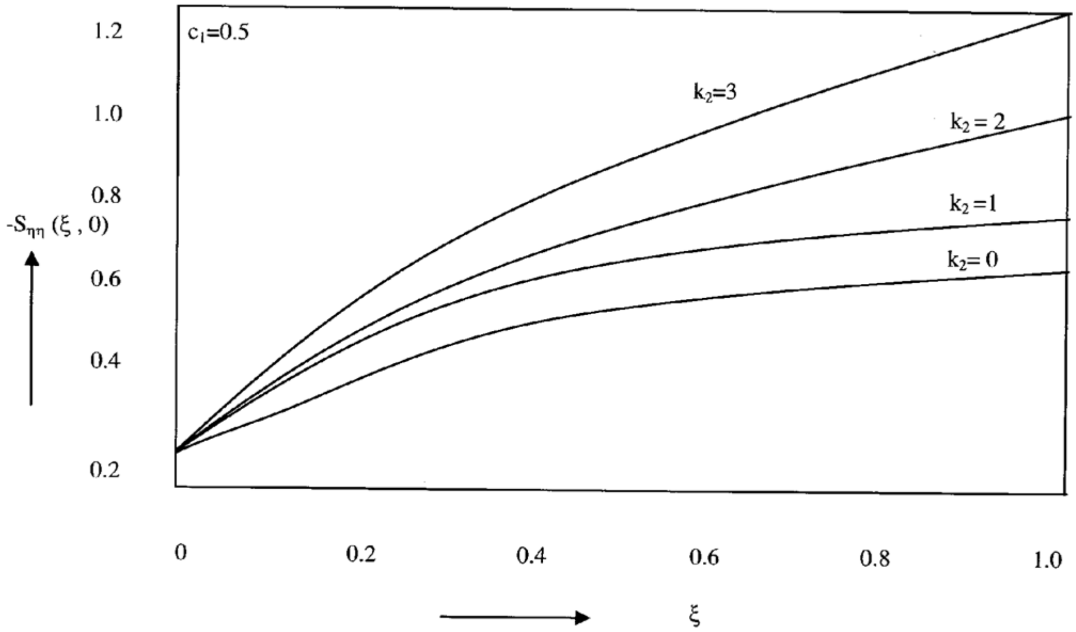

Fig.2. Effect of the permeability parameter k_2 on the surface shear stress in the *Y*-direction, i.e., $S_{\eta\eta}$ (ξ, *0*) for $0 \le \xi \le 1$ and for $Mn=1.0$.

The variations of surface shear stresses in *x* and *y* directions with dimensionless time ξ, for several values of the stretching parameter $C_1(0 \le C_1 \le 1)$ when $k_2=1.0$ and $Mn=1.0$ and Pr=*0.7* are presented in Figs 3 and 4 when $C_1=0$, $S_{\eta\eta}(\xi, 0)=0$, as the problem reduces to the two dimensional case. The surface shear stresses in the *x* and *y* directions ($-f_{\eta\eta}(\xi, 0)$, $-S_{\eta\eta}(\xi, 0)$ for $0 \le C_1 \le 1$ increase with ξ almost linearly. The effect is significantly pronounced on the surface shear stress in the *y* direction ($-S_{\eta\eta}(\xi, 0)$). For $Mn=k_2=1.0$, Pr=*0.7* the surface shear stresses in the *y* and *x* directions increase as C_1 increases from *0.0* to *0.75*.

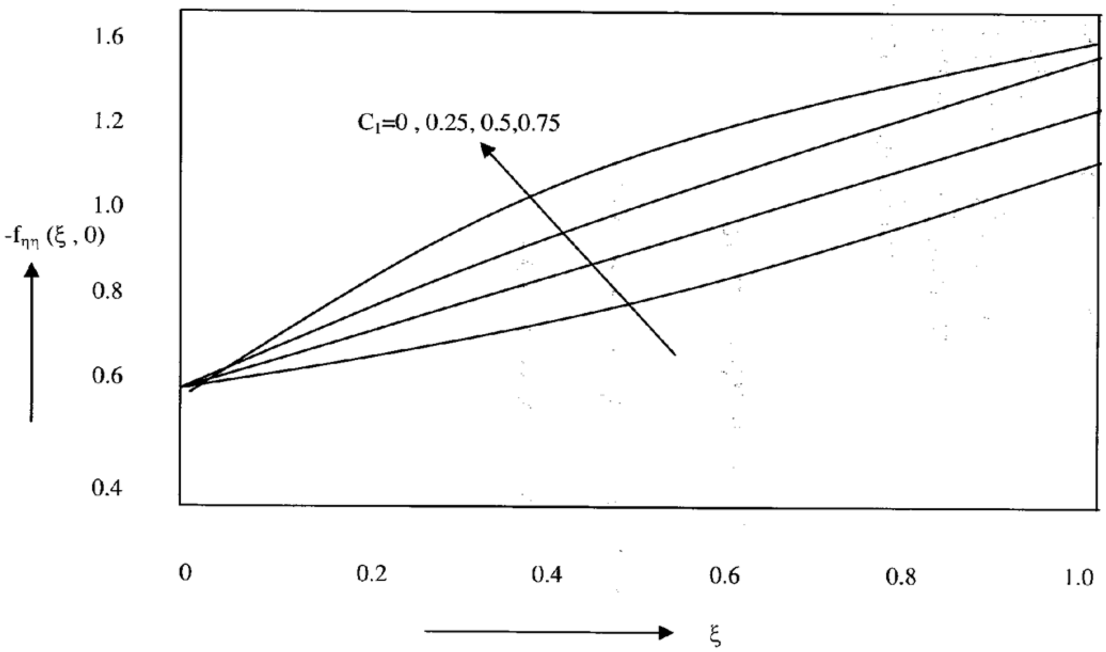

Fig.3. Effect of the stretching parameter c_1 on the surface shear stress in the *x*-direction, $-f_{\eta\eta}$ (ξ, *0*) for $0 \le \xi \le 1$ for $k_2 = 1.0$ and $Mn = 1.0$.

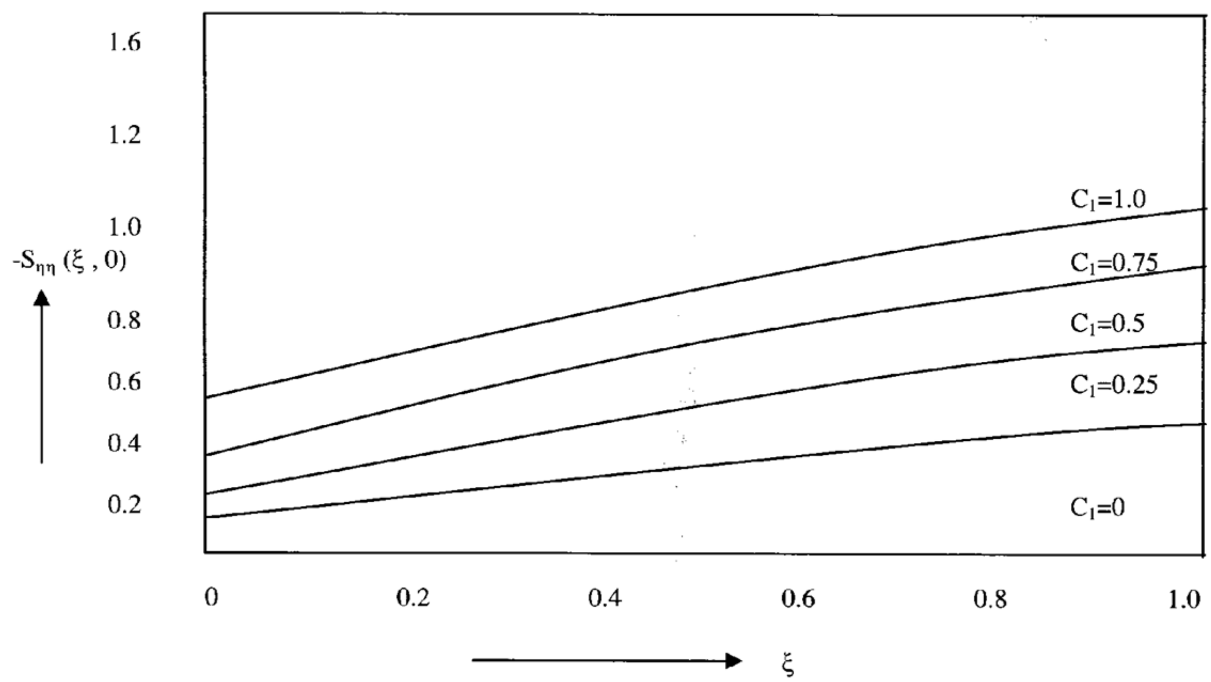

Fig.4. Effect of the stretching parameter C_1 on the surface shear stress in the *Y*-direction $-S_{\eta\eta}$ (ξ, *0*) for $0 \le \xi \le 1$ and $k_2=1.0$ and $Mn=1.0$.

In Figs 5a and 5b we have plotted graphs for the surface heat transfer $-g_\eta(\xi, 0)$ for $0\le\xi\le1$ with $C_1=0.5$, Pr=*0.7* for various values of the permeability parameter k_2 and magnetic parameter *Mn* respectively. It is noticed from the figures that the surface heat transfer decreases with increasing values of k_2 and *Mn*. This is due to the reduction in the functions *f* and *S* with increasing k_2 and *Mn* which increases the thermal boundary layer thickness. This results in the reduction of the surface heat transfer $\left(-g_\eta\left(\xi,0\right)\right)$ as k_2 and *Mn* increase in the respective figures. As k_2 increases from *0* to *3*, $-g_\eta(\xi, 0)$ decreases by about *33%*. Also as *Mn* increases from zero to *5*, $-g_\eta(\xi, 0)$ decreases by about *36%*. Also, for all k_2 and *Mn* there is a smooth transition from the short-time solution to the long-time solution. A similar trend was observed by Williams and Rhyne (1980) for the impulsive flow over a wedge in the absence of a porous medium and magnetic field.

The variation of the surface mass transfer $-h_\eta(\xi, 0)$ for various values of the permeability parameter k_2, magnetic parameter *Mn* with the Schmidt number Sc=*3.0*, $C_1=0.5$ is shown in Figs 6a and 6b, respectively. It is observed from the figures that the surface mass transfer decreases with increasing values of k_2 and *Mn*. It can be explained that the surface mass transfer $-h_\eta(\xi, 0)$ increases with ξ for all k_2 (Fig.6a) and for all *Mn* (Fig.6b) except $-h_\eta(\xi, 0)$ for $\xi>\xi_0$ which depends on k_2 and *Mn* ($\xi_0=0.42$ when $k_2=0$; $Mn=0$; $\xi_0=0.8$ for $k_2=Mn=1.0$).

The effect of the stretching parameter C_1 on the surface mass transfer is plotted in Fig.7a for $k_2=Mn=1.0$ and Sc=*3.0*. It is found from the figure that the surface mass transfer $-h_\eta(\xi, 0)$ increases with ξ for $\xi\le\xi_0$ ($\xi_0=0.8$ when $k_2=Mn=1$), $C_1=0.25$. Beyond this value, it decreases. The surface mass transfer also increases with increasing values of the stretching parameter C_1, because increasing values of C_1 implies a higher surface velocity which accelerates the fluid in the boundary layer.

In Fig. 7b a graph of the surface mass transfer $-h_\eta(\xi, 0)$ for various values of the Schmidt number is plotted and it is noticed from the figure that the surface mass transfer increases with increasing values of the Schmidt number Sc.

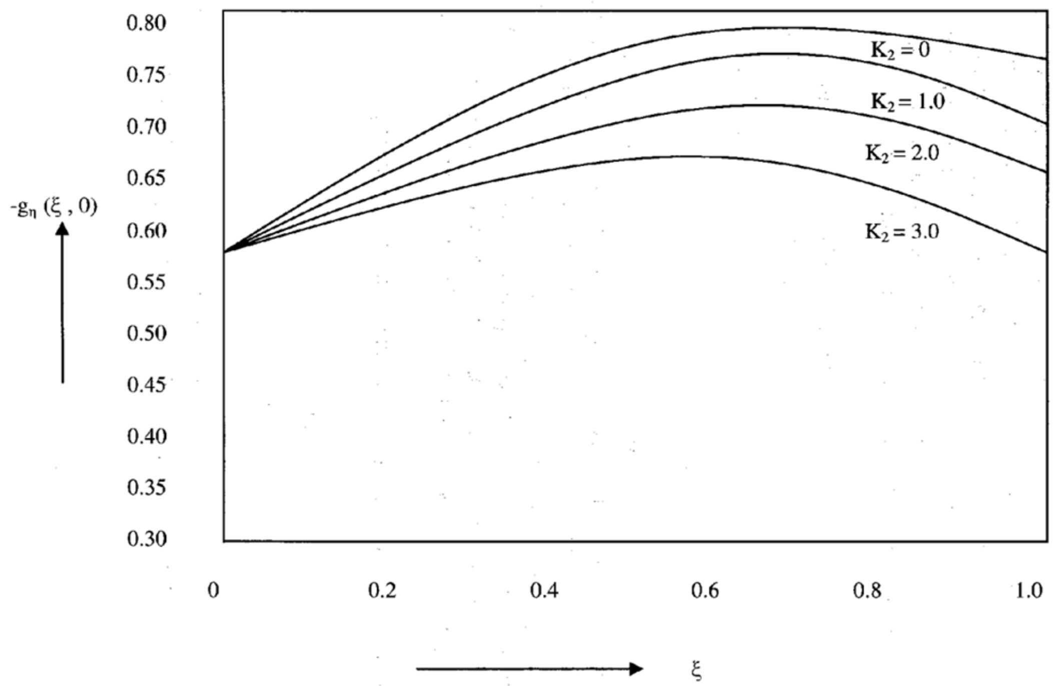

Fig.5a. Effect of the permeability parameter k_2 on the surface heat transfer - h_η (ξ, *0*) for $0 \le \xi \le 1$ with $C_1=0.5$, Pr=*0.7* and $Mn=1.0$.

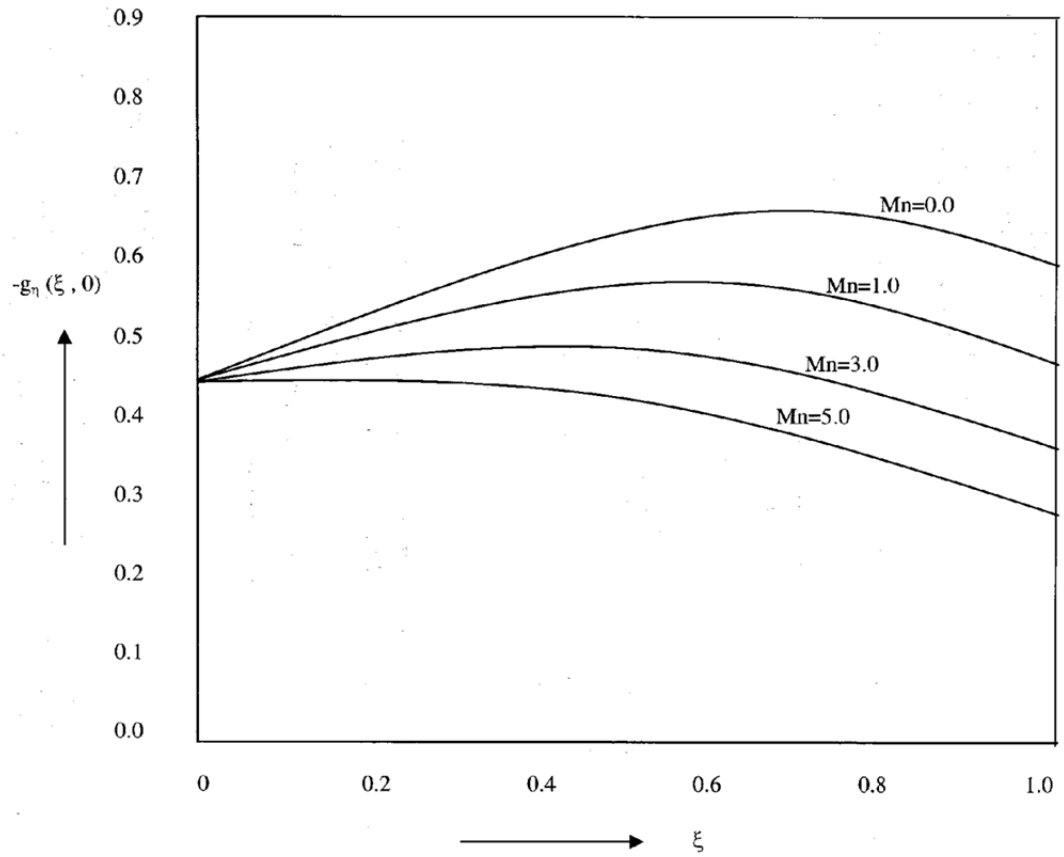

Fig.5b. Effect of the magnetic parameter *Mn* on the surface heat transfer $-g_\eta$ (ξ, *0*) for $0 \le \xi \le 1$ with $c_1=0.5$ and Pr=*0.7* and $k_2=1$.

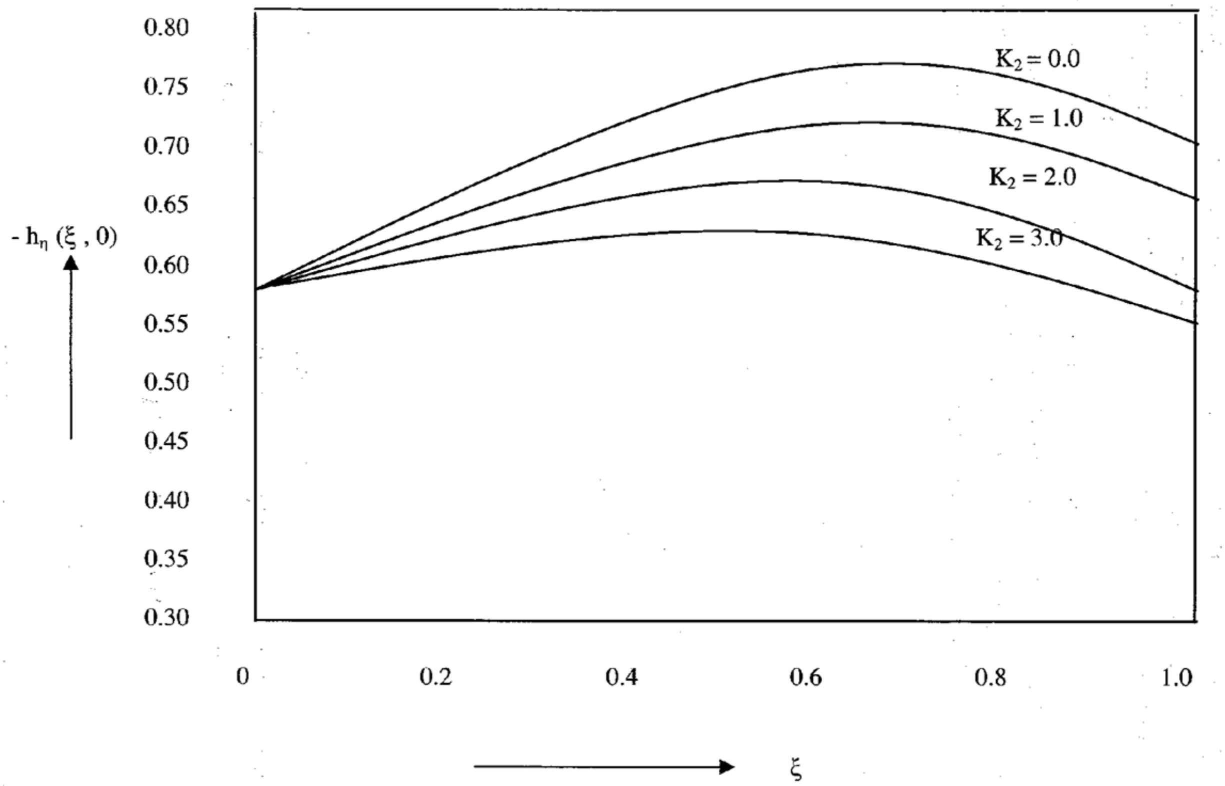

Fig.6a. Effect of the permeability parameter K_2 on the surface heat transfer - h_η (ξ, *0*) for $0 \leq \xi \leq 1$ with $C_1=0.5$, Sc=*3* and $Mn=1.0$.

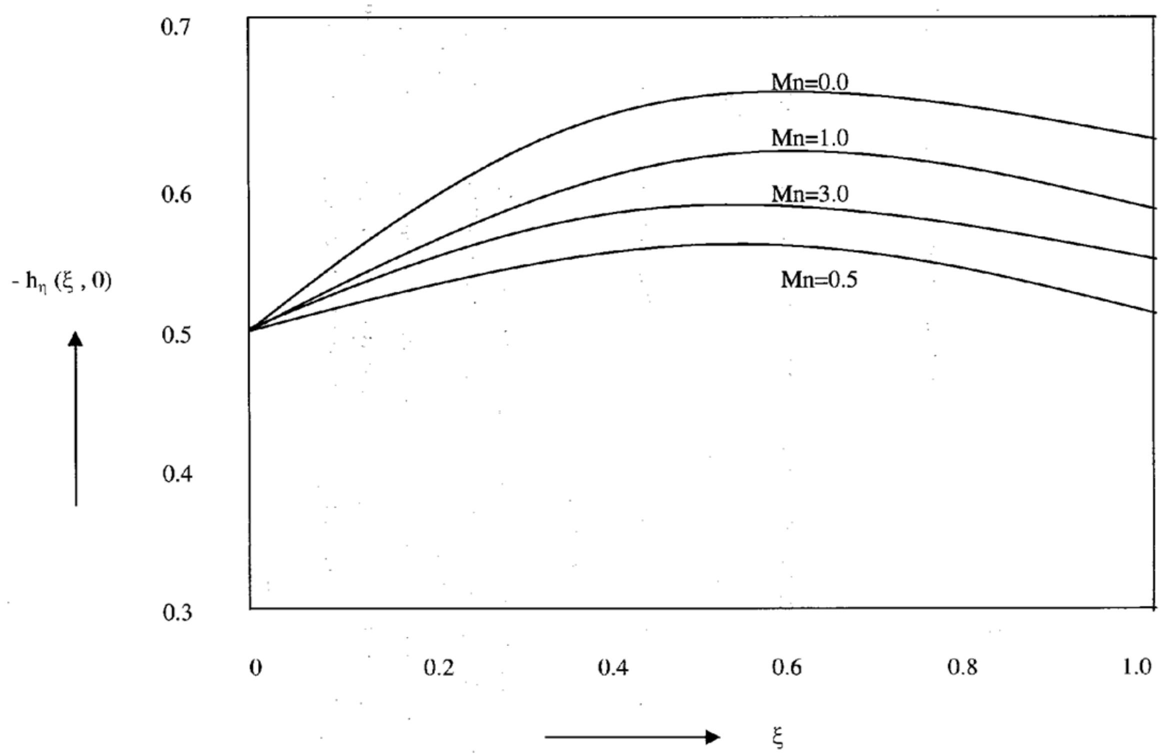

Fig.6b. Effect of the magnetic parameter *Mn* on the surface mass transfer - h_η (ξ, *0*) for $0 \leq \xi \leq 1$ with $C_1=0.5$ Sc=*3* and $K_2=1.0$.

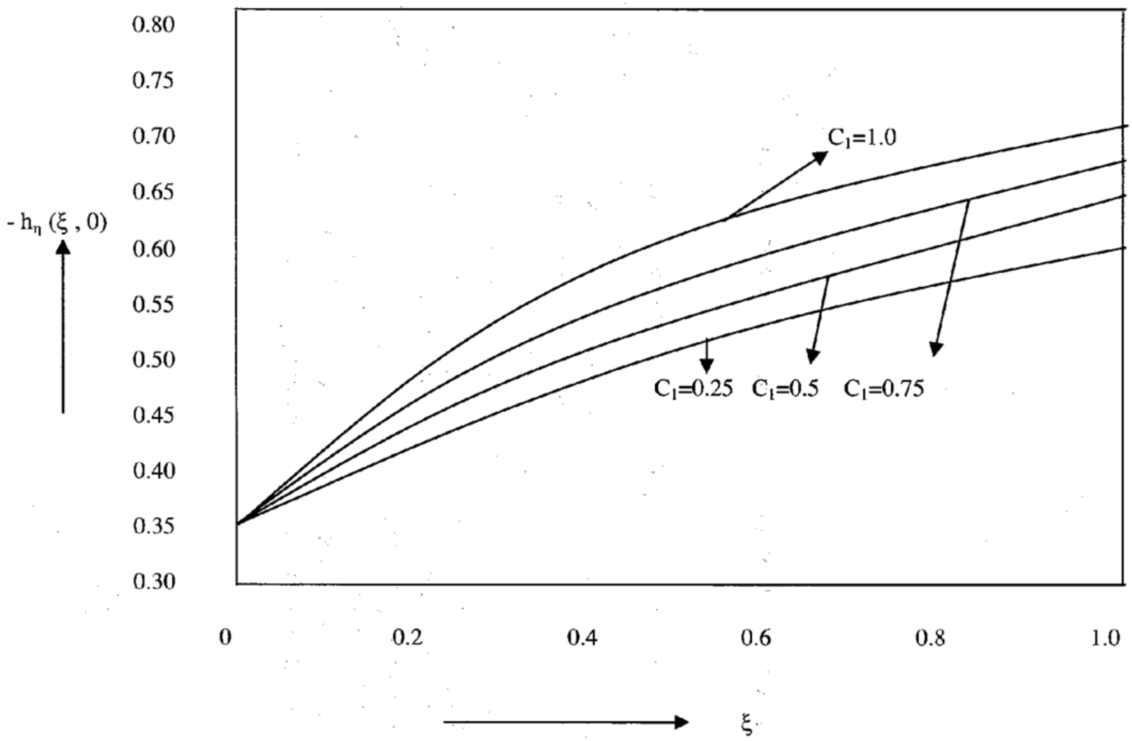

Fig.7a. Effect of the stretching parameter C_1 on the surface mass transfer - h_η (ξ, *0*) for $0 \leq \xi \leq 1$ with $Mn=1.0$, Sc=3.0, $k_2=1.0$.

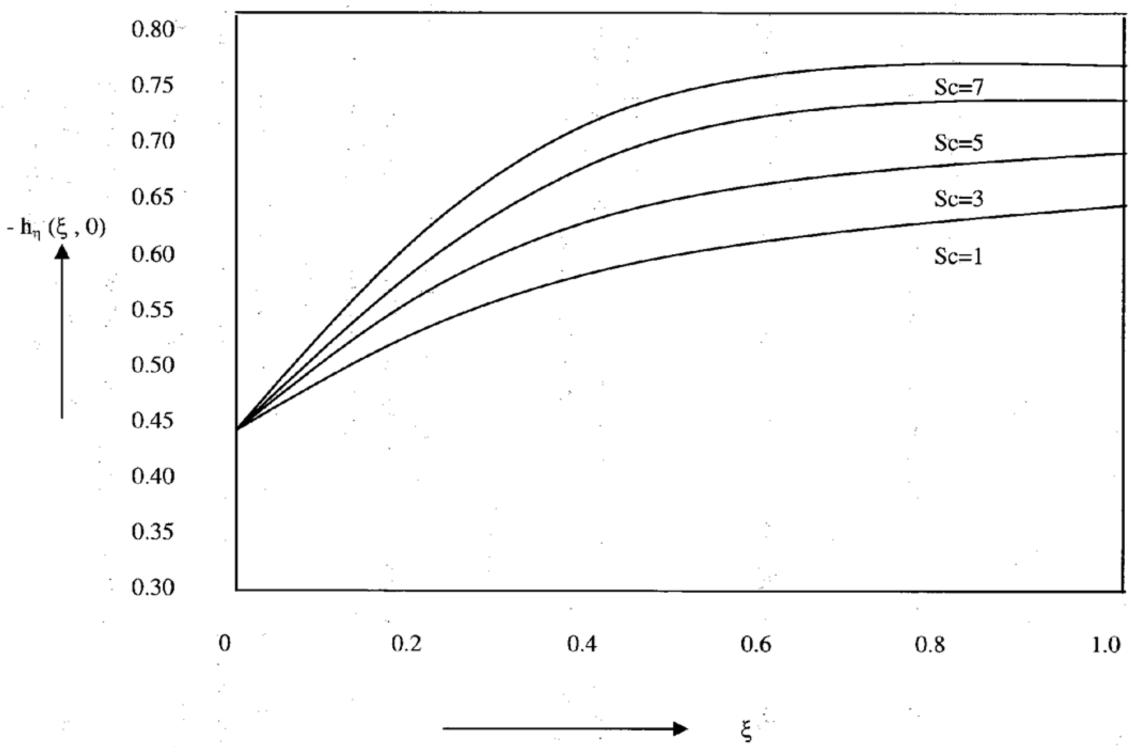

Fig.7b. Effect of the Schmidt number on the surface mass transfer - h_η (ξ, *0*) for $0 \leq \xi \leq 1$ with $Mn=1.0$, $k_2=1.0$, $c1=0.5$.

Conclusions

The surface shear stresses, the surface heat transfer and the surface mass transfer increase with the stretching parameter C_1, permeability parameter k_2 and magnetic parameter *Mn* and there is a smooth transition from the short-time solution to the long-time solution. The surface shear stresses increase with time ξ, but the surface heat transfer and mass transfer increases upto a certain instant of time, but beyond this time they decrease. The stretching parameter C_1, permeability parameter k_2, magnetic parameter *Mn* and Schmidt number Sc affect most significantly the surface shear stress and surface mass transfer.

Nomenclature

a_0&b_0 – velocity gradients in x and y directions
B_0 – magnetic field strength
C – concentration
C_1 – ratio of the surface velocity gradients along y and x directions
C_∞ – species concentration far away from wall
C_w – species concentration at the wall
D – mass diffusivity
F_x, F_y – magnetic forces in x and y directions
f_η&S_η – dimensionless velocity components along x and y directions
g – dimensionless temperature
h – dimensionless concentration
k' – co-efficient of porosity
k_2 – permeability parameter
L – characteristic length
Mn – magnetic parameter
Pr – Prandtl number
Rm – Reynolds number
Sc – Schmidt number
T – temperature
T_w – temperature at the wall
T_∞ – temperature far away from the wall
t – time
t^*, ξ – dimensionless times
u, v, w – velocity components along x, y and z direction
V – characteristic velocity
x, y, z – space variables along longitudinal, transverse and normal directions
α – thermal diffusivity
η – dimensionless transformed similarity variable
μ_0 – magnetic permeability
ν – kinematic viscosity
ρ – density
σ – electrical conductivity

References

Abromowitz M. and Stegun I.A. (1972): *Handbook of mathematical functions.* – 55, Providence National Bureau of Standards.Amer. Math. Soci..

Andersson H.I. (1995): *An exact solution of the Navier-Stokes equations for MHD flow.* – Acta. Mech., vol.113, pp.241-244.

Carraghar P. and Crane L.J. (1982): *Heat transfer on a continuous stretching sheet.* – ZAMM, vol.62, pp.564-565.

Chaim T.C. (1996): *Heat transfer with thermal conductivity in a stagnation point flow towards a stretching sheet*. – Int. Comm. Heat Mass Transfer, vol.23, pp.239-248.

Chakrabarti A. and Gupta A.S. (1979): *Hydromagnetic flow, heat and mass transfer over a stretching sheet*. – Quart. Appl. Maths, vol.33, pp.73-78.

Crane L.J. (1970): *Flow past a stretching plate*. – ZAMP, vol.21, pp.645-647.

Dennis S.C.R. (1972): *The motion of a viscous fluid past an impulsively started semi-infinite flat plate*. – J. Inst. Math. Appl., vol.10, pp.105-117.

Dutta B.K. (1989): *Heat transfer from a stretching sheet with uniform suction and blowing*. – Acta. Mech., vol.78, pp.255-262.

Dutta B.K., Roy P. and Gupta A.S. (1985): *Temperature field in flow over a stretching sheet with uniform heat flux*. – Int. Comm. Heat Mass Transfer, vol.28, pp.1234-1237.

Eringen A.C. and Maugin G.A. (1990): *Electrodynamics of continua*, 2, New York Springer.

Gupta P.S. and Gupta A.S. (1977): *Heat and mass transfer on a stretching sheet with suction and blowing*. – J. Chem. Engng., vol.55, pp.744-746.

Hall M.G. (1969): *The boundary layer over an impulsively started flat plate*. – Proc. Roy. Soc., vol.310A, pp.401-414.

Jeng D.R., Chang T.C.A. and Dewitt K.J. (1986): *Momentum and heat transfer on a continuous moving surface*. – J. Heat Transfer, vol.108, pp.532-539.

Nanbu K. (1971): *Unsteady Falker-Skan flow*. – ZAMP, vol.22, pp.1167-1172.

Rajagopal K., Veena P.H. and Pravin V.K. (): Heat transfer in a fluid over a Linearly a

Siddappa B., Subhas Abel and Veena Hongunti (1995): *Oscillatory motion of a visco-elastic fluid past a stretching sheet*. – IL NUOVO CIMENTO, 17D, N1, pp.53-60.

Smith S.A. (1967): *The impulsive motion of a wedge in a viscous fluid*. – ZAMP, vol.18, pp.508-522.

Takhar H.S., Chamkha A.J. and Nath G. (2001): *Unsteady three-dimensional MHD boundary layer flow due to the impulsive motion of a stretching surface*. – Acta. Mechanica., vol.146, pp.59-71.

Vajravelu K.A. and Hadjinicolaou A. (1997): *Convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream*. – Int. J. Engg. Sci., vol.35, pp.1237-1244.

Wang C.Y. (1984): *The three-dimensional flow due to a stretching flat surface*. – Phys. of Fluids, vol.27, pp.1915-1917.

Watkins C.A. (1975): *Heat transfer in the boundary layer over an impulsively started flat plate*. - J. Heat Transfer, vol.97, pp.482-484.

Williams J.C. and Rhyne T.H. (1980): *Boundary layer development on a wedge impulsively set into motion*. - SIAM. J. Appl. Math., vol.38, pp.215-224.

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