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ON DISCONTINUTIES IN THERMOELASTIC PLANE WAVES WITHOUT ENERGY DISSIPATION DUE TO A THERMO-MECHANICAL SHOCK

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The present paper deals with the thermoelastic plane waves due to a thermo-mechanical shock in the form of pulse at the boundary of a homogeneous, isotropic thermoelastic half-space. The field equations of the Green-Naugdhi theory without energy dissipation for an thermoelastic solid in the generalized thermoelasticity theory are written in the form of a vector-matrix differential equation using Laplace transform techniques and then solved by an eigenvalue approach. Exact expressions for the considered field variables are obtained and presented graphically for copper-like material. The characteristic features of the present theory are analyzed by comparing these solutions with their counterparts in other generalized thcrmoelasticity theories.

Key words: generalized thrmoelasticity, Green-Naghdi model, thermoelastic half-space, pulse function, eigenvalue approach

1. Introduction

During the second-half of the twentieth century, non-isothermal problems of the theory of elasticity became increasingly important. This is due mainly to their numerous applications in widely diverge fields. First, the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stress, reducing the strength of the aircraft structure. Secondly, in the nuclear field, the extremely high temperatures and temperature gradients originating inside nuclear reactors influence their design and operations (1978).

Problems of plane wave propagation in the generalized thermoelsticity theories which admit finite speed of thermal signals (second sound effect), in elastic solids have become the most interesting research topic in the last three decades. In contrast to the conventional coupled thermoelasticity theory based on a parabolic heat conduction equation (1960) predicting an infinite speed for the propagation of heat, these theories involve a hyperbolic heat equation and are usually referred to as generalized thermoelasticity theories. Among these generalized theories, the extended thermoelasticity theory (ETE) developed by Lord and Shulman (1967) and temperature-rate-dependent thermoelasticity theory (TRDTE) developed by Green and Lindsay (1972) have been investigated by many authors. Due to the experimental evidence in favour of finiteness of heat propagation speed, the generalized thermoelasticity theories are supposed to be more realistic than the conventional coupled thermoelasticity theory (CTE) in dealing with the case of those practical problems involving very large heat flux and/or short time intervals, like those occurring in laser units and energy channels, see (1986; 1988; 1989).

There are some engineering materials such as metals that are not suitable for use in experiments concerning second sound propagation because they have a relatively high rate of thermal damping. But, given the state of recent advances in material science, it may be possible in the foreseeable future to identify (or even manufacture for laboratory purposes) an idealized material for the purpose of studying the propagation of thermal waves at finite speed. The relevant theoretical developments on the subject are due to

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Green and Naghdi (1991; 1992; 1993) and provide sufficient basic modifications in the constitutive equations that permit treatment of a much wider class of heat flow problems, labeled as types I, II, III. Among these models, type-I is the same as the classical heat equation (based on Fourier's law) when the respective theories are linearized, whereas type-II and type-III models permit the thermal wave propagation at finite wave speed. An important feature of type-II models, which is not present in type-I or type-III models, is that it does not accommodate dissipation of thermal energy whereas type-III model accommodate dissipation of energy. The entropy flux vector in type II (thermo-elasticity without energy dissipation (TEWOED)) and type-III (thermo-elasticity with energy dissipation (TEWED)) theories are determined in terms of the potential that also determines stresses. When Fourier conductivity is dominant the temperature equation reduces to the classical Fourier law of heat conduction and when the effect of conductivity is negligible the equation has undamped thermal wave solutions without energy dissipation. The uniqueness of solution for TEOWED theory has been studied by Chandrasekharaiah (1996a; 1996b). Several investigations relating to TEWOED were presented by Roychoudhuri and Datta (2005), Sharma and Chouhan (1999), Roychoudhury and Bandyapadhyay (2004; 2005), Mukhopadhyay (2002; 2004) and Mukhopadhyay and Kumar (2008).

The purpose of the present paper is to study some characteristic features of the theory of thermoelasticity without energy dissipation (TEOWED), developed in Green and Nagdhi (1992), by considering a one-dimensional thermoelastic disturbance in a homogeneous isotropic half-space. We suppose that the thermoelastic plane waves are generated due to the application of a thermo-mechanical shock in the form of pulse at the boundary of the half-space. Here we prefer to adopt the eigenvalue approach of Sarkar and Lahiri (2011) and Lahiri *et al.* (2010) for the solution of the problem with the help of the Lapalce transform. Interestingly, we find that the solution in the Laplace transform domain takes such a simple from that the transform-inversion poses no difficulties whatsoever. We thus obtain exact expression, in a closed form, for the displacement, temperature and stress field. These expressions show that the disturbances consist of two coupled waves, one following the other and propagating with finite waves speeds; one of these is predominantly elastic and the other predominantly thermal and both the waves experiences un-attenuation in contrast to the results derived by Dhaliwal and Rokne (1988; 1989). We observed that physically unrealistic features of TRDTE are not present in TEWOED. At appropriate stages of our analysis we compare our results with those obtained by them. At the end of the paper we present some numerical results applicable to a copper-like material. The graphical representations show that the the points of the medium that are beyond the faster wave front do not experience any disturbance which means that all the field variables are identically zero at the positions beyond the faster wave front and this is in agreement with the analytical results derived here. These similar results can also be seen in ETE, TRDTE theory; see Dhaliwal and Rokne (1988; 1989) for details. The study made in these analyses thus brings to light some similarities and differences between the ETE, TRDTE and TEWOED theories.

2. Governing equations and formulation of the problem

In the context of the G-N theory without energy dissipation the field equations for a linear, homogeneous and isotropic thermoelastic solid, in the absence of body forces and heat sources are as follows (Green and Nagdhi, 1992):

(i) The equation of motion

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{2.1}$$

(ii) Modified equation of heat conduction

$$K^* \nabla^2 T = \rho c_E \frac{\partial^2 T}{\partial t^2} + \gamma T_0 \frac{\partial^2 e}{\partial t^2}, \tag{2.2}$$

(iii) The Duhamel-Neumann constitutive equations are

$$\sigma_{ij} = \lambda u_{i,i}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) - \gamma T_0 \delta_{ij} . \tag{2.3}$$

In the above equations u_i are the displacement components, T is the temperature change above a uniform reference temperature T_0, e is the dilatation, $T\rho$ is the mass density, c_E is the specific heat at constant strain, λ and μ are Lame' constants, $\gamma = (3\lambda + 2\mu)\beta^*$, β^* is the coefficient of volume expansion, and K^* is a material constant characteristic of the theory. In all the above equations the tensor notation is employed. The coupled Eqs (2.1)–(2.3) represent a hyperbolic system that permits finite speed for both elastic and thermal disturbances.

In the case of one-dimensional disturbances where (i) the displacement has only one component u in the x-direction, and (ii) and u,T depend only on x and t, the system of Eqs (2.1), (2.2) and the relation (2.3) reduce to the following system of equations

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} , \tag{2.4}$$

$$K^*\nabla^2 T = \rho c_E \frac{\partial^2 T}{\partial t^2} + \gamma T_0 \frac{\partial^3 u}{\partial x \partial t^2} , \tag{2.5}$$

$$\sigma = \sigma(x,t) = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \gamma T . \tag{2.6}$$

Consider the following non-dimensional variables

$$x' = \frac{1}{L}x, \qquad t' = \frac{v}{L}t, \qquad \Theta = \frac{T}{T_0}, \qquad \sigma' = \frac{\sigma}{\gamma T_0}, \qquad u' = \frac{1}{L}\frac{\lambda + 2\mu}{\gamma T_0}u, \tag{2.7}$$

where L stands for a standard length and v is a standard speed.

Now, the non-dimensional form of equations of motion (2.4), heat Eq.(2.5) and stress-strain relation (2.6) obtained by introducing the non-dimensional variables given in Eq.(2.7), are

$$C_P{}^2\left(\frac{\partial^2 u}{\partial x^2} - \frac{\partial \Theta}{\partial x}\right) = \frac{\partial^2 u}{\partial t^2}, \tag{2.8}$$

$$C_T{}^2\frac{\partial^2 \Theta}{\partial x^2} = \frac{\partial^2 \Theta}{\partial t^2} + \varepsilon\frac{\partial^3 u}{\partial x \partial t^2}, \tag{2.9}$$

$$\sigma = \frac{\partial u}{\partial x} - \Theta \tag{2.10}$$

where $\qquad C_P{}^2 = \dfrac{\lambda + 2\mu}{\rho v^2}, \qquad C_T{}^2 = \dfrac{K^*}{\rho C_E v^2}, \qquad \varepsilon = \dfrac{\gamma^2 T_0}{\rho C_E(\lambda + 2\mu)}.$

In the above equations we suppress primes for simplicity in the notation. We note that C_P and C_T represent the non-dimensional speeds of purely elastic dilatational wave and purely thermal wave respectively, and ε is the usual thermoelastic coupling factor.

We now proceed to study one-dimensional thermoelastic disturbances in an elastic half-space governed by Eqs (2.8)–(2.10) subjected to the following initial and boundary conditions.

The initial conditions of the problem are assumed to be

$$u(x,0)=\Theta(x,0)=\sigma(x,0)=\dot{u}(x,0)=\dot{\Theta}(x,0)=\dot{\sigma}(x,0)=0 \tag{2.11}$$

where the dot stands for differentiation with respect to time t.

We shall consider the non-dimensional boundary conditions in the form of a thermo-mechanical shock with pulse given by

$$\Theta(0,t)=\Theta_0\chi(t) \qquad \text{and} \qquad \sigma(0,t)=\sigma_0\chi(t) \tag{2.12}$$

where $\chi(t)$ is the pulse function defined as

$$\chi(t)=H(t)-H(t-l)=\begin{cases} 1 & \text{if} \quad 0\le t\le l, \\ 0 & \text{otherwise} \end{cases}$$

l is a constant and $H(.)$ is the Heaviside Unit Step function. Also the regularity conditions are

$$u(x,t), \quad \Theta(x,0), \quad \text{and} \quad \sigma(x,t)\to 0 \quad \text{when} \quad x\to\infty. \tag{2.13}$$

3. Solution in the Laplace transform domain

Introducing the Laplace transform defined by the following formula

$$\overline{f}(x,s)=\int_0^\infty f(x,t)e^{-st}dt$$

where $\mathrm{Re}(s)>0$, into the Eqs (2.8)–(2.10) and using the initial conditions (2.11), we get

$$D^2\overline{u}(x,s)=\frac{s^2}{C^2{}_P}\overline{u}(x,s)+D\overline{\Theta}(x,s), \tag{3.1}$$

$$D^2\overline{\Theta}(x,s)=\frac{s^2}{C^2{}_T}\left[\overline{\Theta}(x,s)+D\overline{u}(x,s)\right], \tag{3.2}$$

$$\overline{\sigma}(x,s)=D\overline{u}(x,s)-\overline{\Theta}(x,s), \tag{3.3}$$

where $D\equiv\dfrac{d}{dx}$.

As in Sarkar and Lahiri (2011) and Lahiri *et al*. (2010), the resulting Eqs (3.14) and (3.15) can be written in the form of a vector-matrix differential equation as follows

$$Dv(x,s) = Av(x,s), \tag{3.4}$$

where

$$v(x,s) = \left(\theta \;\; u \;\; D\theta \;\; \mathrm{Du}\right)^T, \qquad A(s) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ c_{31} & 0 & 0 & c_{34} \\ 0 & c_{42} & c_{43} & 0 \end{pmatrix}, \tag{3.5}$$

and $\qquad c_{31} = \dfrac{s^2}{C^2{}_T}, \qquad c_{34} = \varepsilon\dfrac{s^2}{C^2{}_T}, \qquad c_{42} = \dfrac{s^2}{C^2{}_P}, \qquad c_{43} = 1.$

Following the solution methodology through the eigenvalue approach of Sarkar and Lahiri (2011) and Lahiri *et al.* (2010), we now proceed to solve the vector-matrix differential Eq.(3.4).The characteristic equation of the matrix $A(s)$ takes the following form

$$\lambda^4 - \left(c_{31} + c_{42} + c_{34}c_{43}\right)\lambda^2 + c_{31}c_{42} = 0\,. \tag{3.6}$$

Let $\lambda_1{}^2$ and $\lambda_2{}^2$ be the roots of the above characteristic Eq.(3.6) with positive real parts. Then all the four roots of the characteristic Eq.(3.6) which are also the eigenvalue of the matrix $A(s)$ are of the form $\lambda = \pm\lambda_i \;\; (i = 1,2),$ where

$$\lambda_1{}^2 + \lambda_2{}^2 = \left(c_{31} + c_{42} + c_{34}c_{43}\right) \qquad \text{and} \qquad \left(\lambda_1\lambda_2\right)^2 = c_{31}c_{42}.$$

The right eigenvector X , (say) corresponding to the eigenvalue λ can be written as

$$X = \left[\left(c_{42} - \lambda^2\right), -\lambda c_{43}, \lambda\left(c_{42} - \lambda^2\right), -\lambda^2 c_{43}\right]. \tag{3.7}$$

From Eq.(3.7), we can easily calculate the eigenvector X_i corresponding to the eigenvalues $\lambda = \pm\lambda_i \;\; (i = 1,2),$. For our further reference we shall use the following notations

$$X_1 = [X]_{\lambda=\lambda_1}, \quad X_2 = [X]_{\lambda=-\lambda_1}, \quad X_3 = [X]_{\lambda=\lambda_2}, \quad X_4 = [X]_{\lambda=-\lambda_2}. \tag{3.8}$$

The solution of Eq.(3.4) subjected to the boundary condition (2.13) can be written as Sarkar and Lahiri (2011)

$$v(x,s) = A_1 X_2 e^{-\lambda_1 x} + A_2 X_4 e^{-\lambda_2 x}, \tag{3.9}$$

where $A_i \; (i = 1,2)$ are constants to be determined from the boundary conditions (2.12).

Thus the displacement and temperature fields can be written from Eqs (3.5) and (3.7)–(3.9) as

$$\bar{u}(x,s) = A_1\lambda_1 e^{-\lambda_1 x} + A_2\lambda_2 e^{-\lambda_2 x}, \tag{3.10}$$

$$\bar{\Theta}(x,s) = A_1\left(c_{42} - \lambda_1{}^2\right)e^{-\lambda_1 x} + A_2\left(c_{42} - \lambda_2{}^2\right)e^{-\lambda_2 x}. \tag{3.11}$$

Using Eqs (3.10) and (3.11) in the Eq.(2.10), we obtain the stress σ as

$$\bar{\sigma}(x,s) = -c_{42}\left(A_1 e^{-\lambda_1 x} + A_2 e^{-\lambda_2 x}\right). \tag{3.12}$$

Taking the Laplace transform of the boundary conditions (2.12) and then using in Eqs (3.11) and (3.12) and after simplifying, we get the constants A_i as

$$A_i = (-1)^{(i+1)}\frac{\left(1-e^{-sl}\right)}{s\left(\lambda_1^2-\lambda_2^2\right)}\left(\frac{\sigma_0^2}{c_{42}}\lambda_{\left(\frac{2}{i}\right)} - \Theta_0 - \sigma_0\right)(i = 1,2). \tag{3.13}$$

4. Exact solution

Solving Eq.(3.6), we get

$$\lambda_j = sm_j, \quad \text{where} \quad m_j = \frac{1}{\sqrt{2}C_P C_T}\left[\left(C^2{}_P + C^2{}_T + \varepsilon C^2{}_T\right) + (-1)^{j+1}\sqrt{\Omega}\right],$$

and

$$\Omega = \left[\left(C^2{}_P + C^2{}_T + \varepsilon C^2{}_T\right) - 4C^2{}_P C^2{}_T\right](j = 1,2). \tag{4.1}$$

Substituting the values of λ_j from Eq.(4.1) into the transform solutions (3.10)–(3.12) and using Eq.(3.13) and then taking the inverse Laplace transform of the resulting expressions for temperature, displacement and stress, we get the exact solutions in a closed form of the above field variables in the space time domain as follows

$$\begin{aligned} u(x,t) &= m_1\Lambda_2\left[(t-m_1x)H(t-m_1x) - (t-m_1x-l)H(t-m_1x-l)\right] + \\ &\quad + m_2\Lambda_1\left[(t-m_2x)H(t-m_2x) - (t-m_2x-l)H(t-m_2x-l)\right], \end{aligned} \tag{4.2}$$

$$\begin{aligned} \Theta(x,t) &= m_1\Gamma_1\Lambda_2\left[H(t-m_1x) - H(t-m_1x-l)\right] + \\ &\quad + m_2\Gamma_2\Lambda_1\left[H(t-m_2x) - H(t-m_2x-l)\right], \end{aligned} \tag{4.3}$$

$$\begin{aligned} \sigma(x,t) &= \frac{-1}{C_P^2}[m_1\Lambda_2\left\{H(t-m_1x) - H(t-m_1x-l)\right\} + \\ &\quad + m_2\Lambda_1\left\{H(t-m_2x) - H(t-m_2x-l)\right\}] \end{aligned} \tag{4.4}$$

where $$\Lambda_j = \frac{(-1)^j}{\left(m_1^2 - m_2^2\right)}\left[\sigma_0\left(m_j^2 C_P^2\right) - \Theta_0\right] \quad \text{and} \quad \Gamma_j = \frac{1}{C_P^2} - m_j^2 \ (j = 1, 2).$$

As a verification of our solutions, we directly show that the solutions (4.2)–(4.4) satisfy the governing Eqs (2.8)–(2.10), the homogeneous initial conditions (2.11), the boundary conditions (2.12) and (2.13). Thus the solutions given in Eqs (4.28)–(4.4) are indeed the exact solutions in a closed form for $u(x,t), \Theta(x,t)$, and $\sigma(x,t)$ in our present problem.

From the exact solutions in closed form given in Eqs (4.28)–(4.4), we observe that each of $u(x,t), \Theta(x,t)$, and $\sigma(x,t)$ is made up of two parts and that each part corresponds to a wave propagating with a finite speed. The terms involving $H(t - m_1 x)$ and $H(t - m_1 x - l)$ represent the contribution of the wave travelling with the speed $V_E = \dfrac{1}{m_1}$, (say) at the wave front $x = V_E t$ for two different times t and $t - l$. The terms associated with $H(t - m_2 x)$ and $H(t - m_2 x - l)$ represent the contribution of the wave travelling with the speed $V_T = \dfrac{1}{m_2}$, (say) at the wave front $x = V_T t$ for two different times t and $t - l$. We thus observe that the speed of the wave corresponding to the first part is V_E and that corresponding to the second part is V_T.
From Eq.(4.1) we find out that

(i) $V_E < V_T$.

(ii) For those materials in which $C_T > C_P$, $V_E \to C_P$, and $V_T \to C_T$ as $\varepsilon \to 0$. Thus when $\varepsilon \neq 0$, V_E and V_T correspond respectively to the modified elastic wave (or the e-wave) and the modified thermal wave (or the Θ−wave). The faster wave is predominantly a thermal wave and the slower wave is predominantly an elastic wave.

(iii) For those material in which $C_T < C_P$, $V_E \to C_T$, and $V_T \to C_P$ as $\varepsilon \to 0$. Thus when $\varepsilon \neq 0$, V_E and V_T correspond respectively to the modified thermal wave (or the Θ−wave) and the modified elastic wave (or the e-wave). The faster wave is predominantly an elastic wave and the slower wave is predominantly thermal wave.

Accordingly, the disturbances being considered consist of two distinct, coupled waves, one following the other; the faster wave propagating with the speed V_T and the slower wave with the speed V_E. Further, the faster wave is a predominantly elastic wave (*e*-wave) or a predominantly thermal wave (Θ-wave) and the slower wave is a Θ-wave or a e-wave as $C_T < C_P$ or $C_T > C_P$.

It has to be pointed out that in Sarkar and Lahiri (2011), where the counterparts of our problem in the contexts of ETE and TRDTE have been considered, only approximate solutions valid for small time have been constructed. This is due to the fact that the roots of the counterparts of our characteristic Eq.(3.6) in ETE and TRDTE are irrational functions of *s* and consequently the exact transform-inversion for all *s* is not as simple as in our present problem. It may be added that the same difficulty arises even in the context of the conventional thermoelasticity theory (CTE). But in the case of the TEWOED, such problems do not arise and we can easily obtain the exact solutions in a closed form. This novel feature of the TEWOED stems from the fact that in this theory the roots of the characteristic equation are directly proportional to the transform parameter; see Eq.(4.1).

From the solutions given in Eqs (4.2)–(4.4), we observe two more interesting features. First, neither the *e*-wave nor the Θ-wave experiences any decay with distance (attenuation). Secondly, all of $u(x,t), \Theta(x,t)$, and $\sigma(x,t)$ are identically zero for $x > tV_T$; where V_T is the speed of the faster wave. This means that, at a given instant of time $t^* > 0$ the points of the half-space that are beyond the faster wavefront $\left(x = t^* V_T\right)$ do not experience any disturbance. But the solutions obtained in Dhaliwal and Rokne (1988;

1989) indicate that in the contexts of both the ETE and TRDTE theories, both e-wave and Θ-waves experience attenuation. This difference between the predictions of the TEWOED and those of ETE and TRDTE stems from the fact that whereas TEWOED does not sustain energy dissipation, both ETE and TRDTE do accommodate energy dissipation due to the present of temperature-term in the heat transport equation in both the ETE and TRDTE theory. Again the second feature has also been noted in Dhaliwal and Rokne (1988; 1989). Indeed, this second phenomenon is a characteristic of all the generalized thermoelasticity theories. Our observation thus verifies that the TEWOED is a generalized thermoelasticity theory.

5. Analysis of discontinuities

By direct inspection of the solutions (4.2)–(4.3), we find that u is continuous but Θ and σ suffers discontinuity at both the wavefronts; namely, $t=\dfrac{x}{V_E}$ and $t=\dfrac{x}{V_T}$. Now, we can easily write down the discontinuities experienced by $u(x,t), \Theta(x,t)$, and $\sigma(x,t)$ at the wavefronts $t=\dfrac{x}{V_E}$ and $t=\dfrac{x}{V_T}$ as follows

$$\left[u^{+}-u^{-}\right]_{x=\eta V_E}=\left[u^{+}-u^{-}\right]_{x=\eta V_T}=0, \tag{5.1}$$

$$\left[\Theta^{+}-\Theta^{-}\right]_{x=\eta V_E}=m_1\Gamma_1\Lambda_2, \qquad \left[\Theta^{+}-\Theta^{-}\right]_{x=\eta V_T}=m_2\Gamma_2\Lambda_1, \tag{5.2}$$

$$\left[\sigma^{+}-\sigma^{-}\right]_{x=\eta V_E}=\frac{-m_1\Lambda_2}{C_P^2}, \qquad \left[\sigma^{+}-\sigma^{-}\right]_{x=\eta V_T}=\frac{-m_2\Lambda_1}{C_P^2}, \tag{5.3}$$

for the two instants of time $\eta=t, t-l$.

Expression (5.1) indicates that the displacement u is continuous at both the wavefronts. This is also the same situation in the context of the ETE (Dhaliwal and Rokne, 1988). But in the context of the TRDTE, the displacement is discontinuous at both the wavefronts (Dhaliwal and Rokne, 1989). A discontinuity in displacement means that one portion of matter penetrates into another, and this phenomenon is physically unrealistic; indeed, it violates the continuum hypothesis. Thus, the TRDTE predicts a non-acceptable physical behavior for the displacement field near the wavefronts, whereas the TEWOED (similarly as ETE) does not make such a prediction.

From expression (5.2) we see that the temperature is discontinuous at both the wavefronts. This is also the case in the contexts of the ETE and TRDTE (Dhaliwal and Rokne, 1988; 1989).

Expression (5.3) shows that the stress is discontinuous at both the wavefronts, however, there is a significant difference. Whereas the magnitudes of discontinuities are finite in our present analysis and in the ETE (Dhaliwal and Rokne, 1988) theory, these are infinite in the TRDTE (Dhaliwal and Rokne, 1989) case and also in this case the stress exhibits a Dirac delta behavior near the wavefronts. Thus, the TRDTE predicts a physically unrealistic behavior for the stress field near the wavefronts, whereas the TEWOED (like ETE) does not make such a prediction.

We also observe from Eqs (5.2) and (5.3) that the jumps in temperature and stress fields are all finitely constants. However, in the contexts of ETE and TRDTE (Dhaliwal and Rokne, 1988; 1989) the corresponding jumps decay with time because of attenuation. Thus, we observe that the predictions of the TEWOED are qualitatively similar to those of the ETE but dissimilar to those of the TRDTE, in general.

6. Numerical example

Now, we consider a numerical example to illustrate the analytical procedure presented earlier. For this purpose we chose a copper material for which

$$C_P^2 = 1, \qquad C_T^2 = \frac{1}{0.5}, \qquad \varepsilon = 0.0168, \qquad l = 3.5\,.$$

For the material chosen, we have $C_T > C_P$ and as such the faster wave happens to be the $\Theta-$wave and the slower wave the $e-$wave. By using expression (4.3), we obtain the dimensionless waves as $V_T = 1.0326$ and $V_E = 0.4842$, respectively. In Dhaliwal and Rokne (1988), the authors analyzed the behavior of all the considered field variables at dimensionless time $t = 0.25$ with distance x. But in our present numerical analysis, we take a wide range of values for both x and t.

Figures 1–3 display the displacement, temperature and stress distributions for a wide range of $x(0 \le x \le 3)$ for different values of dimensionless time t with a wide range $0 \le t \le 3$.

Figure 1 shows that the displacement is continuous at all positions including the locations of the wavefronts, as predicted by the theoretical result obtained in Section 5. From Fig.1, we also noticed that the displacement increases steadily between the boundary and the position just beyond the slower wavefront $x = tV_E$, decreases thereafter up to the location of the faster wavefront $x = tV_T$ and finally becomes identically zero beyond this location, as predicted by the theoretical result.

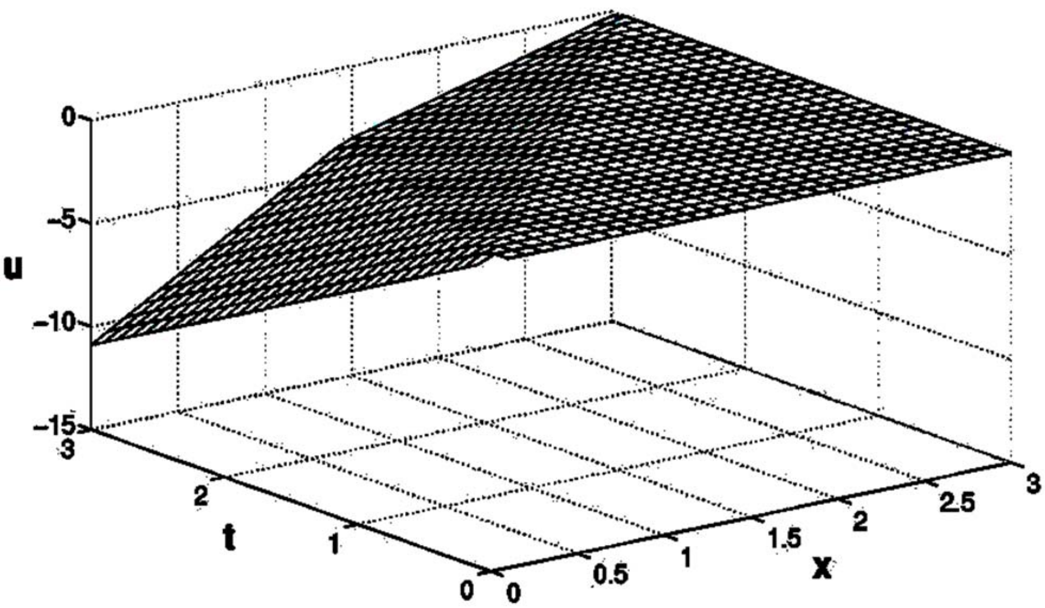

Fig.1. Displacement distribution.

Figures 2–3 indicates that the temperature and stress are discontinuous at both the wavefronts as predicted by the theoretical results obtained earlier.

Figures 4–6 display the displacement, temperature and stress distributions for a wide range of $x(0 \le x \le 3)$, at time $t = 2.5$, for different values of dimensionless Green-Naugdhi parameter C_T with a wide range $\left(1.5 \le C_T \le 16.5\right)$.

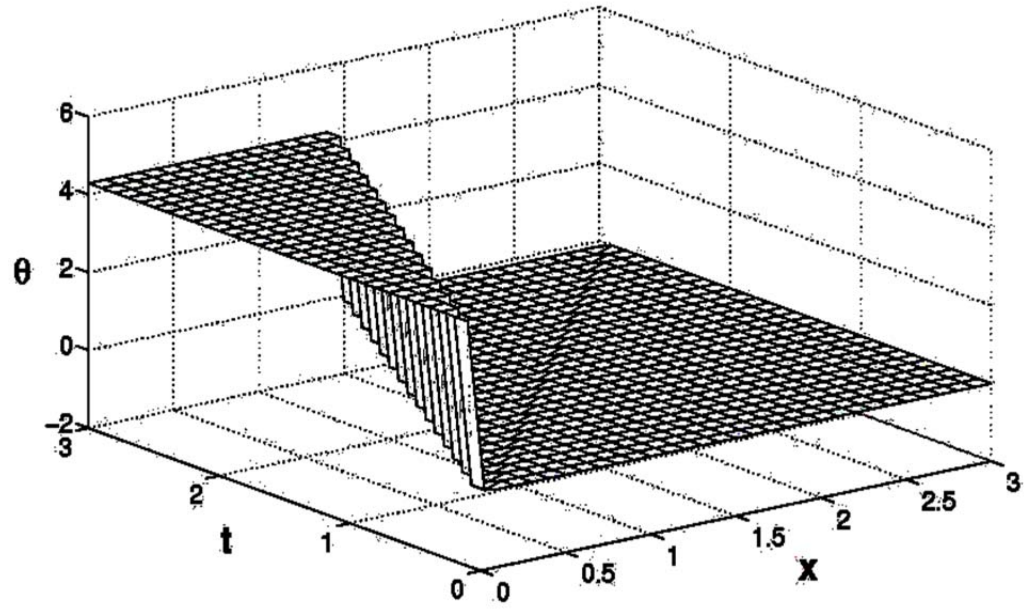

Fig.2. Temperature distribution.

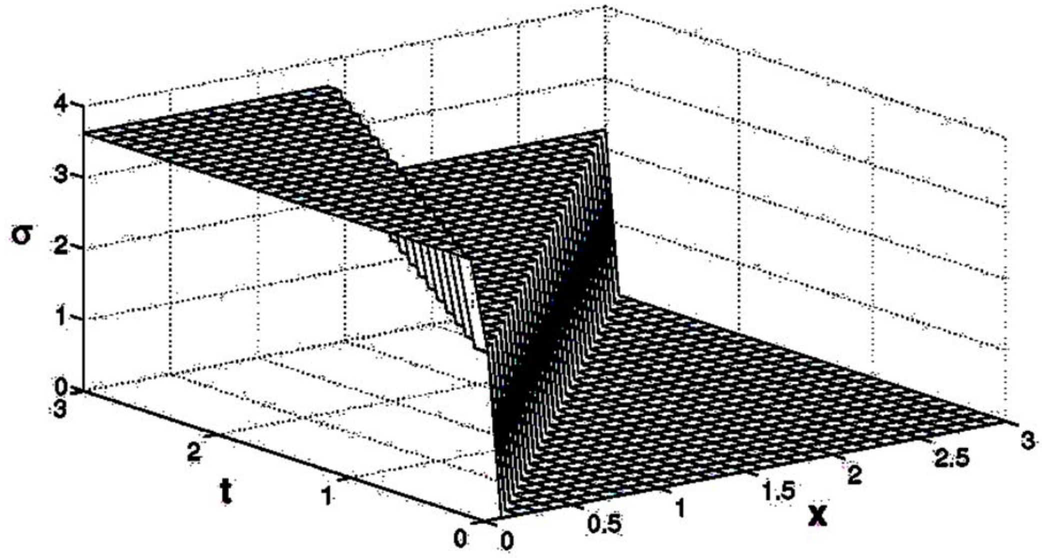

Fig.3. Stress distributionat.

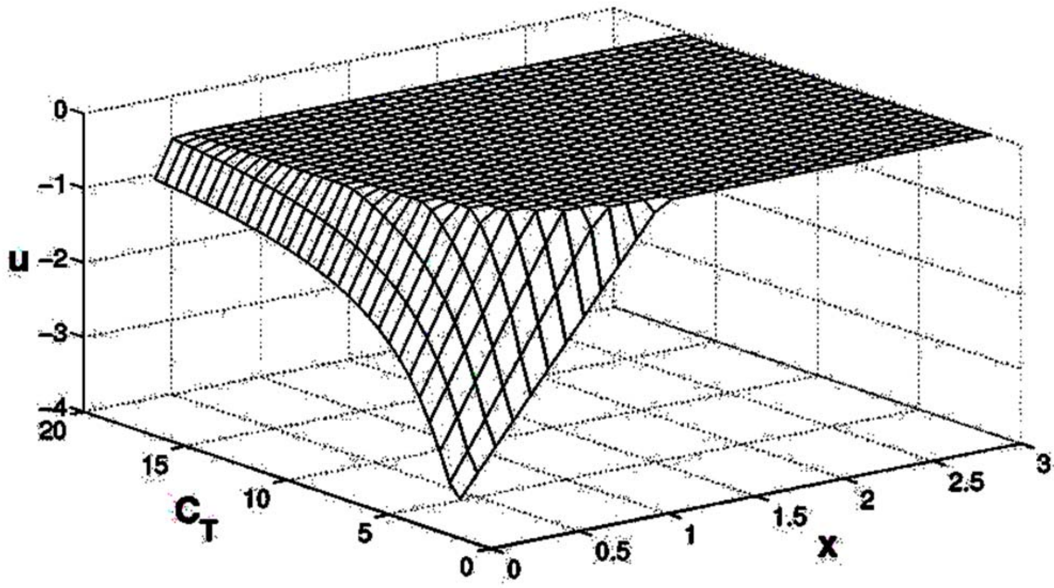

Fig.4. Displacement distribution.

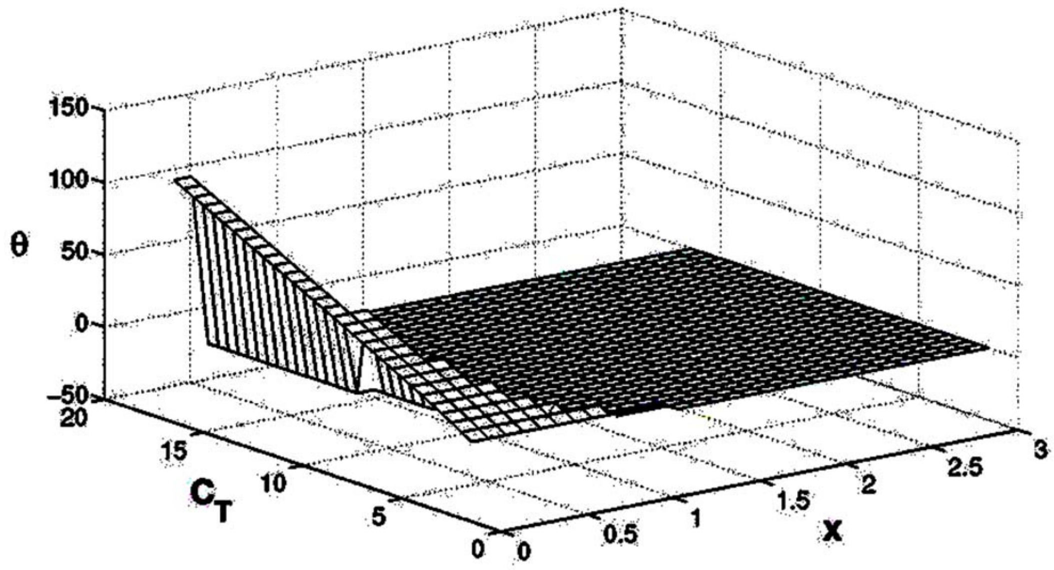

Fig.5. Temperatrue distribution.

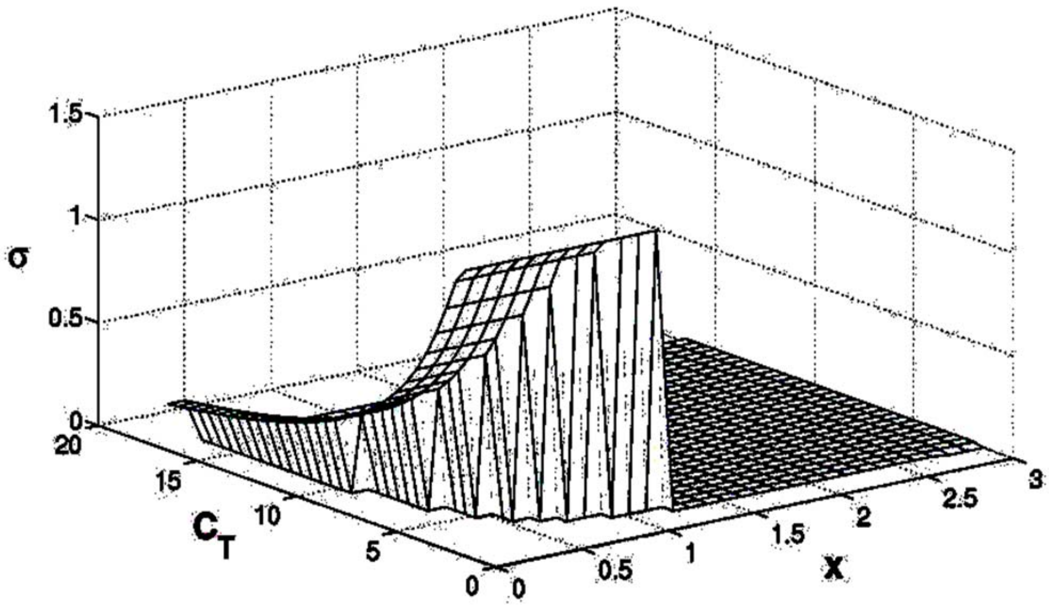

Fig.6. Stress distribution.

Figures 7 is plotted to show the variation of V_E for a wide range of $x(0 \le x \le 3)$ at $t = 2.5$ for different values of the dimensionless Green-Naugdhi parameter C_T with a wide range $(1.5 \le C_T \le 16.5)$.

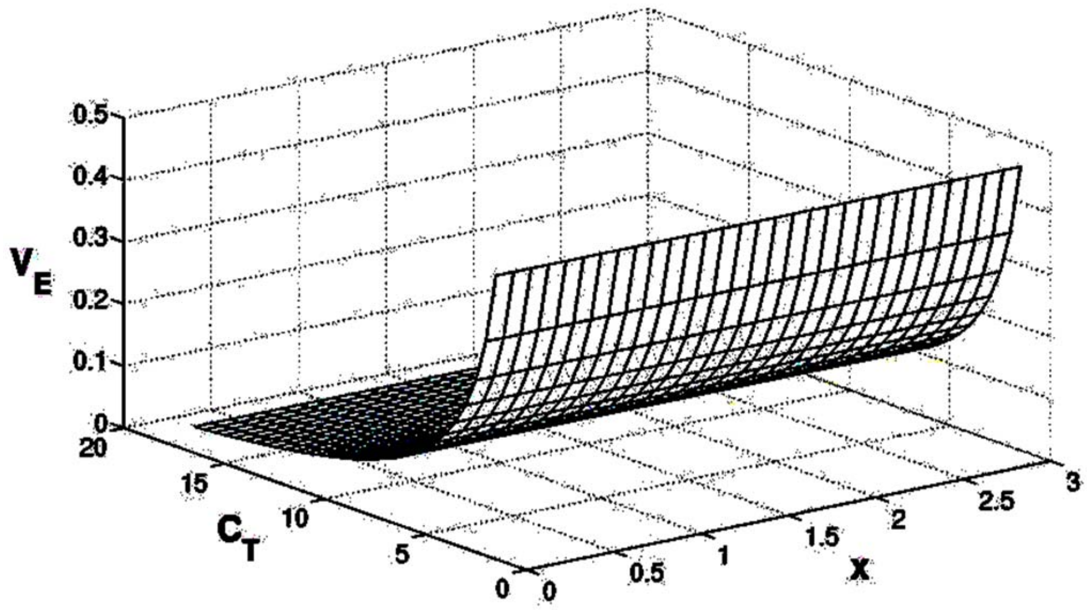

Fig.7. Distribution of V_E with C_T and x.

Figures 8–9 are plotted to show the distributions of V_T for a wide range of $x(0 \le x \le 3)$ at $t = 2.5$, for different values of the dimensionless Green-Naugdhi parameter C_T with a wide range $(1.5 \le C_T \le 16.5)$. Figure 8 shows the variation of V_T for $\varepsilon = 0.0168$ while Fig.9 displays the variation of V_T for $\varepsilon = 0.0$.

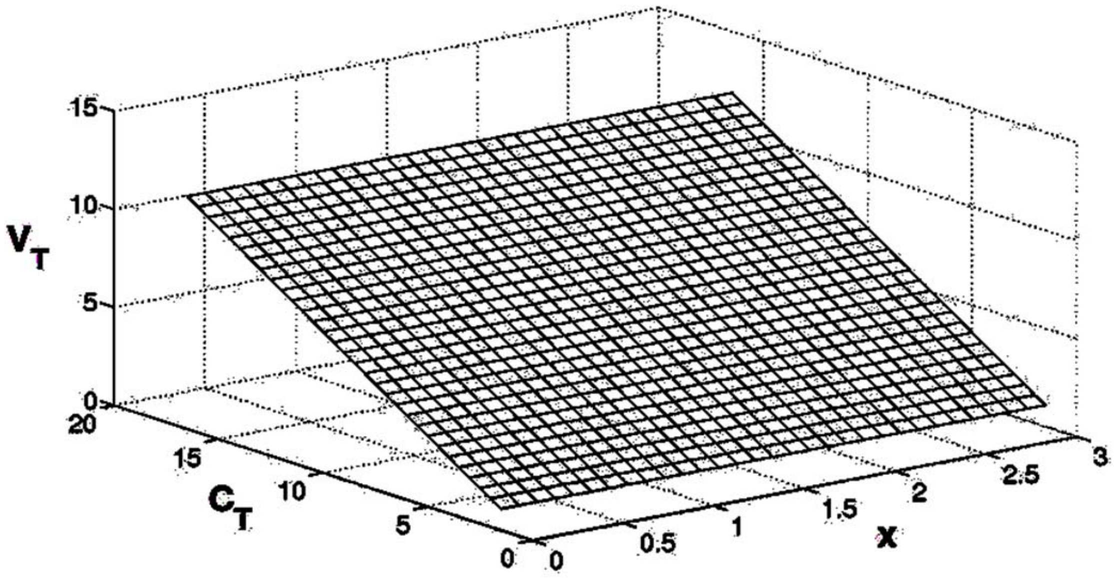

Fig.8. Distribution of V_T for $\varepsilon = 0.0168$.

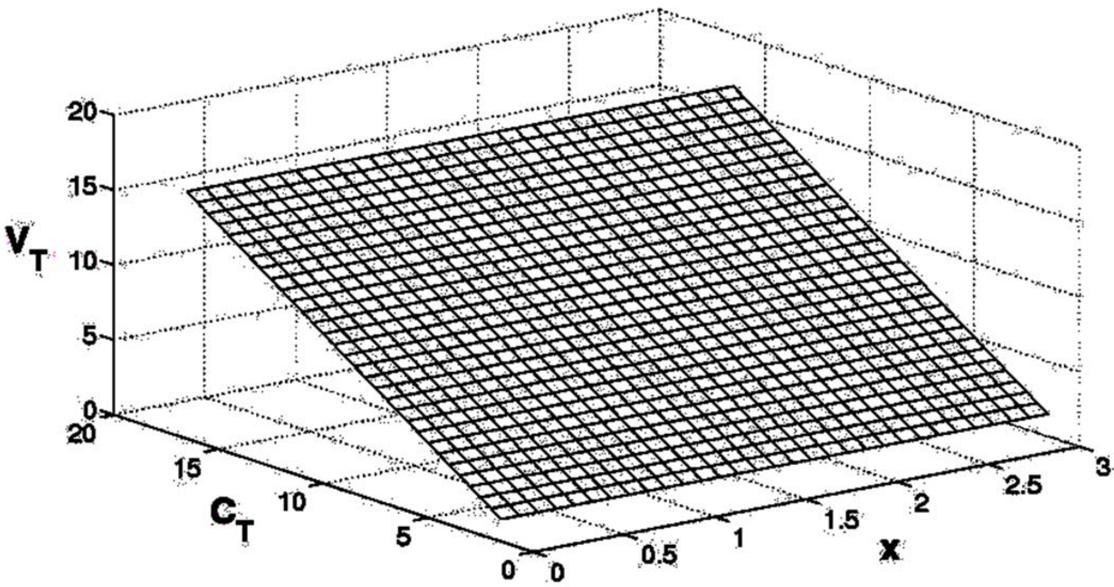

Fig.9. Distribution of V_T for $\varepsilon = 0.0$.

Figures 10–12 display the displacement, temperature and stress distributions for a wide range of $x(0 \le x \le 3)$ at a small value of time $t = 0.25$. Here, Figs 10–12 are plotted to show the the position of the e −wavefront and Θ −wavefront for particular times $t = 0.5$, and 0.25.

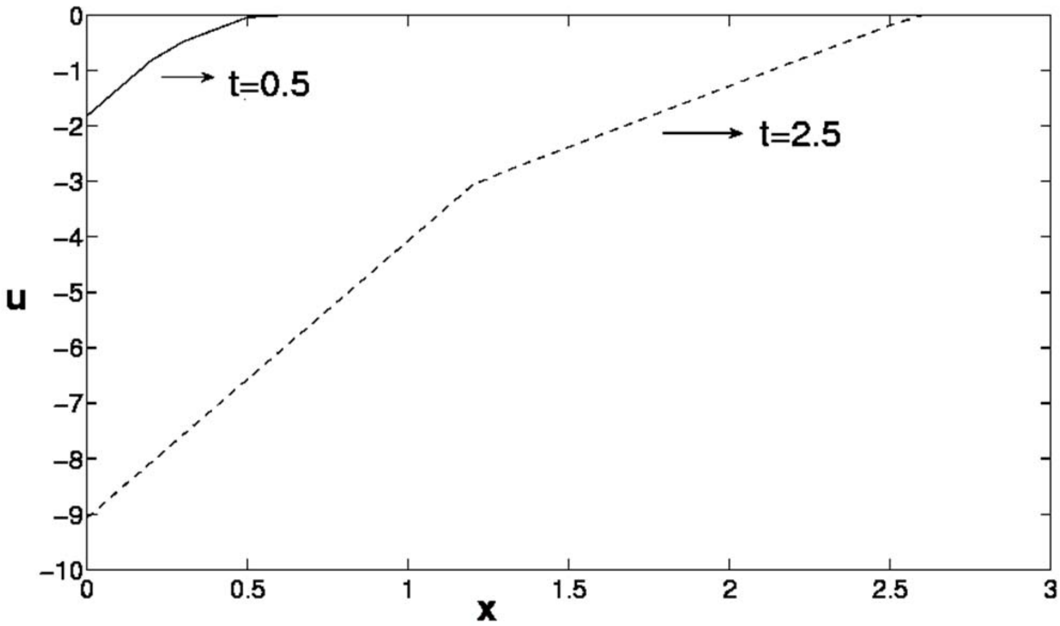

Fig.10. Displacement distribution.

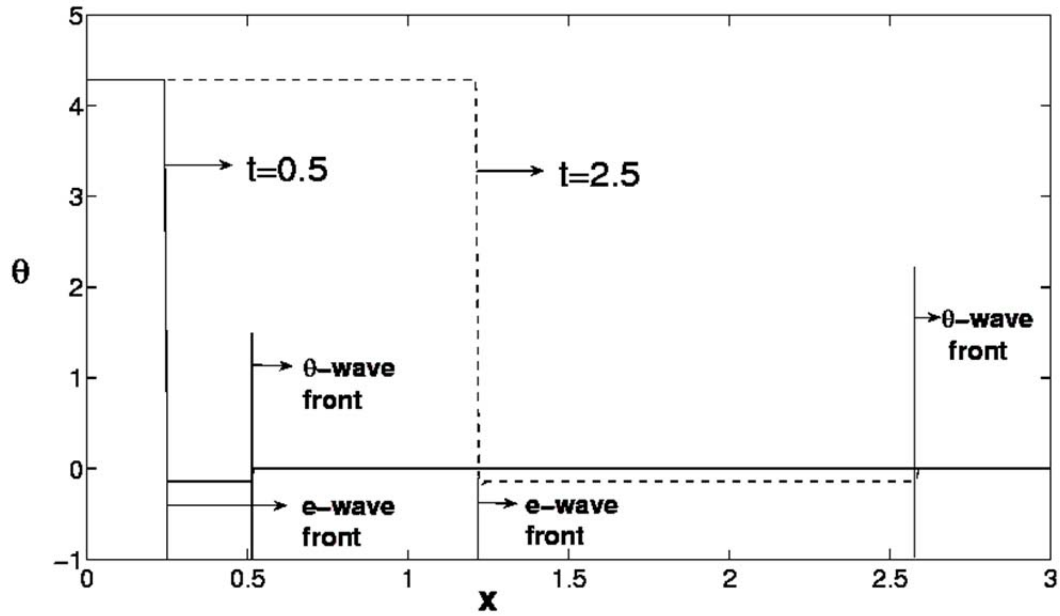

Fig.11. Temperatrue distribution.

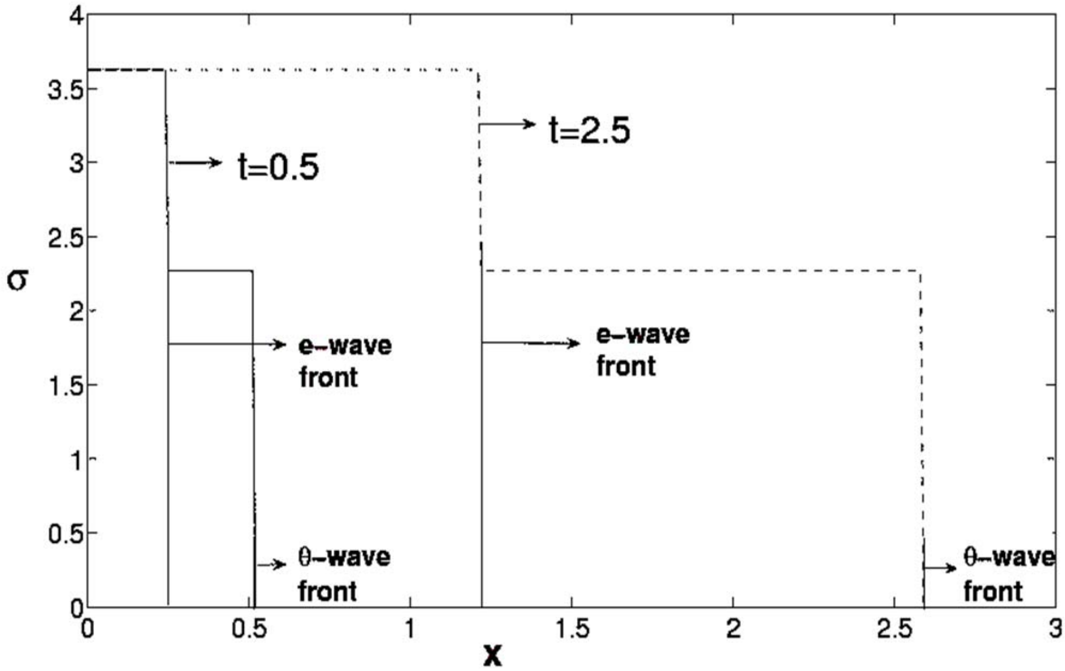

Fig.12. Stress distribution.

According to the observations made in the above paragraphs, all of $u(x,t), \Theta(x,t)$, and $\sigma(x,t)$ vanish identically at all points that are beyond the faster wavefront, indicating that the effects of disturbances are restricted to the domain $0 \le x \le V_T$. This observation agrees with our theoretical results obtained in section 4.

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Nomeclature

C_P – the non-dimensional speeds of purely elastic dilatational wave
C_T – the non-dimensional speeds of purely thermal wave
c_E – specific heat at constant strain
D – $\equiv \dfrac{d}{dx}$
e – cubical dilatation
$H(.)$ – heaviside unit step function
K^* – a material constant characteristic of the theory
T – absolute temperature
T_0 – uniform reference temperature
u_i – displacement components
β^* – coefficient of volume expansion

γ – $= (3\lambda + 2\mu)\beta^*$
ε – thermoelastic coupling factor
λ, μ – Lame’ constants
ρ – mass density

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