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Brief note

MHD AND THERMAL RADIATION EFFECTS ON EXPONENTIALLY ACCELERATED ISOTHERMAL VERTICAL PLATE WITH UNIFORM MASS DIFFUSION

R. MUTHUCUMARASWAMY*
Department of Applied Mathematics
Sri Venkateswara College of Engineering
Sriperumbudur 602 105, INDIA
E-mail: msamy@svce.ac.in

V. VISALAKSHI
Department of Mathematics
Ganadipathy Tulsis' Jain Engineering College
Kaniyambadi, Vellore 632102, INDIA

Thermal radiation effects on an unsteady free convective flow of a viscous incompressible flow of a past an exponentially accelerated infinite isothermal vertical plate with uniform mass diffusion in the presence magnetic field are considered. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised to T_w and the concentration level near the plate is also raised to C'_w. An exact solution to the dimensionless governing equations is obtained by the Laplace transform method, when the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane against gravitational field. The effects of velocity, temperature and concentration fields are studied for different physical parameters such as the magnetic field parameter, thermal radiation parameter, Schmidt number, thermal Grashof number, mass Grashof number and time. It is observed that the velocity increases with decreasing magnetic field parameter or radiation parameter. But the trend is just reversed with respect to a or t.

Key words: radiation, isothermal, vertical plate, mass diffusion, magnetic field.

1. Introduction

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines. Various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation exists in space technology. In such cases, one has to take into account the effect of thermal radiation and mass diffusion.

England and Emery (1969) studied thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Radiation effects on mixed convection along an isothermal vertical plate were studied by Hossain and Takhar (1996). Raptis and Perdikis (1999) studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das *et al.* (1996) analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

* To whom correspondence should be addressed

Magnetoconvection plays an important role in agriculture,petroleum industries, geophysics and in astrophysics. Important applications are found in the study of geological formations, in exploration and thermal recovery of oil and in the assessment of aquifers,geothermal reservoirs and underground nuclear waste storage sites. MHD flow has applications in metrology, solar physics and in motion of the earth's core. Also, it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. The effects of a transversely applied magnetic field on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate were studied by Soundalgekar *et al*. (1979). MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of a transverse magnetic field were studied by Soundalgekar *et al.* (1979). The dimensionless governing equations were solved using the Laplace transform technique.

Free convection effects on flow past an exponentially accelerated vertical plate were studied by Singh and Kumar (1984). The skin friction for an accelerated vertical plate was studied analytically by Hossain and Shayo (1986). Jha *et al.* (1991) analyzed mass transfer effects on an exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion.

However, a combined study of MHD and thermal radiation effects on an exponentially accelerated infinite isothermal vertical plate with uniform mass diffusion has not been made. It is proposed to study the unsteady flow past an exponentially accelearted infinite isothermal vertical plate, in the presence of a magnetic field and thermal radiation. The dimensionless governing equations are tackled using the Laplace transform technique. The solutions are in terms of an exponential and complementary error function.

2. Formulation of the problem

Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞ is studied. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the temperature from the plate is raised to T_w and the concentration level near the plate is also raised to C'_w. The plate is also subjected to a uniform magnetic field of strength B_0 assumed to be applied normal to the plate. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. It is assumed that the effect of viscous dissipation is negligible in the energy equation. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u, \tag{2.1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \tag{2.2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2}, \tag{2.3}$$

with the following initial and boundary conditions

$$t' \le 0: \quad u = 0, \qquad T = T_\infty, \qquad C' = C'_\infty \quad \text{for all} \quad y,$$

$$t' > 0: \quad u = u_0 \exp(a't'), \quad T = T_w, \qquad C' = C'_w \quad \text{at} \quad y = 0, \tag{2.4}$$

$$u = 0, \qquad T \to T_\infty, \qquad C' \to C'_\infty \quad \text{as} \quad y \to \infty.$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma \left(T_\infty^4 - T^4 \right). \tag{2.5}$$

It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 . \tag{2.6}$$

By using Eqs (2.5) and (2.6), Eq.(2.2) reduces to

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3 \left(T_\infty - T \right). \tag{2.7}$$

Introducing the following non-dimensional quantities

$$U = \frac{u}{u_0}, \qquad t = \frac{t' u_0^2}{\nu}, \qquad Y = \frac{y u_0}{\nu}, \qquad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\mathrm{Gr} = \frac{g\beta\nu\left(T_w - T_\infty\right)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad \mathrm{Gc} = \frac{\nu g \beta^* \left(C'_w - C'_\infty\right)}{u_0^3}, \quad a = \frac{a'\nu}{u_0{}^2}, \tag{2.8}$$

$$R = \frac{16a^* \nu^2 \sigma T_\infty^3}{k u_0^2}, \qquad \mathrm{Pr} = \frac{\mu C_p}{k}, \qquad \mathrm{Sc} = \frac{\nu}{D}, \qquad M = \frac{\sigma B_0^2 \nu}{\rho u_0^2},$$

into Eqs (2.1) to (2.4), leads to

$$\frac{\partial U}{\partial t} = \mathrm{Gr}\,\theta + \mathrm{Gc}\,C + \frac{\partial^2 U}{\partial Y^2} - M\,U, \tag{2.9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\mathrm{Pr}} \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\mathrm{Pr}}\,\theta, \tag{2.10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{\mathrm{Sc}} \frac{\partial^2 C}{\partial Y^2}. \tag{2.11}$$

The initial and boundary conditions in a non-dimensional form are

$$
\begin{aligned}
& U=0, \quad \theta=0, \quad C=0, \quad \text{for all} \quad Y,t\le 0,\\
t>0:\ & U=\exp(\mathrm{at}), \quad \theta=1, \quad C=1, \quad \text{at} \quad Y=0,\\
& U=0, \quad \theta\to 0, \quad C\to 0, \quad \text{as} \quad Y\to\infty.
\end{aligned} \tag{2.12}
$$

All the physical variables are defined in the nomenclature. The solutions are obtained for a hydrodynamic flow field in the presence of a first order chemical reaction. Equations (2.9) to (2.11), subject to the boundary conditions (2.12), are solved by the usual Laplace-transform technique and the solutions are derived as follows

$$
\begin{aligned}
U=&\frac{\exp(at)}{2}\Big[\exp\left(2\eta\sqrt{(M+a)t}\right)\mathrm{erfc}\left(\eta+\sqrt{(M+a)t}\right)+\exp\left(-2\eta\sqrt{(M+a)t}\right)\mathrm{erfc}\left(\eta-\sqrt{(M+a)t}\right)\Big]+\\
&+(e+f)\Big[\exp\left(2\eta\sqrt{Mt}\right)\cdot\mathrm{erfc}\left(\eta+\sqrt{Mt}\right)+\exp\left(-2\eta\sqrt{Mt}\right)\cdot\mathrm{erfc}\left(\eta-\sqrt{Mt}\right)\Big]-2f\cdot\mathrm{erfc}\left(\eta\sqrt{\mathrm{Sc}}\right)+\\
&-e\exp(ct)\Big[\exp\left(-2\eta\sqrt{(M+c)t}\right)\cdot\mathrm{erfc}\left(\eta-\sqrt{(M+c)t}\right)+\\
&+\exp\left(2\eta\sqrt{(M+c)t}\right)\cdot\mathrm{erfc}\left(\eta+\sqrt{(M+c)t}\right)\Big]+\\
&-f\exp(dt)\Big[\exp\left(-2\eta\sqrt{(M+d)t}\right)\cdot\mathrm{erfc}\left(\eta-\sqrt{(M+d)t}\right)+\\
&+\exp\left(2\eta\sqrt{(M+d)t}\right)\cdot\mathrm{erfc}\left(\eta+\sqrt{(M+d)t}\right)\Big]+\\
&-e\Big[\exp\left(2\eta\sqrt{Rt}\right)\cdot\mathrm{erfc}\left(\eta\sqrt{pr}+\sqrt{at}\right)+\exp\left(-2\eta\sqrt{Rt}\right)\cdot\mathrm{erfc}\left(\eta\sqrt{pr}-\sqrt{at}\right)\Big]+\\
&-e\exp(ct)\Big[\exp\left(-2\eta\sqrt{\Pr(b+c)t}\right)\cdot\mathrm{erfc}\left(\eta\sqrt{\Pr}-\sqrt{(b+c)t}\right)+\\
&+\exp\left(2\eta\sqrt{\Pr(b+c)t}\right)\cdot\mathrm{erfc}\left(\eta\sqrt{\Pr}+\sqrt{(b+c)t}\right)\Big]+\\
&+f\exp(dt)\Big[\exp\left(-2\eta\sqrt{dt\,\mathrm{Sc}}\right)\cdot\mathrm{erfc}\left(\eta\sqrt{\mathrm{Sc}}-\sqrt{dt}\right)+\\
&+\exp\left(2\eta\sqrt{dt\mathrm{Sc}}\right)\cdot\mathrm{erfc}\left(\eta\sqrt{\mathrm{Sc}}+\sqrt{dt}\right)\Big],
\end{aligned} \tag{2.13}
$$

$$
\theta=\frac{1}{2}\Big[\exp\left(2\eta\sqrt{Rt}\right)\mathrm{erfc}\left(\eta\sqrt{\Pr}+\sqrt{at}\right)+\exp\left(-2\eta\sqrt{Rt}\right)\mathrm{erfc}\left(\eta\sqrt{\Pr}-\sqrt{at}\right)\Big], \tag{2.14}
$$

$$
C=\mathrm{erfc}\left(\eta\sqrt{\mathrm{Sc}}\right) \tag{2.15}
$$

where

$$
b=\frac{R}{\Pr},\quad c=\frac{M-R}{\Pr-1},\quad d=\frac{M}{\mathrm{Sc}-1},\quad e=\frac{\mathrm{Gr}}{2c(1-\Pr)},\quad f=\frac{\mathrm{Gc}}{2d(1-\mathrm{Sc})},\quad \eta=\frac{Y}{2\sqrt{t}}.
$$

3. Discussion of results

In order to get a physical view of the problem the numerical values of the velocity, tempaertaure and concentration fields are studied for different values of the thermal radiation parameter, magnetic field parameter, thermal Grashof number, mass Grashof number, Schmidt number and time. The purpose of the calculations given here is to assess the effects of the parameters $M, R, \mathrm{Gr}, \mathrm{Gc}, t$ and Sc upon the nature of the flow and transport. The solutions are in terms of an exponential and complementary error function.

The velocity profiles for different $(a = 0, 0.5, 1)$, $\mathrm{Gr} = \mathrm{Gc} = 5$ and $R = 2$ at time $t = 0.2$ are presented in Fig.1. It is observed that the velocity increases with increasing values of a. The effect of velocity for different values of the radiation parameter (R=*2*, *5*, *15*), M=*0.2*, Gr=*5*, Gc=*10*, a=*1*, Pr=*0.71*, Sc=*2.01* and t=*0.6* is shown in Fig.2. The trend shows that the velocity increases with decreasing the radiation parameter. This shows that the velocity decreases in the presence of high thermal radiation. The velocity profiles for different values of the magnetic field parameter (M=*0.2*, *5*, *10*), Gr=*5*, Gc=*10*, a=*1*, R=*15*, Sc=*2.01*, Pr=*0.71* and t=*0.2* are presented in Fig.3. It is clear that the velocity increases with decreasing the magnetic field parameter.

The effects of velocity profiles for different time (t=*0.2*, *0.4*, *0.6*), M=*0.2*, Gr=*5*, Gc=*10*, R=*5*, a=*1*, Sc=*2.01* and Pr=*0.71* are are shown in Fig.4. In this case, the velocity increases gradually with respect to time t. The effect of velocity for different values of the thermal Grashof number (Gr=*5,10*), mass Grashof number (Gc=*5*, *10*), R=*2*, a=*1*, M=*0.2*, Sc=*2.01*, Pr=*0.71* and t=*0.6* is presented in Fig.5. It is observed that the velocity increases with increasing the thermal Grashof number or mass Grashof number.

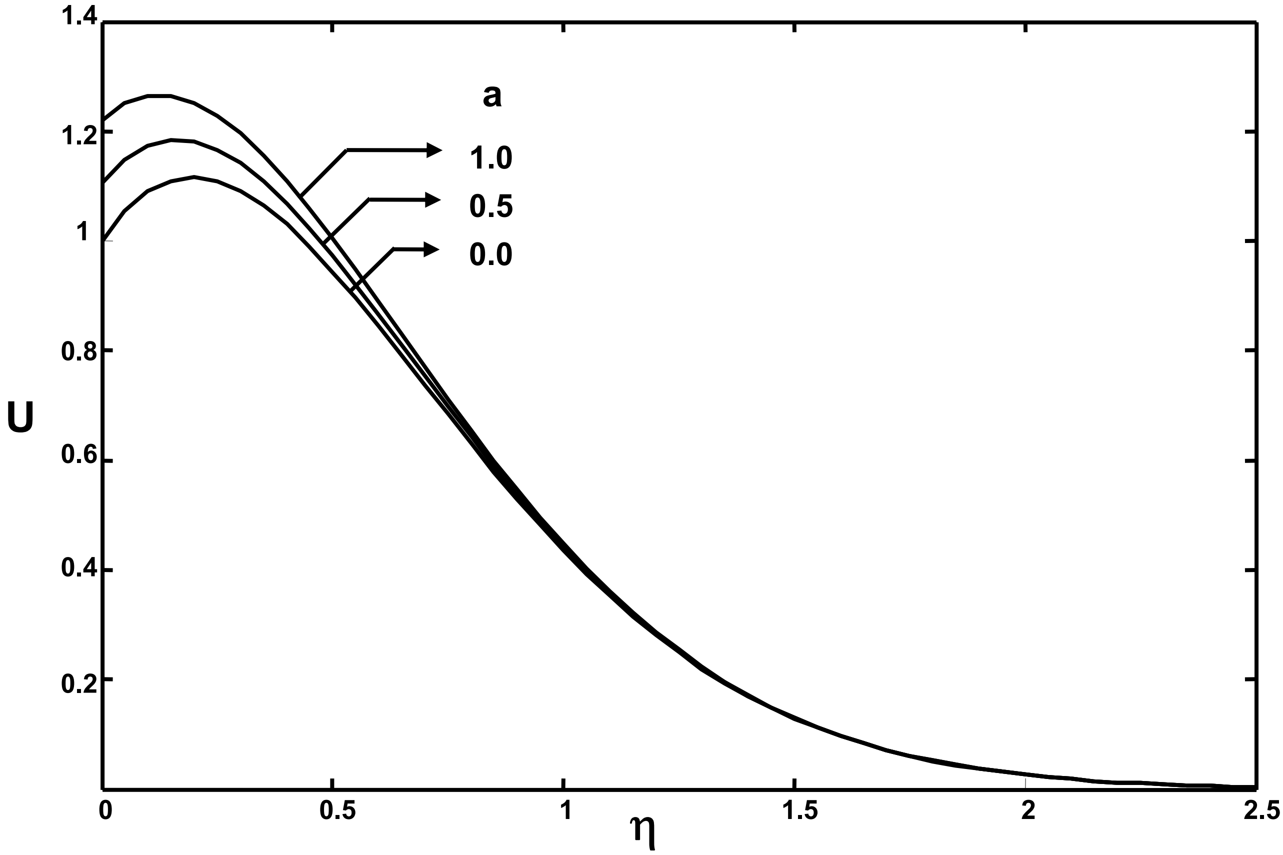

Fig.1. Velocity profiles for different *a*.

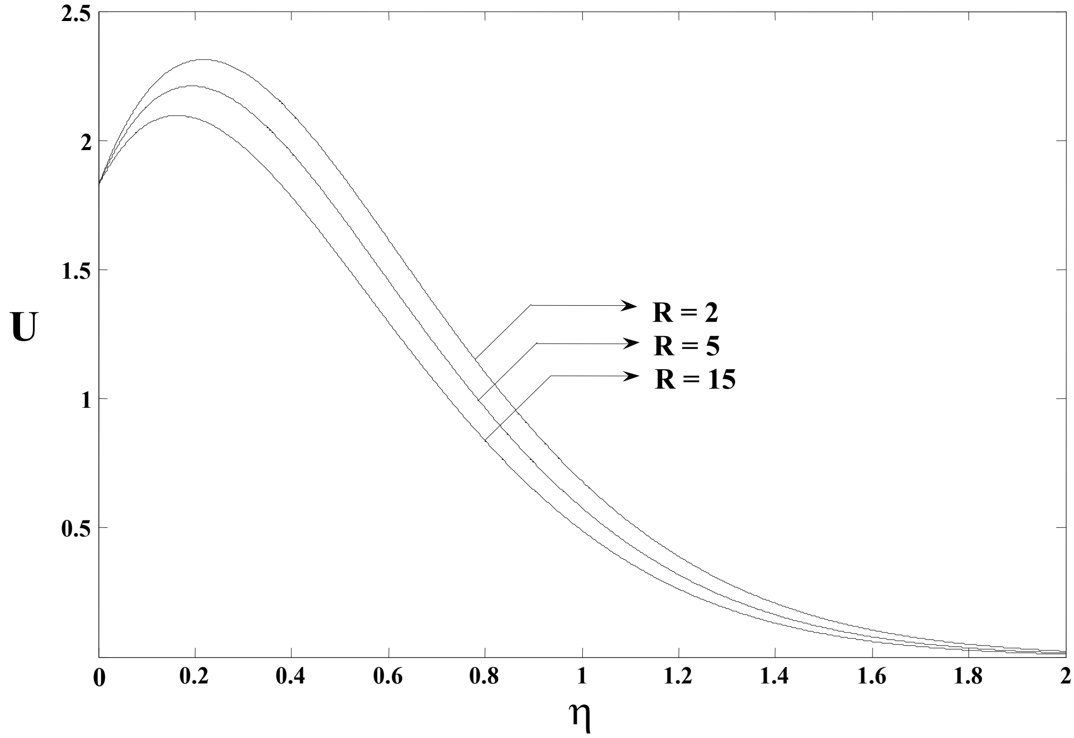

Fig.2. Velocity profiles for different values of *R*.

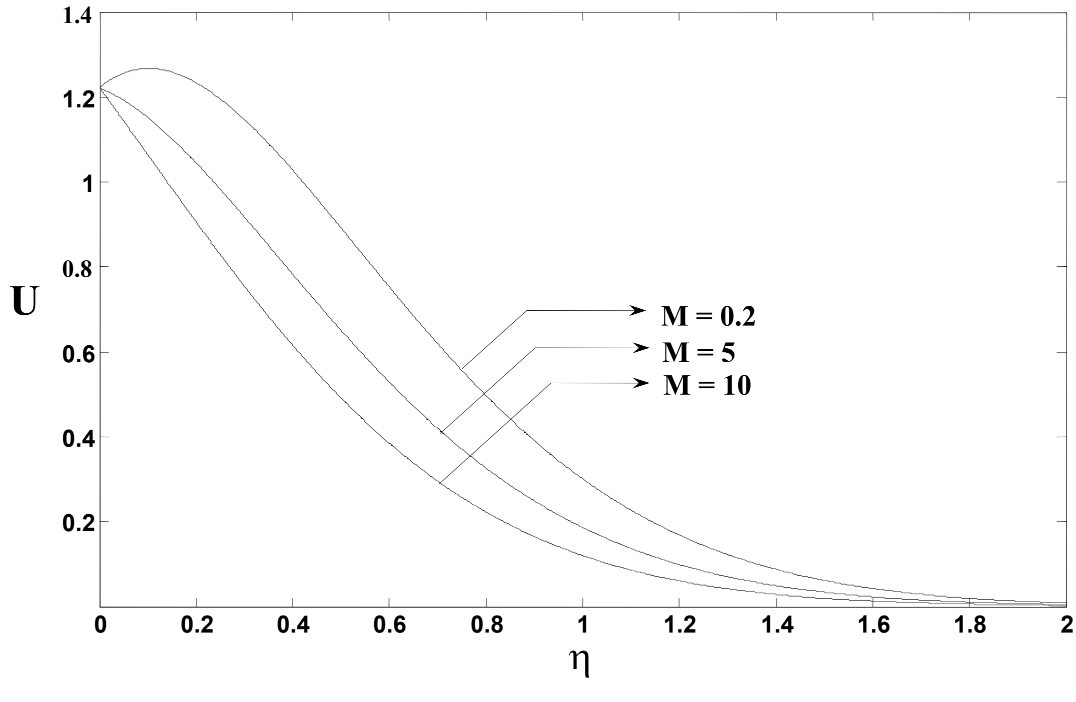

Fig.3. Velocity profiles for different values of *M*.

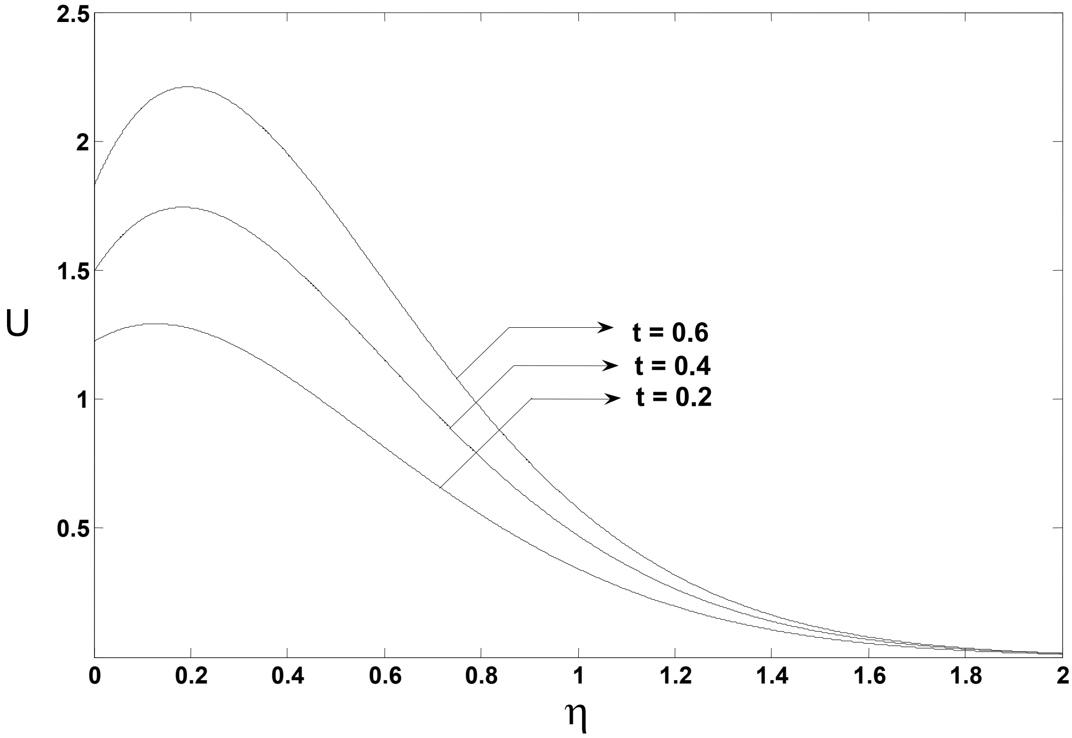

Fig.4. Velocity profiles for different values of *t*.

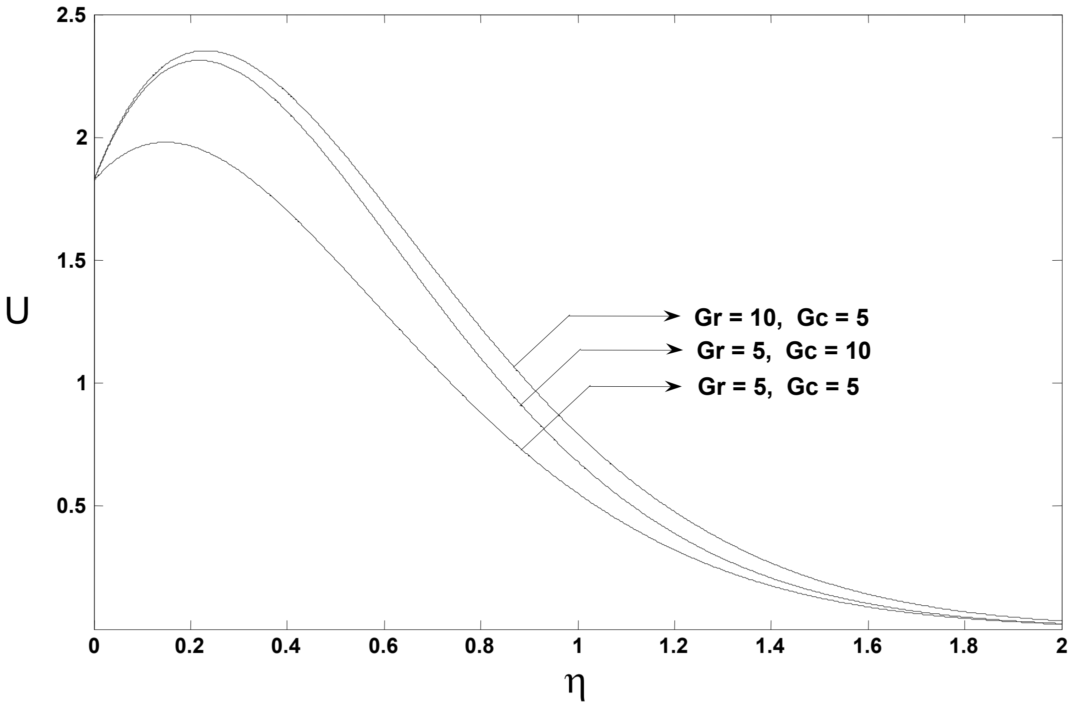

Fig.5. Velocity profiles for different values of Gr and Gc.

Figure 6 represents the effect of concentration profiles at time $t=1$ for different values of the Schmidt number (Sc=*0.16*, *0.3*, *0.6*, *2.01*). The effect of concentration is important in the concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

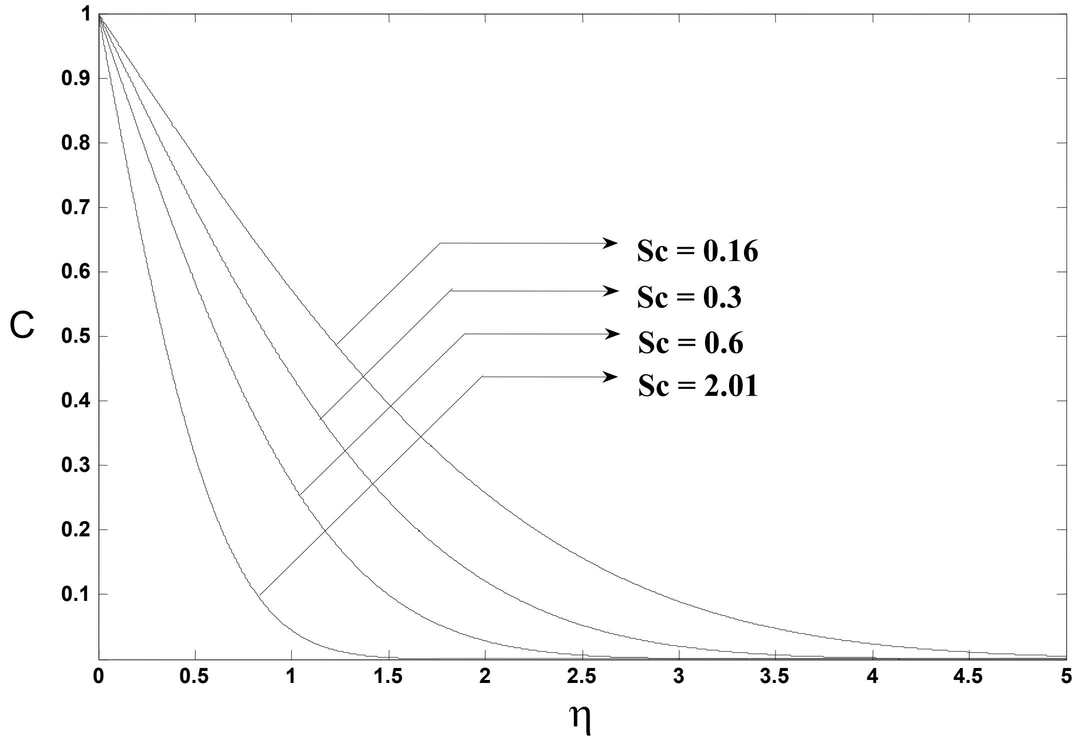

Fig.6. Concentration profiles for different values of Sc.

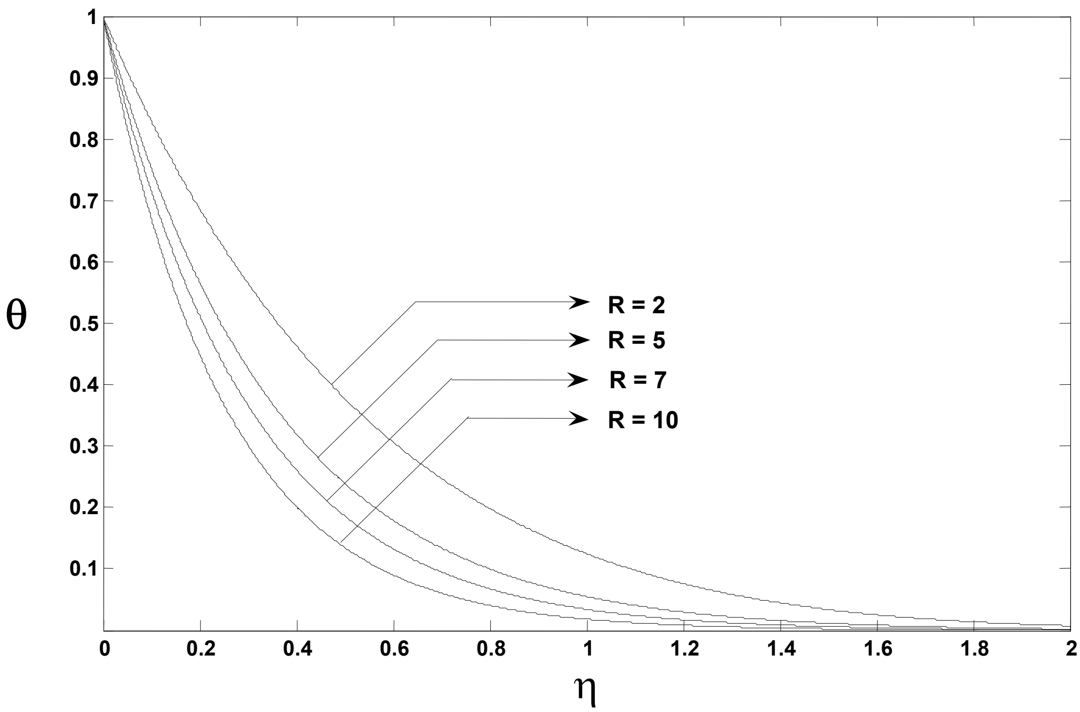

Fig.7. Temperature profiles for different values of *R*.

The temperature profiles are calculated for different values of the thermal radiation parameter(R=*2*, *5*, *7*, *10*) from Eq.(2.14) and these are shown in Fig.7. for air (Pr=*0.71*) at t=*0.4*. The effect of the thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing the radiation parameter.

4. Conclusion

MHD and thermal radiation effects on flow past an exponentially accelearted infinite isothermal vertical plate with uniform mass diffusion were studied. The dimensionless equations are solved using the Laplace transform technique. The effect of velocity, temperature and concentration for different parameters such as M, R, Gr, Gc, Sc and t are studied. The study yields the following results

(i) The velocity increases with decreasing the magnetic field parameter M and thermal radiation parameter R. The trend is just reversed with respect to time t.
(ii) The plate temperature decreases due to high thermal radiation.
(iii) It is observed that the concentration increases with decreasing the Schmidt number.

Nomenclature

a – constant
a^* – absorbtion constants
C – dimensionless concentration
C' – species concentration in the fluid
C_p – specific heat at constant pressure
C'_w – concentration of the plate
C'_∞ – concentration in the fluid far away from the plate
D – mass diffusion coefficient
Gc – mass Grashof number
Gr – thermal Grashof number
g – acceleration due to gravity
k – thermal conductivity
M – magnetic field parmeter
Pr – Prandtl number
R – thermal radiation parameter
Sc – Schmidt number
T – temperature of the fluid near the plate
T_w – temperature of the plate
T_∞ – temperature of the fluid far away from the plate
t' – time
t – dimensionless time
U – dimensionless velocity
u – velocity of the fluid in the x'-direction
u_0 – velocity of the plate
Y – dimensionless coordinate axis normal to the plate
y – coordinate axis normal to the plate
α – thermal diffusivity
β – volumetric coefficient of thermal expansion
β^* – volumetric coefficient of expansion with concentration
η – similarity parameter
θ – dimmensionless temperature
μ – cofficient of viscosity
ν – kinematic viscosity

ρ – density of the fluid
τ – dimensionless skin-friction
erfc – complementary error function

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