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Int. J. of Applied Mechanics and Engineering, 2013, vol.18, No.2, pp.329-339
DOI: 10.2478/ijame-2013-0019

EFFECTS OF CHEMICAL REACTION ON MHD FLOW PAST AN IMPULSIVELY STARTED INFINITE VERTICAL PLATE WITH UNIFORM HEAT AND MASS FLUX

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Finite difference solutions of the unsteady MHD flow past an impulsively started infinite vertical plate with uniform heat and mass flux are presented here, taking into account the homogeneous chemical reaction of first order. The dimensionless governing equations are solved by an efficient, more accurate, unconditionally stable and fast converging implicit scheme. The effects of velocity, temperature and concentration for different parameters such as chemical reaction parameter, Schmidt number, Prandtl number, thermal Grashof number, mass Grashof number and time are studied. It is observed that due to the presence of a first order chemical reaction, the velocity increases during the generative reaction and decreases in the destructive reaction. It is observed that the velocity decreases in the presence of the magnetic field, as compared to its absence.

Key words: MHD, chemical reaction, vertical plate, heat and mass flux.

1. Introduction

The influence of the magnetic field on a viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of rayon and nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads, etc. in the presence of the magnetic field.

Chemical reactions can be classified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. In many chemical engineering processes, there does occur a chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications, e.g., polymer production, manufacturing of ceramics or glassware and food processing. Bourne ad Dixon (1971) analyzed the cooling of fibres in the formation process.

Chambre and Young (1958) analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. Das *et al*. (1994) studied the effect of a homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on a moving isothermal vertical plate in the presence of a chemical reaction were studied by Das *et al*. (1999). The dimensionless governing equations were solved by the usual Laplace transform technique and the solutions are valid only at lower time level.

The effects of a transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate were studied by Soundalgekar *et al.* (1979). MHD effects on impulsively started vertical infinite plate with variable temperature were studied by Soundalgekar *et al*. (1981). The dimensionless governing equations were solved using the Laplace transform technique. Muthucumaraswamy and Ganesan (2001) studied the effects of a first order homogeneous chemical reaction on the flow past an impulsively started semi-infinite vertical plate with uniform heat flux and mass diffusion. The governing equations were solved numerically.

The problem of an unsteady natural convection flow past an impulsively started infinite vertical plate with uniform heat and mass flux in the presence of chemical reaction and magnetic field has not received attention of any researcher. Hence, the present study is to investigate the MHD flow past an impulsively started infinite vertical plate with uniform heat and mass flux in the presence of a homogeneous first order chemical reaction by an implicit finite-difference scheme of Crank-Nicolson type.

2. Mathematical analysis

Here the hydromagnetic flow of a viscous incompressible fluid past an impulsively started infinite vertical plate with uniform heat and mass flux is studied. It is assumed that there is a first order chemical reaction between the diffusing species and the fluid. The *x*-axis is taken along the plate in the vertically upward direction and the *y*-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate starts moving impulsively in the vertical direction with constant velocity u_0 against the gravitational filed. At the same time, the heat is supplied from the plate to the fluid at a uniform rate and the concentration level near the plate is also raised at an uniform rate. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the plate. The induced magnetic field and viscous dissipation are assumed to be negligible. Then, under the usual Boussinesq's approximation, the unsteady flow is governed by the following equations

$$\frac{\partial u}{\partial t'} = g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho}, \tag{2.1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = \frac{\partial^2 T'}{\partial y^2}, \tag{2.2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} - K_\ell C. \tag{2.3}$$

The initial and boundary conditions are

$$t' \le 0: \quad u = 0, \quad T' = T'_\infty, \quad C' = C'_\infty,$$

$$t' > 0: \quad u = u_0, \quad \frac{\partial T'}{\partial y} = -\frac{q}{k}, \quad \frac{\partial C'}{\partial y} = -\frac{j''}{D} \quad \text{at} \quad y = 0, \tag{2.4}$$

$$u = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \text{at} \quad x = 0,$$

$$u \to 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y \to \infty.$$

On introducing the following non-dimensional quantities

$$X = \frac{x u_0}{\nu}, \quad Y = \frac{y u_0}{\nu}, \quad U = \frac{u}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad T = \frac{T' - T'_\infty}{\left(q\nu / k u_0\right)},$$

$$\mathrm{Gr} = \frac{g\beta q \nu^2}{k u_0^4}, \quad C = \frac{C' - C'_\infty}{j''\nu / \left(D u_0\right)}, \quad \mathrm{Gc} = \frac{g\beta^* \nu^2 j''}{D u_0^4}, \tag{2.5}$$

$$\mathrm{Sc} = \frac{\nu}{D}, \quad M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, \quad K = \frac{\nu K_\ell}{u_0^2}.$$

Equations (2.1) to (2.3) are reduced to the following non-dimensional form

$$\frac{\partial U}{\partial t} = \mathrm{Gr}\, T + \mathrm{Gc}\, C + \frac{\partial^2 U}{\partial Y^2} - M U, \tag{2.6}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\mathrm{Pr}} \frac{\partial^2 T}{\partial Y^2}, \tag{2.7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{\mathrm{Sc}} \frac{\partial^2 C}{\partial Y^2} - K C. \tag{2.8}$$

The corresponding initial and boundary conditions in the non-dimensional form are

$$\begin{aligned}
&t \le 0: \quad U = 0, \quad T = 0, \quad C = 0, \\
&t > 0: \quad U = 1, \quad \frac{\partial T}{\partial Y} = -1, \quad \frac{\partial C}{\partial Y} = -1 \quad \text{at} \quad Y = 0, \\
&\qquad\quad U = 0, \quad T = 0, \quad C = 0 \quad \text{at} \quad X = 0, \\
&\qquad\quad U \to 0, \quad T \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty.
\end{aligned} \tag{2.9}$$

3. Numerical technique

The unsteady, non-linear coupled Eqs (2.6) to (2.8) with the condition (2.9) are solved by employing an implicit finite difference scheme of Crank-Nicolson type. The finite difference equations corresponding to Eqs (2.6) to (2.8) are as follows

$$\begin{aligned}
\frac{\left[U_{i,j}^{n+1} - U_{i,j}^{n}\right]}{\Delta t} = &\frac{\mathrm{Gr}}{2}\left[T_{i,j}^{n+1} + T_{i,j}^{n}\right] + \frac{\mathrm{Gc}}{2}\left[C_{i,j}^{n+1} + C_{i,j}^{n}\right] - \frac{M}{2}\left[U_{i,j}^{n+1} + U_{i,j}^{n}\right] + \\
&+ \frac{\left[U_{i,j-1}^{n+1} - 2U_{i,j}^{n+1} + U_{i,j+1}^{n+1} + U_{i,j-1}^{n} - 2U_{i,j}^{n} + U_{i,j+1}^{n}\right]}{2(\Delta Y)^2},
\end{aligned} \tag{3.1}$$

$$\frac{\left[T_{i,j}^{n+1}-T_{i,j}^{n}\right]}{\Delta t}=\frac{1}{\Pr}\frac{\left[T_{i,j-1}^{n+1}-2T_{i,j}^{n+1}+T_{i,j+1}^{n+1}+T_{i,j-1}^{n}-2T_{i,j}^{n}+T_{i,j+1}^{n}\right]}{2(\Delta Y)^{2}}. \tag{3.2}$$

The thermal boundary condition at $Y = 0$ in the finite difference form is

$$\frac{1}{2}\frac{\left[T_{i,1}^{n+1}+T_{i,1}^{n}-T_{i,-1}^{n+1}-T_{i,-1}^{n}\right]}{2\Delta Y}=-1. \tag{3.3}$$

At $Y = 0$ (i.e., $j = 0$) Eq.(3.2), becomes

$$\frac{\left[T_{i,0}^{n+1}-T_{i,0}^{n}\right]}{\Delta t}=\frac{1}{\Pr}\frac{\left[T_{i,-1}^{n+1}-2T_{i,0}^{n+1}+T_{i,1}^{n+1}+T_{i,-1}^{n}-2T_{i,0}^{n}+T_{i,1}^{n}\right]}{2(\Delta Y)^{2}}. \tag{3.4}$$

After eliminating $T_{i,-1}^{n+1}+T_{i,-1}^{n}$ using Eq.(3.3), Eq.(3.4) reduces to the form

$$\frac{\left[T_{i,0}^{n+1}-T_{i,0}^{n}\right]}{\Delta t}=\frac{1}{\Pr}\frac{\left[T_{i,1}^{n+1}-T_{i,0}^{n+1}+T_{i,1}^{n}-T_{i,0}^{n}+2\Delta Y\right]}{(\Delta Y)^{2}}, \tag{3.5}$$

$$\frac{\left[C_{i,j}^{n+1}-C_{i,j}^{n}\right]}{\Delta t}=\frac{1}{\mathrm{Sc}}\frac{\left[C_{i,j-1}^{n+1}-2C_{i,j}^{n+1}+C_{i,j+1}^{n+1}+C_{i,j-1}^{n}-2C_{i,j}^{n}+C_{i,j+1}^{n}\right]}{2(\Delta Y)^{2}}-\frac{K}{2}\left(C_{i,j}^{n+1}+C_{i,j}^{n}\right). \tag{3.6}$$

The boundary condition at $Y = 0$ for the concentration in the finite difference form is

$$\frac{1}{2}\frac{\left[C_{i,1}^{n+1}+C_{i,1}^{n}-C_{i,-1}^{n+1}-C_{i,-1}^{n}\right]}{2\Delta Y}=-1. \tag{3.7}$$

At $Y = 0$ (i.e., $j=0$), Eq.(3.6) becomes

$$\frac{\left[C_{i,0}^{n+1}-C_{i,0}^{n}\right]}{\Delta t}=\frac{1}{\mathrm{Sc}}\frac{\left[C_{i,-1}^{n+1}-2C_{i,0}^{n+1}+C_{i,1}^{n+1}+C_{i,-1}^{n}-2C_{i,0}^{n}+C_{i,1}^{n}\right]}{2(\Delta Y)^{2}}-\frac{K}{2}\left(C_{i,0}^{n+1}+C_{i,0}^{n}\right). \tag{3.8}$$

After eliminating $C_{i,-1}^{n+1}+C_{i,-1}^{n}$ using Eq.(3.7), Eq.(3.8) reduces to the form

$$\frac{\left[C_{i,0}^{n+1}-C_{i,0}^{n}\right]}{\Delta t}=\frac{1}{\mathrm{Sc}}\frac{\left[C_{i,1}^{n+1}-C_{i,0}^{n+1}+C_{i,1}^{n}-C_{i,0}^{n}+2\Delta Y\right]}{(\Delta Y)^{2}}-\frac{K}{2}\left(C_{i,0}^{n+1}+C_{i,0}^{n}\right). \tag{3.9}$$

The region of integration is a rectangle with sides $X_{\max}$ (= 1) and $Y_{\max}$(= 14), where $Y_{\max}$ corresponds to $Y = \infty$ which lies very well outside the momentum, energy and concentration boundary layers. The maximum of Y was chosen as 14 after some preliminary investigations so that the last two of the boundary conditions (2.9) are satisfied. Here the subscript i-designates the grid point along the X-direction, j-along the Y-direction and the superscript n along the t-direction.

The computations of U, T and C at time level ($n+1$) using the values at previous time level (n) are carried out as follows: The finite-difference Eqs (3.1), (3.5) and (3.9) at every internal nodal point on a particular i-level constitute a tridiagonal system of equations. Such a system of equations is solved by using Thomas algorithm as discussed in Carnahan *et al*. (2.2). Thus, the values of C are found at every nodal point for a particular i at $(n+1)^{th}$ time level. Similarly, the values of T and U are calculated from Eqs (3.5) and (3.1) respectively. This process is repeated for various i-levels. Thus the values of C, T and U are known, at all grid points in the rectangle region at $(n+1)^{th}$ time level.

Computations are carried out for different time levels until the steady-state is reached. The steady-state solution is assumed to have been reached, when the absolute difference between the values of U as well as temperature T and concentration C at two consecutive time steps are less than 10^{-5} at all grid points.

4. Results and discussion

The effects of velocity, temperature and concentration are studied for different parameters The velocity profiles for different magnetic parameters are shown in Fig.1. It is observed that for $M = 0, 2, 5, 10$, $K = 2$, Gr $=2$, Gc $=5$, Pr $= 0.71$ and Sc$=0.6$, the velocity decreases in the presence of the magnetic field. This shows that an increase in the magnetic field parameter leads to a fall in the velocity.

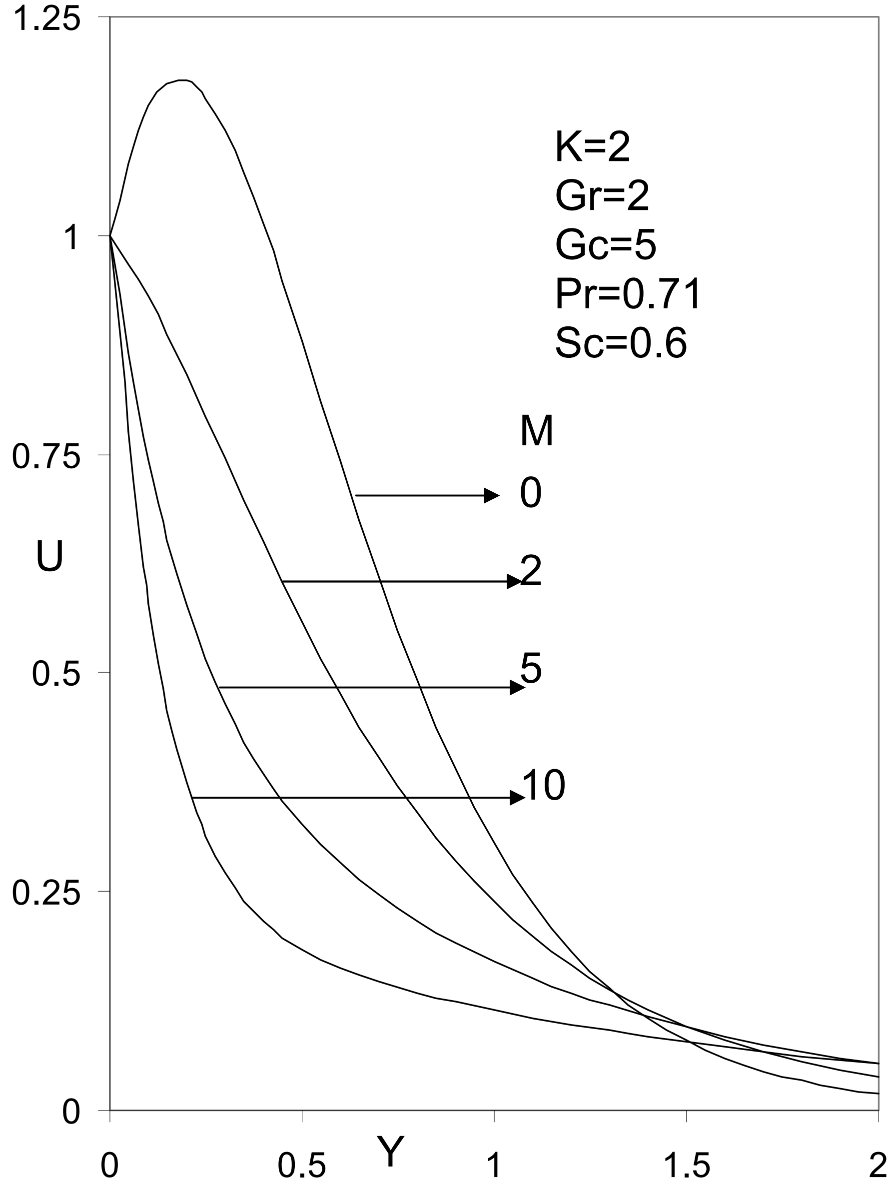

Fig.1. Velocity profiles for different M.

The velocity profiles for different values of the chemical reaction parameter ($K = -1, 0.2, 2$), $M = 2$, Gr = 2, Gc = 5, Pr = 0.71 and (Sc = 0.16, 0.6, 2.01) are shown in Fig.2. It is observed that the velocity increases with the decreasing chemical reaction parameter. This shows that velocity increases during the generative reaction and decreases in the destructive reaction. It is also observed that the velocity decreases with the increasing Schmidt number.

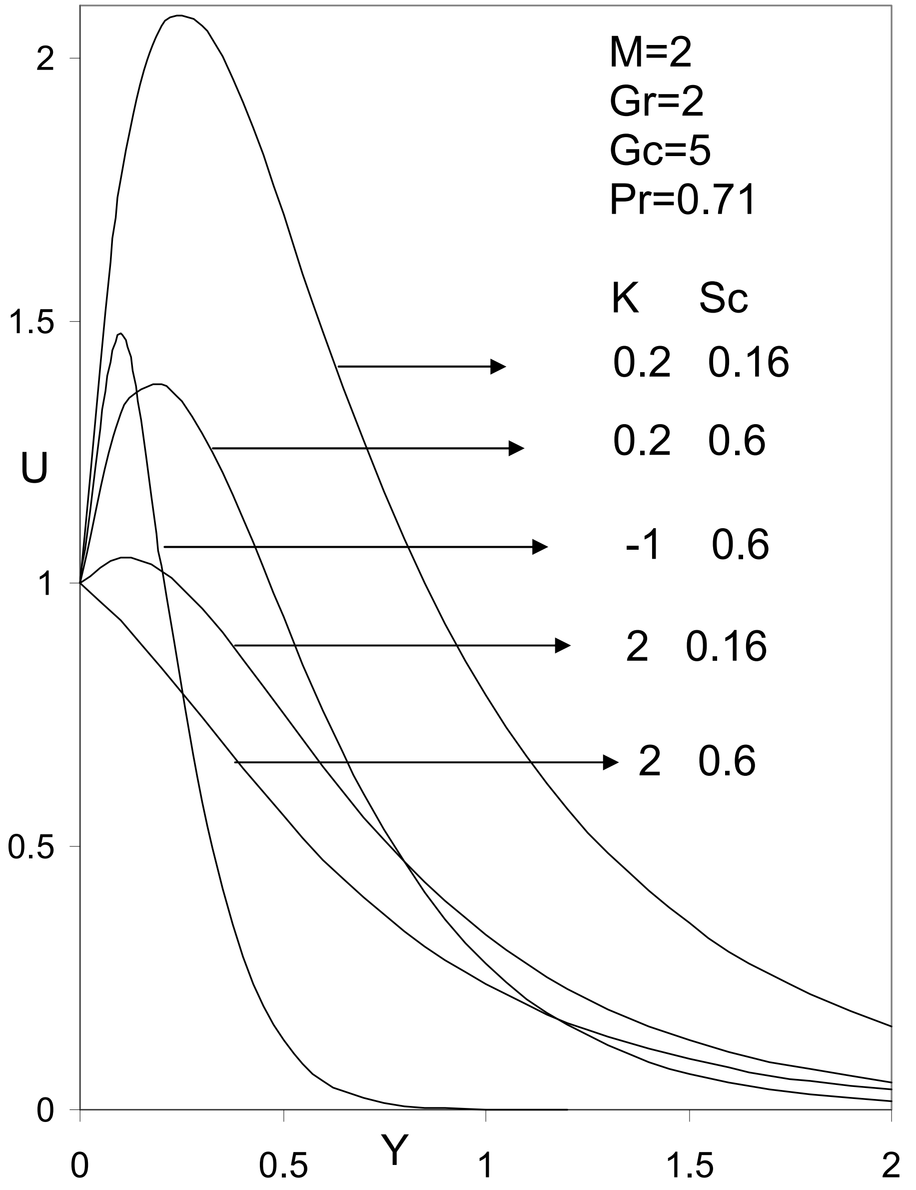

Fig.2. Velocity profiles for different *K* and Sc.

In Fig.3, the velocity profiles for different values of thermal Grashof number and mass Grashof number are shown graphically. The Figure shows that the velocity increases with the increasing thermal Grashof number or mass Grashof number.

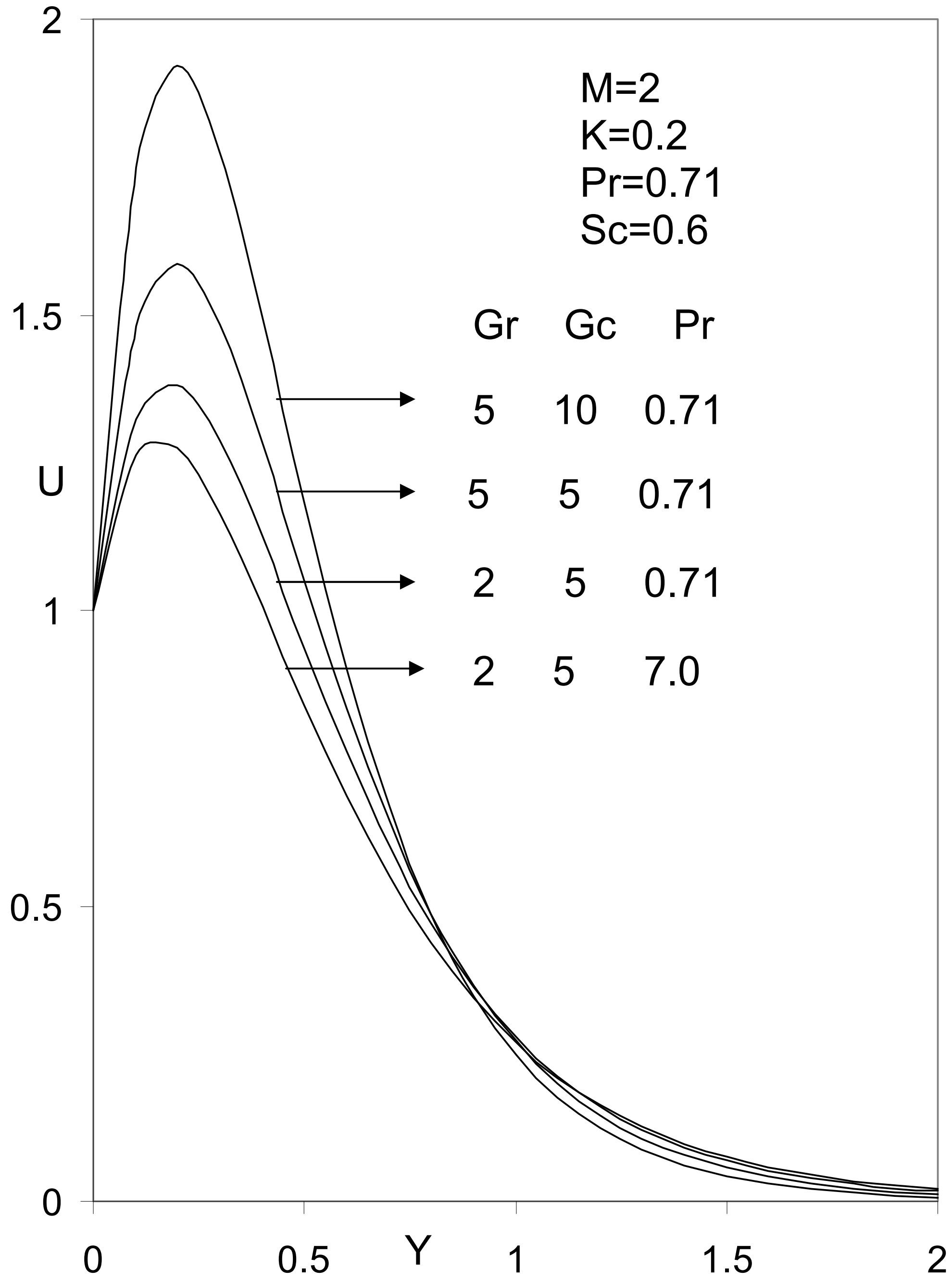

Fig.3. Velocity profiles for different Gr, Gc and Pr.

The temperature profiles for different values of the chemical reaction parameter and Prandtl number are shown in Fig.4. It is observed that the temperature increases with the decreasing Prandtl number. This shows that the buoyancy effect on the temperature distribution is very significant in air (Pr= *0.71*) compared to water (Pr = *7.0*). The temperature trend is reversed with respect to the chemical reaction parameter.

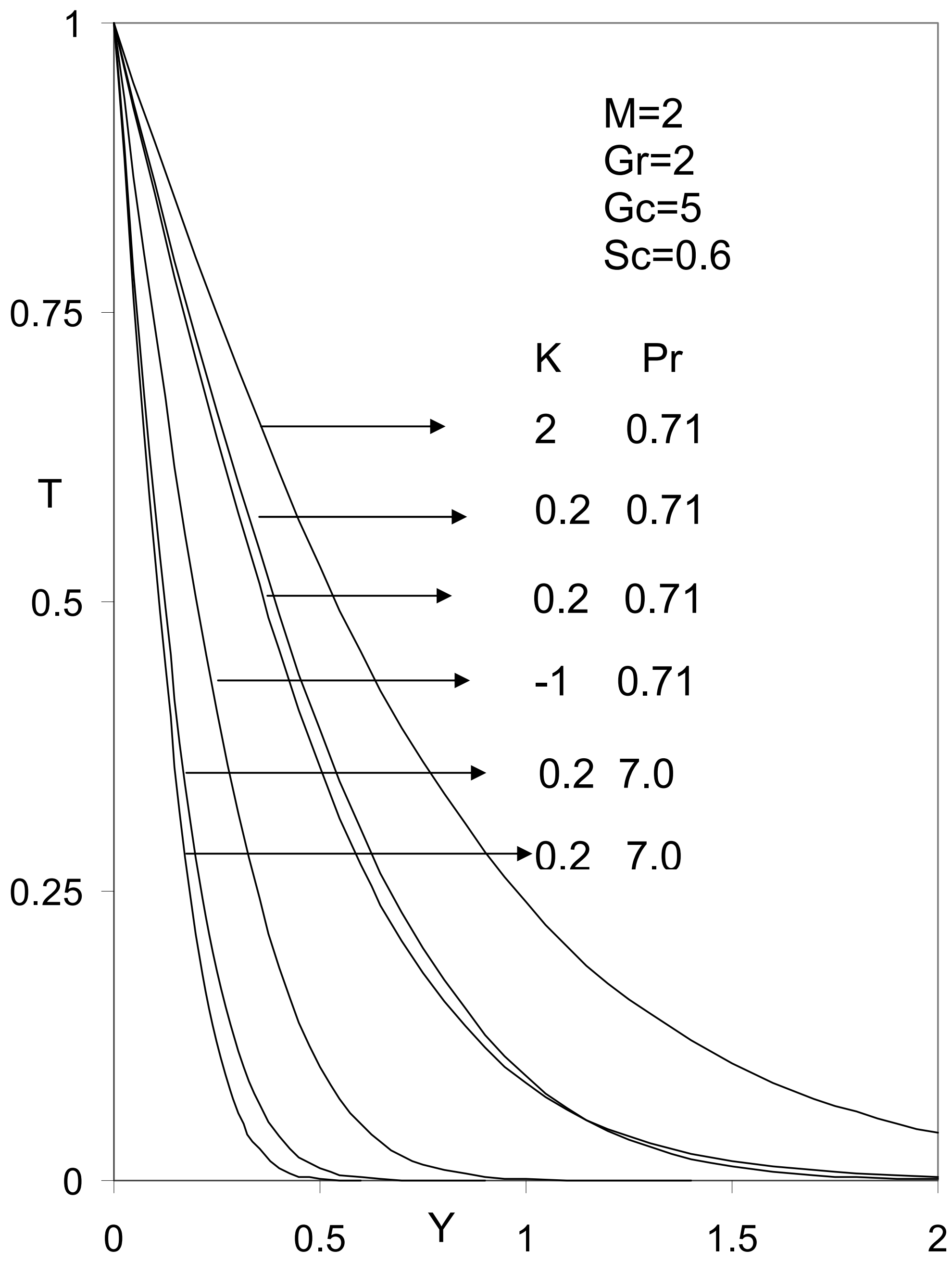

Fig.4. Temperature profiles for different *K* and Pr.

The effect of the chemical reaction parameter and the Schmidt number is very important for concentration profiles. The steady-state concentration profiles for different values of the chemical reaction parameter and Schmidt number are shown in Fig.5. There is a fall in concentration due to increasing the values of the chemical reaction parameter or Schmidt number.

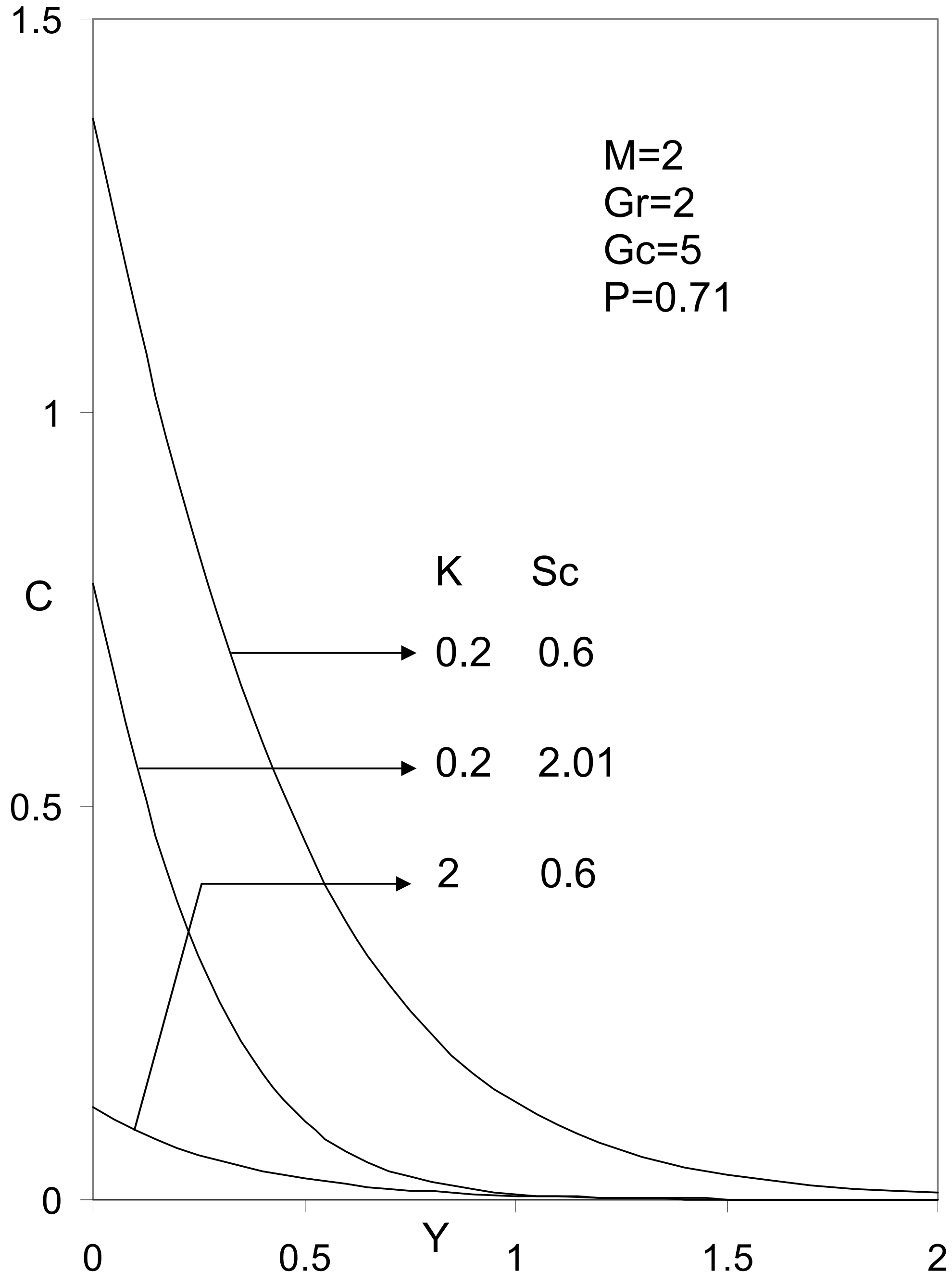

Fig.5. Concentration profiles for different *K* and Sc.

5. Conclusions

A numerical study has been carried out for the unsteady hydromagnetic flow past an impulsively started infinite vertical plate with uniform heat and mass flux in the presence of a homogeneous chemical reaction of first order. The dimensionless governing equations are solved by an implicit finite difference scheme of Crank-Nicolson type. It is observed that the velocity decreases in the presence of the magnetic field. It is also observed that the velocity and concentration increases during the generative reaction and decreases in the destructive reaction.

Nomenclature

B_0 – magnetic field strength
C – dimensionless concentration
C' – concentration
D – mass diffusion coefficient
Gr – thermal Grashof number
Gc – mass Grashof number
g – accelaration due to gravity
j'' – mass flux per unit area at the plate
K – dimensionless chemical reaction parameter
K_l – chemical reaction parameter
k – thermal conductivity of the fluid
M – magnetic field parameter
Pr – Prandtl number
q – heat flux per unit area at the plate
Sc – Schmidt number
T – dimensionless temperature
T' – temperature
t – dimensionless time
t' – time
U – dimensionless velocity components in X-direction respectively
u – velocity components in x –directions, respectively
u_0 – velocity of the plate
X – dimensionless spatial coordinate along the plate
x – spatial coordinate along the plate
Y – dimensionless spatial coordinate normal to the plate
y – spatial coordinate normal to the plate
α – thermal diffusivity
β – coefficient of volume expansion
β^* – volumetric coefficient of expansion with concentration
μ – coefficient of viscosity
ν – kinematic viscosity
σ – Stefan-Boltzmann constant

Subscripts

i – grid point along the X-direction
j – grid point along the Y-direction
w – conditions at the wall
∞ – conditions in the free stream

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Received: October 17, 2012

Revised: March 21, 2013