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RADIATION EFFECTS ON OSCILLATING VERTICAL PLATE WITH UNIFORM HEAT AND MASS FLUX

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Thermal radiation effects on flow past an impulsively started infinite vertical oscillating plate with uniform heat and mass flux is studied. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The dimensionless governing equations are solved using the Laplace-transform technique. The velocity, temperature and concentration are studied for different physical parameters such as the radiation parameter, phase angle, Schmidt number and time. The variation of the skin-friction for different values of the parameters is also shown in a table.

Key words: oscillating vertical plate, radiation, heat and mass flux.

1. Introduction

Radiative convective flows are encountered in countless industrial and environment processes, e.g., heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Radiative heat transfer plays an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications.

The first exact solution of the Navier-stokes equation was given by Stokes (1851) who studied the flow of a viscous incompressible fluid past an infinite horizontal plate oscillating in its own plane in an infinite mass of stationary fluid. Such a flow past an infinite vertical plate oscillating in its own plane was first studied by Soundalgekar (1979) for an isothermal plate. Mansour (1990) studied the interaction of free convection with thermal radiation of the oscillatory flow past a vertical plate. Zhang *et al*. (2004) studied the free convection effects on a heated vertical plate subjected to a periodic oscillation. The effects of thermal radiation on flow past an oscillating plate with variable temperature were studied by Pathak *et al*. (2006). Free convection effects on a vertical oscillating porous plate with constant heating were studied by Toki (2009). Chandrakala and Bhaskar (2009) studied the effects of thermal radiation on the flow past an infinite vertical oscillating isothermal plate in the presence of a transversely applied magnetic field. Chandrakala (2011) studied the effects of thermal radiation on the flow past an infinite vertical oscillating plate with uniform heat flux.

The unsteady flow past a moving infinite vertical oscillating plate in the presence of radiation with uniform heat and mass flux has not received much attention from contemporary researchers. It is now proposed to study the thermal radiation effects on the oscillatory flow past a vertical plate with uniform heat and mass flux.

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2. Mathematical formula

Consider the flow of an incompressible viscous radiating fluid past an impulsively started infinite vertical plate with uniform heat and mass flux. The x' - axis is taken along the plate in the vertical direction and the y' - axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature and concentration in a stationary condition. At time $t' > 0$, the plate is given an impulsive motion in the vertical direction against the gravitational field with constant velocity u_0. At the same time, the heat is supplied from the plate to the fluid at a uniform rate and the concentration level near the plate is also raised at a uniform rate. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations

$$\frac{\partial u'}{\partial t'} = g\beta\left(T' - T'_\infty\right) + v\frac{\partial^2 u'}{\partial y'^2}, \tag{2.1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'}, \tag{2.2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2}, \tag{2.3}$$

with the following initial and boundary conditions

$$\begin{aligned} &t' \le 0: \quad u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \qquad \text{for all} \quad y' \\ &t' > 0: \quad u' = u_0 \cos\omega t, \quad \frac{\partial T'}{\partial y} = -\frac{q}{k}, \quad \frac{\partial C'}{\partial y} = -\frac{j''}{D} \qquad \text{at} \quad y' = 0 \\ &u' = 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty \qquad \text{as} \quad y' \to \infty \end{aligned} \tag{2.4}$$

The local radiant for the case of an optically thin gray gas is expressed by

$$q_r = -\frac{4\sigma}{3\kappa^*}\frac{\partial T'^4}{\partial y'}. \tag{2.5}$$

It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3_\infty T' - 3T^4_\infty. \tag{2.6}$$

By using Eqs (2.5) and (2.6), Eq. (2.2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T_\infty'^3}{3\kappa^*} \frac{\partial^2 T'}{\partial y'^2} . \tag{2.7}$$

Introduction of the following dimensionless quantities

$$u = \frac{u'}{u_0}, \qquad t = \frac{t' u_0^2}{\nu}, \qquad y = \frac{y' u_0}{\nu}, \qquad \theta = \frac{T' - T'_\infty}{\left(q\nu / k u_0\right)}$$

$$\mathrm{Gr} = \frac{g\beta q \nu^2}{k u_0^4}, \qquad \mathrm{Pr} = \frac{\mu C_p}{k}, \qquad N = \frac{\kappa^* k}{4\sigma T_\infty'^3} \tag{2.8}$$

$$C = \frac{C' - C'_\infty}{j''\nu/(D u_0)}, \qquad \mathrm{Gc} = \frac{g\beta^* \nu^2 j''}{D u_0^4}$$

in Eq.(2.1) to Eq.(2.7) leads to

$$\frac{\partial u}{\partial t} = \mathrm{Gr}\,\theta + \frac{\partial^2 u}{\partial y^2}\sqrt{a^2 + b^2} , \tag{2.9}$$

$$3N\,\mathrm{Pr}\,\frac{\partial \theta}{\partial t} = (3N + \mathrm{Pr})\,\frac{\partial^2 \theta}{\partial y^2} , \tag{2.10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{\mathrm{Sc}} \frac{\partial^2 C}{\partial y^2} . \tag{2.11}$$

The initial and boundary conditions in a non-dimensionless form are

$$\begin{aligned} & u = 0, \quad \theta = 0, \quad C = 0 \qquad \text{for all} \quad y,\ t' \le 0, \\ & t' > 0 : u = u_0 \cos \omega t , \qquad \frac{\partial \theta}{\partial y} = -1, \quad \frac{\partial C}{\partial y} = -1 \qquad \text{at} \quad y = 0, \\ & u = 0, \quad \theta \to 0, \quad C \to 0 \qquad \text{as} \quad y \to \infty. \end{aligned} \tag{2.12}$$

Equations (2.9), (2.10) and (2.11), subject to the boundary conditions (2.12), are solved by the usual Laplace-transform technique and the solutions are derived as follows

$$\theta = 2\sqrt{t}\left[\frac{\exp\left(-\eta^2 a\right)}{\sqrt{\pi}} - \eta\sqrt{a}\,\mathrm{erfc}\left(\eta\sqrt{a}\right)\right], \tag{2.13}$$

$$C = 2\sqrt{t}\left[\frac{\exp\left(-\eta^2 \mathrm{Sc}\right)}{\sqrt{\pi}} - \eta\sqrt{\mathrm{Sc}}\,\mathrm{erfc}\left(\eta\sqrt{\mathrm{Sc}}\right)\right], \tag{2.14}$$

$$u=\frac{\exp(-i\omega t)}{4}\left[\exp\left(-2\eta\sqrt{-i\omega t}\ \operatorname{erfc}\left(\eta-\sqrt{-i\omega t}\right)\right)+\exp\left(2\eta\sqrt{-i\omega t}\ \operatorname{erfc}\left(\eta+\sqrt{-i\omega t}\right)\right)\right]+$$
$$+\frac{\exp(i\omega t)}{4}\left[\exp\left(-2\eta\sqrt{i\omega t}\ \operatorname{erfc}\left(\eta-\sqrt{i\omega t}\right)\right)+\exp\left(2\eta\sqrt{i\omega t}\ \operatorname{erfc}\left(\eta+\sqrt{i\omega t}\right)\right)\right]+$$
$$-\frac{\mathrm{Gr}\,t^{3/2}}{3\sqrt{a}\,(1-a)}\left[\frac{4}{\sqrt{\pi}}\left(1+\eta^2\right)\exp\left(-\eta^2\right)-\eta\left(6+4\eta^2\right)\operatorname{erfc}(\eta)\right]+$$
$$-\frac{\mathrm{Gc}\,t^{3/2}}{3(1-\mathrm{Sc})}\left[\frac{4}{\sqrt{\pi}}\left(1+\eta^2\right)\exp\left(-\eta^2\right)-\eta\left(6+4\eta^2\right)\operatorname{erfc}(\eta)\right]+$$
$$+\frac{\mathrm{Gr}\,t^{3/2}}{3\sqrt{a}\,(1-a)}\left[\frac{4}{\sqrt{\pi}}\left(1+\eta^2 a\right)\exp\left(-\eta^2 a\right)-\eta\sqrt{a}\left(6+4\eta^2 a\right)\operatorname{erfc}\left(\eta\sqrt{a}\right)\right]+$$
$$+\frac{Gc\,t^{3/2}}{3(1-\mathrm{Sc})}\left[\frac{4}{\sqrt{\pi}}\left(1+\eta^2\mathrm{Sc}\right)\exp\left(-\eta\mathrm{Sc}^2\right)-\eta\sqrt{\mathrm{Sc}}\left(6+4\eta^2\mathrm{Sc}\right)\operatorname{erfc}\left(\eta\sqrt{\mathrm{Sc}}\right)\right] \tag{2.15}$$

where $a=\dfrac{3N\,\mathrm{Pr}}{3N+4}$ and $\eta=\dfrac{y}{2\sqrt{t}}$.

3. Discussion of results

The numerical values of the velocity, temperature and wall concentration are computed for different parameters such as the radiation parameter, Grashof number, Schmidt number, time and Pr=*0.71*. The purpose of the calculations given here is to assess the effects of the parameters *N*, *M*, *t* and ωt.

The velocity profiles for different values of the radiation parameter (*N*=*2,30*) are shown in Fig.1. It is observed that the velocity increases with a decreasing radiation parameter. This shows that velocity decreases in the presence of high thermal radiation.

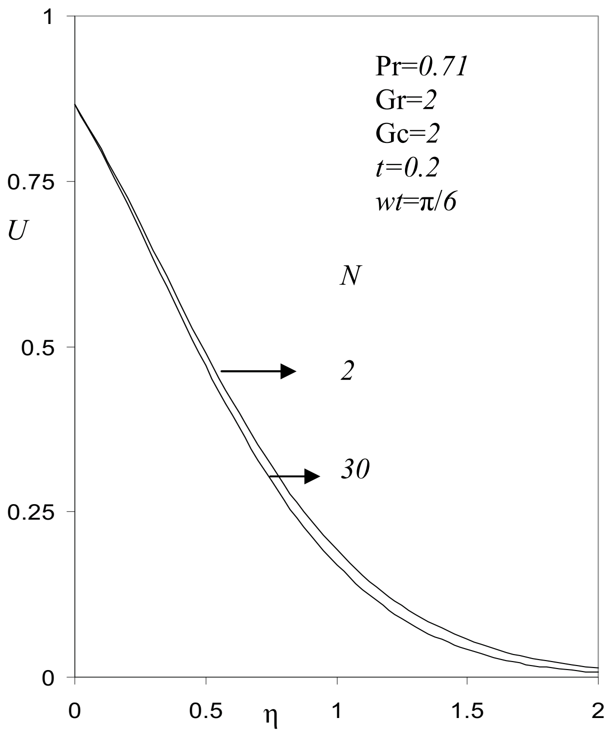

Fig.1. Velocity profiles for different *N*.

The velocity profiles for different values of the time ($t=0.2$, 0.4, 0.6) are shown in Fig.2. This shows that velocity increases with the increasing values of time t.

In Fig.3, the velocity profiles are shown for different values of ωt. It is observed that velocity increases with decreasing phase angle ωt.

The temperature profiles are calculated for different values of the thermal radiation parameter ($N=2$, 5, 30) and are shown in Fig.4. It is observed that temperature increases with a decreasing radiation parameter.

The temperature profiles are calculated for different values of the Prandtl number $\mathrm{Pr}=0.71$ and $\mathrm{Pr}=7.0$ and are shown in Fig.5. The effect of the Prandtl number is important in temperature profiles. It is observed that temperature increases in the presence of air.

Figure 6 represents, the effect of concentration profiles for different Schmidt number ($\mathrm{Sc}=0.16$, 0.3, 0.6, 2.01) and time $t=(0.2,0.4)$. The effect of Schmidt number is important in concentration field. It is observed that the wall concentration increases with decreasing values of the Schmidt parameter. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity.

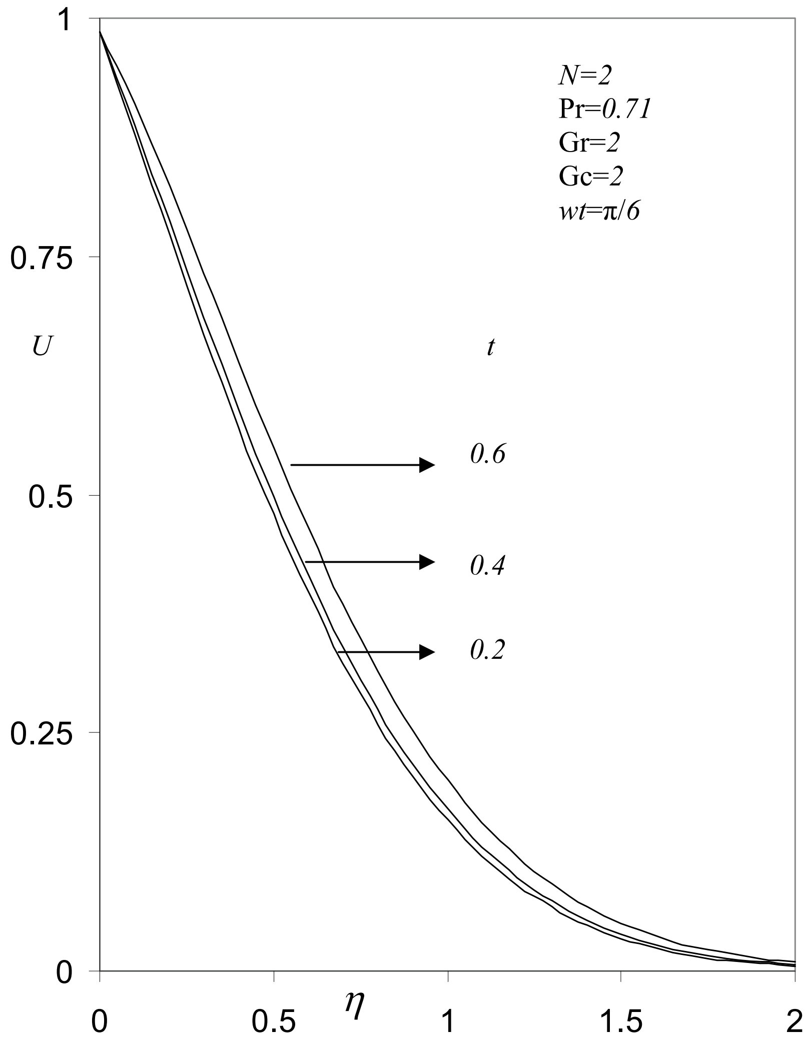

Fig.2. Velocity profiles for different t.

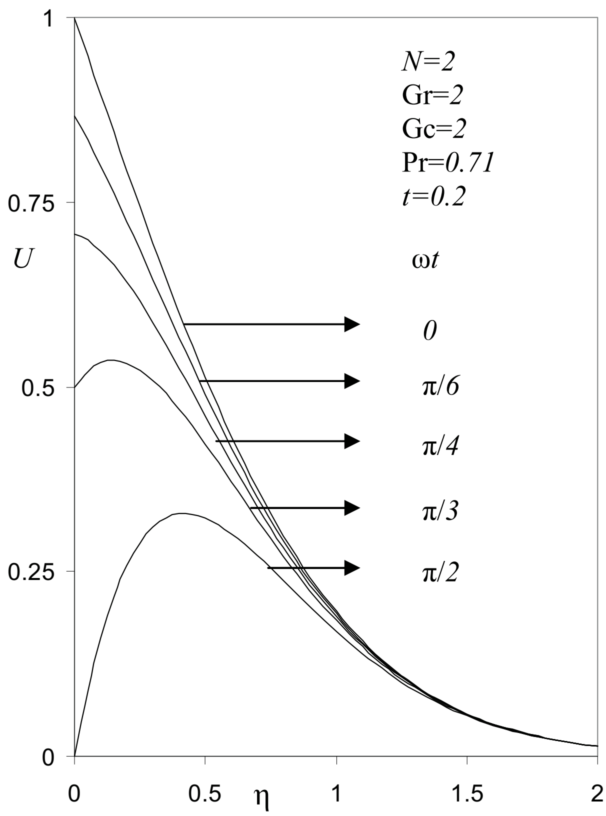

Fig.3. Velocity profiles for different *wt.*

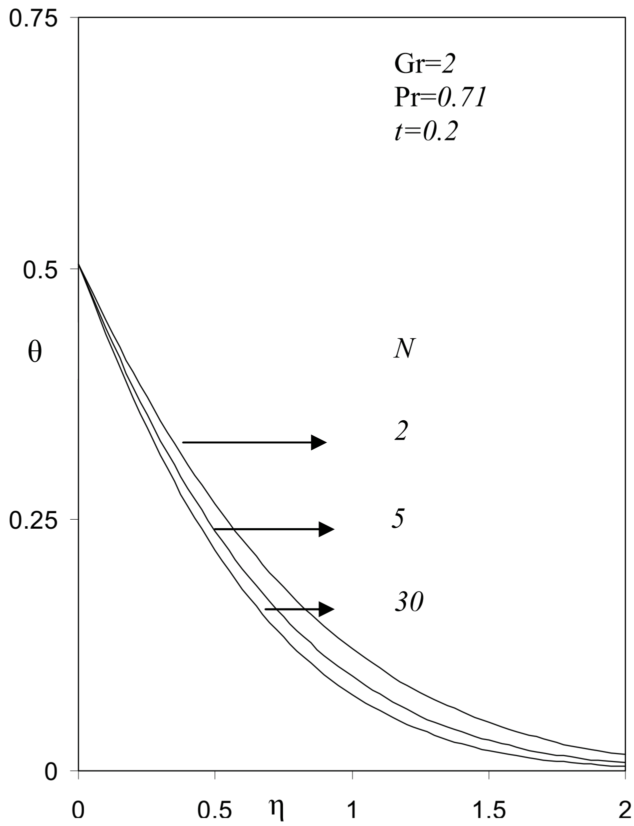

Fig.4. Temperature profiles for different *N*.

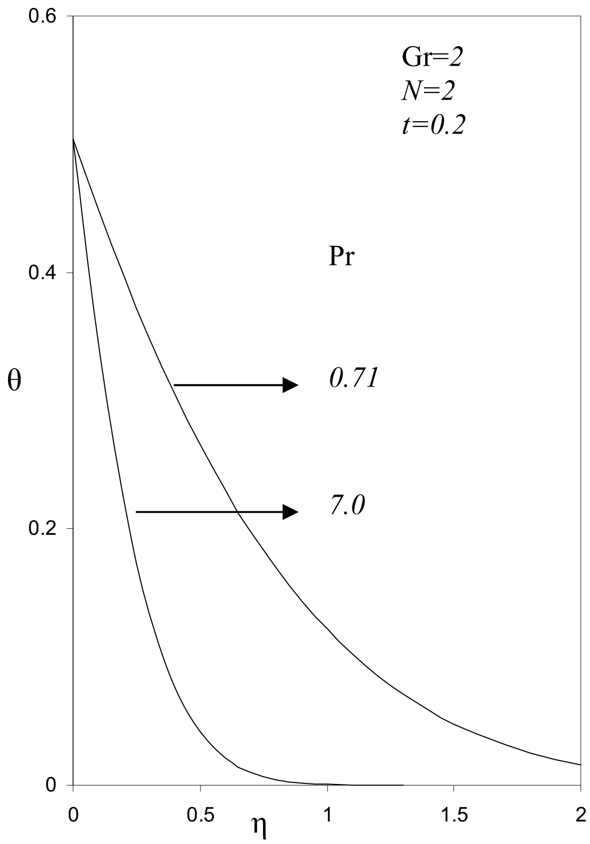

Fig.5. Temperature profiles for different Pr.

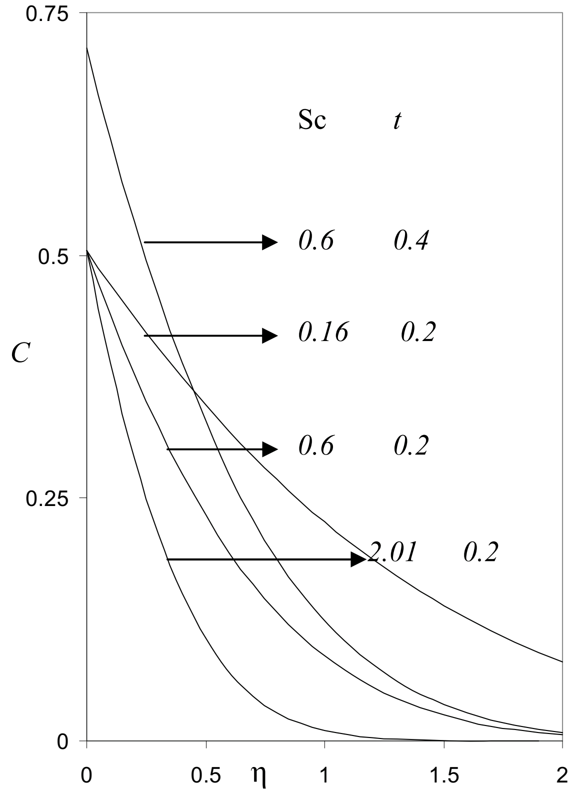

Fig.6. Concentration profiles for different Sc and *t*.

From the velocity field, we now study the skin-friction. It is given by

$$\tau = -\left(\frac{du}{dy}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{du}{d\eta}\right)_{\eta=0}. \tag{3.1}$$

Hence, from Eqs (2.15) and (3.1)

$$\tau = \frac{1}{\sqrt{\pi t}}\left[\frac{\exp(-i\omega t)}{2}\left(1+\sqrt{-i\omega\pi t}\,\mathrm{erf}(\sqrt{-i\omega t})\right)+\right.$$

$$\left.+\frac{\exp(i\omega t)}{2}\left(1+\sqrt{i\omega\pi t}\,\mathrm{erf}(\sqrt{i\omega t}\right) - \frac{\mathrm{Gr}\,t^{3/2}\sqrt{\pi}}{\sqrt{a}\left(1+\sqrt{a}\right)} - \frac{\mathrm{Gc}\,t^{3/2}\sqrt{\pi}}{\left(1+\sqrt{\mathrm{Sc}}\right)}\right]. \tag{3.2}$$

The numerical values of τ are presented in Tab.1. It is observed from this table that an increase in the Schmidt number parameter or radiation leads to rise in the value of the skin-friction. As time advances the value of skin-friction decreases. It is also observed that skin-friction decreases with increasing values of the thermal Grashof number or mass Grashof number.

Table 1. Values of the non-dimensional skin-friction.

N	Gr	Gc	Sc	t	$\omega t = 0$	$\omega t = \pi/6$	$\omega t = \pi/4$	$\omega t = \pi/3$
0.2	2	5	0.6	0.2	1.1912	-0.4950	-2.1043	-3.9863
2	2	5	0.6	0.2	1.2197	-0.4665	-2.0759	-3.9578
2	2	5	0.3	0.2	1.2160	-0.4702	-2.0796	-3.9615
2	2	5	0.6	0.4	.06556	-0.5367	-1.6747	-3.0054
2	5	5	0.6	0.2	1.1949	-0.4914	-2.1007	-3.9827
2	2	2	0.6	0.2	1.2349	-0.4514	-2.0607	-3.9427
5	2	5	0.6	0.2	1.2227	-0.4636	-2.0729	-3.9549
10	2	5	0.6	0.2	1.2237	-0.4625	-2.0719	-3.9538

4. Conclusions

An exact analysis is performed to study the thermal radiation effects on the flow past an impulsively started infinite oscillating vertical plate with uniform heat and mass flux. The dimensionless governing equations are solved by the usual Laplace-transform technique. The velocity, temperature and wall concentration for different physical parameters are studied graphically. The conclusions of the study are as follows.

(I) The presence of radiation causes a fall in the velocity and temperature.
(II) Velocity increases with the decreasing phase angle ωt.
(III) As time increases, it is found that there is a rise in velocity.

Nomenclature

A – constant
C' – species concentration in the fluid
$C_w{}'$ – concentration of the plate
C'_∞ – concentration in the fluid far away from the plate
C – dimensionless concentration
C_p – specific heat at constant pressure
D – mass diffusion coefficient
erfc – complementary error function
Gc – mass Grashof number
Gr – thermal Grashof number
g – acceleration due to gravity
j'' – mass flux per unit area at the plate
k – thermal conductivity of the fluid
k^* – mean absorption coefficient
N – radiation parameter
Pr – Prandtl number
p – pressure
q_r – radiative heat flux in the y-direction
Sc – Schmidt number
T' – temperature of the fluid near the plate
T'_w – temperature of the plate
T'_∞ – temperature of the fluid far away from the plate
t' – time
t – dimensionless time
u – dimensionless velocity
u' – velocity of the fluid in the x-direction
u_0 – velocity of the plate
y – coordinate axis normal to the plate
y' – dimensionless coordinate axis normal to the plate
β – volumetric coefficient of thermal expansion
β^* – volumetric coefficient of expansion with concentration
η – similarity parameter
μ – coefficient of viscosity
ν – kinematic viscosity
ρ – density
$\bar{\tau}$ – dimensionless average skin-friction
θ – dimensionless temperature
η – similarity parameter
ωt – phase angle

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