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FLOW AND HEAT TRANSFER AT A NONLINEARLY SHRINKING POROUS SHEET:THE CASE OF ASYMPTOTICALLY LARGE POWER-LAW SHRINKING RATES

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The boundary layer flow and heat transfer of a viscous fluid over a nonlinear permeable shrinking sheet in a thermally stratified environment is considered. The sheet is assumed to shrink in its own plane with an arbitrary power-law velocity proportional to the distance from the stagnation point. The governing differential equations are first transformed into ordinary differential equations by introducing a new similarity transformation. This is different from the transform commonly used in the literature in that it permits numerical solutions even for asymptotically large values of the power-law index, *m*. The coupled non-linear boundary value problem is solved numerically by an implicit finite difference scheme known as the Keller- Box method. Numerical computations are performed for a wide variety of power-law parameters ($1 < m < 100{,}000$) so as to capture the effects of the thermally stratified environment on the velocity and temperature fields. The numerical solutions are presented through a number of graphs and tables. Numerical results for the skin-friction coefficient and the Nusselt number are tabulated for various values of the pertinent parameters.

Key words: boundary layer flow, porous shrinking sheet, Keller-Box method, similarity solutions, heat transfer.

1. Introduction

The study of a two-dimensional boundary layer flow and heat transfer induced by a stretching boundary has received considerable interest because of its extensive applications in manufacturing processes (Altan and Gegel, 1979; Fischer, 1976; Klein and Tadmor, 1970). Such flows have promising applications in industries, for example, in the extrusion of polymer sheet from a die or in the drawing of plastic films.

During the manufacture of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. The mechanical properties of the final product strictly depend on the stretching and the cooling rates in the process. In these situations, it is very important to control the drag and the heat flux for better product quality. The physical situation was recognized as a backward boundary layer problem by Sakiadis (1961). He was the first, among others, to investigate the flow behavior for this class of boundary layer problems. In his pioneering paper, solutions were obtained to the boundary layer flows on

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continuous moving surfaces which are substantially different from those of boundary layer flows on stationary surfaces. The thermal behavior of the problem was studied by Erickson *et al*. (1966) using finite difference and integral methods, and experimentally verified by Tsou *et al*. (1967). Crane (1970) extended the work of Sakiadis (1961) to the flow caused by an elastic sheet moving in its own plane with a velocity varying linearly with the distance from a fixed point. Thereafter various aspects of the above boundary layer problem on continuous moving surface were considered by many researchers (Grubka and Bobba, 1985), (Vleggar, 1977), (Soundalgekar and Murthy, 1980), (Gupta and Gupta, 1977), (Chen and Char, 1988), (Ali, 1994). A new solution branch of both impermeable and permeable stretching sheet was found recently by Liao, 2005; 2007). This indicates that multiple solutions for the stretching surface problems are possible under certain conditions.

All the above investigators restricted their analyses to flow induced by a linear stretching sheet. However, (Gupta and Gupta, 1977) have noted that stretching of the sheet may not necessarily be linear (they find applications in polymer and electrochemical industries); the concept of linear stretching is further extended to non-linear stretching sheet $u_w = bx^n$, $n > 0$ by many authors. To mention a few, (Vajravelu, 2001) studied the boundary-layer flow of a viscous fluid over a nonlinearly stretching sheet and obtained the numerical solution. Cortell (2007) extended the work of Vajravelu (2001) to obtain more realistic solutions on the flow and heat transfer over a non-isothermal stretching sheet in the presence of viscous dissipation. Sajid *et al*. (2008) analyzed the axisymmetric flow over a non-linearly stretching sheet. Mathematical properties of such physical systems have been recently considered in Akyildiz *et al*., (2010); Van Gorder and Vajravelu, (2010a; 2010b; 2011); Mahapatra *et al*, (2010).

The physical situation of a stretching sheet is one of the possible cases. Another physical phenomenon is the flow of an incompressible viscous fluid over a shrinking sheet. Such a situation occurs in the flow over a rising shrinking balloon. From the consideration of continuity, Crane's stretching sheet solution induces far field suction toward the sheet, while flow over a shrinking sheet would give rise to a velocity away from the sheet. From a physical point of view, vorticity generated at the shrinking sheet is not confined within a boundary layer and a steady flow is not possible unless adequate suction is applied at the surface. A paper published by Miklavcic and Wang (2006) investigates two-dimensional and axisymmetric viscous flows induced by a shrinking sheet in the presence of uniform surface suction. Fang (2008) analyzed the boundary layer flow of a continuously shrinking sheet with a power-law surface velocity. The shrinking sheet problem was extended to other types of fluids by Hayat *et al*. (2007), Sajid *et al*. (2008). For the flow induced by a shrinking sheet, it is essentially a backward flow discussed by Glodstein (1965); for a backward flow configuration the fluid loses any memory of the perturbation induced by the leading ledge, say the slot.

This flow has quite distinct physical phenomena from the forward stretching flow. Available literature on the flow over a stretching/ shrinking sheet (Sajid and Hayat, 2009; Aman and Ishak, 2010; Cortell, 2007; Hayat *et al*., 2007; Sajid *et al*., 2008) reveals that not much work has been carried out for viscous flow and heat transfer over a non-linear permeable shrinking sheet in a thermally stratified environment.

In the present paper, we extend the shrinking sheet problem to a more general situation with a power law velocity of the permeable sheet shrinking into the slot. This is a generalization of Henkes and Hoogendoorn (1989) to the study of the viscous fluid flow and heat transfer with an arbitrary power-law permeable shrinking sheet. Here the momentum and energy equations are coupled nonlinear partial differential equations. These coupled nonlinear partial differential equations are reduced to couple nonlinear ordinary differential equations by a similarity transformation and are solved numerically by an implicit finite difference method known as the Keller box method. A point of novelty here is that our similarity transform differs from those commonly found in the literature for nonlinear stretching or shrinking sheets (see, for instance Ali, 1994; Liao, 2005; 2007) as our transformation permits numerical solutions for arbitrarily large values of the power-law shrinking parameter. Thus, we are able to obtain similarity solutions for a wider variety of power-law shrinking situations than previously possible.

2. Mathematical formulation

Consider a laminar steady two-dimensional viscous flow over a continuously shrinking sheet in a quiescent fluid. The sheet shrinks in its own plane with a velocity proportional to the power of distance from the origin. The sheet shrinking-velocity is assumed to be $-u_w(x)$ where $u_w(x)$ is a positive function for all values of x and the mass transfer velocity at the wall is $v_w = v_w(x)$. The x-axis runs along the shrinking surface in the direction opposite to the sheet motion and y-axis is perpendicular to it. The governing momentum and energy equations based on the usual boundary layer assumption can be written as (in the absence of viscous dissipation)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}, \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{2.3}$$

where u and v are the velocity components in the x and y directions, respectively. T is the temperature, p the fluid pressure, ν the kinematic viscosity, and α is the thermal diffusivity of the fluid. In addition, since we neglected the dissipation term in the energy equation, the current analysis is applicable to low Eckert number flows. The appropriate boundary conditions for the problem are

$$u=-u_w(x),\quad v=v_w(x),\quad T=T_w(x)\quad \text{at}\quad y=0, \tag{2.4a}$$

$$u=0\qquad T=T_\infty(x)\qquad \text{as}\qquad y\to\infty. \tag{2.4b}$$

In Eqs (2.4) the negative sign indicates the shrinking sheet and $v_w(x)$ is the mass flux velocity, with $v_w(x)<0$ for suction and $v_w(x)>0$ for blowing or injection, respectively. The subscript w denotes the conditions at the wall. Here, $u_w(x)$, $T_w(x)$ and $T_\infty(x)$ are functions of x (and are assumed to vary in powers of x, the distance from the slot) and are defined as follows (for details see Henkes and Hoogendoorn, 1989; Kulkarni *et al.*, 1987)

$$u_w(x)=U_w(Mx+N)^m,\quad T_w(x)=(n+1)\Delta T(Mx+N)^m+T_c,$$
$$T_\infty(x)=n\Delta T(Mx+N)^m+T_c \tag{2.5}$$

where T_c is a constant, m is a power law exponent parameter for the shrinking sheet and is positive , n is the wall temperature parameter describing the environment temperature for $n=0$ and fixed wall temperature for $n=-1$, M and N are (positive and non-negative, respectively) constants. The temperature field can be rewritten as

$$T=\left(n+\theta(\eta)\right)\Delta T\,\xi^{m}+T_c, \tag{2.6}$$

and the transformed coordinates in the above expression are

$$\xi = Mx+N, \quad \xi\geq 0 \quad \text{and} \quad \eta=\left(\frac{U_w M}{\nu}\right)^{\frac{1}{2}}\xi^{\frac{m-1}{2}}\sqrt{m}\quad y. \tag{2.7}$$

A stream function is introduced as

$$\psi=\left(\frac{\nu\,U_w}{M}\right)^{\frac{1}{2}}\xi\,\sqrt{m}\;f(\eta), \tag{2.8}$$

which defines the velocity components u and v as

$$u=U_w\xi^{m}\;m\;f'(\eta), \qquad v=-\frac{1}{2}\left(\frac{\nu\,U_w}{M}\right)^{\frac{1}{2}}\xi^{\frac{m-1}{2}}\;m\;M\left\{(m+1)f(\eta)+(m-1)f'(\eta)\right\}, \tag{2.9}$$

so that we can write

$$v_w(x)=-\frac{m+1}{2}\left(\frac{\nu\,U_w}{M}\right)^{\frac{1}{2}}\xi^{\frac{m-1}{2}}\;M\,m\;f_w$$

where f_w is a constant ($f_w>0$ for suction and $f_w<0$ for injection). Substitution of these transformation expressions into Eqs (2.2)-(2.3) yields the following coupled, nonlinear ordinary differential Eqs for f and θ

$$f'''+\left[\frac{1}{2}\left(1+\frac{1}{m}\right)f\,f''-f'^{2}\right]=0, \tag{2.10}$$

$$\theta''+\Pr\left[\frac{1}{2}\left(1+\frac{1}{m}\right)f\;\;\theta'-(n+\theta)f'\right]=0, \tag{2.11}$$

subject to the boundary conditions

$$f(0)=f_w, \quad f'(0)=-1, \quad \theta(0)=1 \tag{2.12a}$$

$$f'(\eta)\to 0, \quad \theta(\eta)\to 0 \quad \text{as} \quad \eta\to\infty \tag{2.12b}$$

where f_w is the lateral mass transfer parameter showing the strength of the mass at the shrinking sheet.

From these equations, it is clear that the solutions to Eqs (2.10)–(2.12) should be amenable to numerical analysis for any positive value of *m*. This is in contrast to the standard transformations for nonlinearly stretching sheets in which the dependence on *m* is non-vanishing as *m* grows large (again, see Eqs (2.13)-(3.2).

We have that $n = 0$ for the non-stratified environment and $n = -1$ for the fixed wall temperature distribution. The environment is stably stratified for $dT_\infty / dx > 0$ or $mMn > 0$. Hence, if m and n are of the same sign the environment is stably stratified. The physical quantities of interest are the skin friction coefficient C_f and the Nusselt number Nu_ξ, which are defined by

$$C_f = \frac{\tau_w}{\rho u_w^2} = \frac{1}{\rho u_w^2}\mu\left(\frac{\partial u}{\partial y}\right)_{at\ y=0}, \quad \mathrm{Nu}_\xi = \frac{\xi\, q_w}{k(T_w - T_\infty)} = \frac{\xi}{k(T_w - T_\infty)}\left(-k\frac{\partial T}{\partial y}\right)_{at\ y=0}. \tag{2.13a}$$

Using Eqs (2.6)-(2.9), we obtain

$$\left(\frac{\mathrm{Re}_\xi}{M}\right)^{\frac{1}{2}} C_f = f''(0), \quad \left(\mathrm{Re}_\xi\, M\right)^{\frac{-1}{2}} \mathrm{Nu}_\xi = -\theta'(0) \tag{2.13b}$$

where $\mathrm{Re}_\xi = \dfrac{U_w \xi^{m+1}}{v}$ is the local Reynolds number.

In the case of a linearly shrinking sheet, for the value of $f_w = 2$ it is known that the solution to Eq.(2.10) reduces to $f(\eta) = 1 + e^{-\eta}$ (see, e.g., Miklavcic and Wang, 2006). Note that when $f_w < 2$ we have no exponential decaying solutions (see, for instance, Van Gorder and Vajravelu, 2009; Fang and Zhang, 2010) for the linearly shrinking sheet problem while when $f_w > 2$ two solutions are possible. Often, one solution is physically meaningful while the other solution is non-physical. We find that such limitations appear in the case of a nonlinearly shrinking sheet as well, and thus we must restrict our attention to the case of suction ($f_w > 0$).

The solutions obtained numerically are the physically meaningful solutions, and we do not discuss any additional non-physical solutions in the present paper.

3. Stability and numerical method

When computing numerical solutions, we would like to ensure that the numerical solutions converge to the true physically meaningful solution to the problem. This is where stability of the system can be useful. Observe that Eqs (2.10) – (2.11) may be written as a five-dimensional dynamical system

$$\begin{aligned}
Z_1' &= Z_2, \\
Z_2' &= Z_3, \\
Z_3' &= Z_2^2 - \frac{1}{2}\left(1 + \frac{1}{m}\right) Z_1 Z_3, \\
Z_4' &= Z_5, \\
Z_5' &= \mathrm{Pr}\left\{(n + Z_4) Z_2 - \frac{1}{2}\left(1 + \frac{1}{m}\right) Z_1 Z_5\right\}.
\end{aligned} \tag{3.1}$$

Now, the boundary conditions Eqs (2.12a)-(2.12b) imply that $Z_2 \to 0$ and $Z_4 \to 0$ (and also $Z_3 \to 0$ and $Z_5 \to 0$) as $\eta \to \infty$ while $Z_1 \to \varepsilon$ (where ε denotes the boundary layer thickness) as $\eta \to \infty$. The Jacobian for Eq.(3.1), evaluated at the equilibrium $(\varepsilon, 0, 0, 0, 0)$ reads

$$DZ = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{\varepsilon}{2}\left(1+\frac{1}{m}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & n\Pr & 0 & 0 & -\frac{\varepsilon}{2}\left(1+\frac{1}{m}\right) \end{bmatrix}, \tag{3.2}$$

and the eigenvalues are 0 (multiplicity three) and $-\frac{\varepsilon}{2}\left(1+\frac{1}{m}\right)$ (multiplicity two). Taking boundary conditions into account, the solution then scales as

$$Z_1 \sim \left(A_1 + A_2\eta\right)\exp\left(-\frac{\varepsilon}{2}\left(1+\frac{1}{m}\right)\eta\right) + \varepsilon, \tag{3.3}$$

for large η. This shows that the solutions are stable for all $m > 0$ and, furthermore, that the decay rate is exponential and may be given by $-\frac{\varepsilon}{2}\left(1+\frac{1}{m}\right)$. As we shall see in the following section, this manner of decay is indeed confirmed by the obtained numerical solutions.

Equations (2.10) and (2.11) are non-linear coupled ordinary differential equations of third-order and second-order, respectively. Exact analytical solutions are not possible for the complete set of Eqs (2.10) and (2.11) and therefore we have used an efficient numerical method with an implicit finite difference scheme known as the Keller-Box method (Brandshaw and Cebeci, 1984; Keller, 1992; Prasad *et al*., 2010). This method is unconditionally stable and has a second order accuracy with arbitrary spacing. First, we write the transformed differential equations and the boundary conditions in terms of first order system, which is then converted to a set of finite difference equations using central differences. Then the non-linear algebraic equations are linearized by Newton's method and the resulting linear system of equations is then solved by block tri-diagonal elimination technique. For the sake of brevity, the details of the numerical solution procedure are not presented here. For numerical calculations, a uniform step size of $\Delta\eta = 0.01$ is found to be satisfactory and the shooting error was controlled with a relative error tolerance of 10^{-6} in all the cases.

4. Results and discussion

Upon employing the Keller-Box method as discussed in Section 3, we were able to obtain numerical solutions to the boundary value problem in Eqs (2.10)–(2.12). Importantly, we were able to deduce the behavior of solutions for arbitrarily large values of the power-law shrinking parameter *m*. We find that as *m* increases, the magnitudes of both the skin friction and Nusselt number decrease (see Tabs 1 and 2, respectively).

Meanwhile, from Figs 1 and 2 we see that an increase in the power-law shrinking parameter m results in a smoothing of the velocity components, as both profiles for f and f' are moderated due to the increase in m. The temperature profiles, shown in Figs 3 and 4, also show that an increase in the power-law shrinking parameter will tend to moderate the temperature differences over the problem domain: As the power-law shrinking parameter increases, the temperature spike witnessed near the origin is mitigated as shown in Fig.3.

In Fig.4, we see that one culprit behind the temperature spikes near the origin is the parameter n, which is a measure of the difference in the temperature of the ambient fluid and of the wall. Hence, in situations where the wall temperature parameter n is sufficiently different from zero, a manner of nonlinear stretching may be considered a practical means by which to reduce the temperature variation over the problem domain. Another feature of note is that the suction from the surface can drastically influence the temperature profiles. This is best shown in Fig.3, where an increase in the suction parameter results in an amplified temperature spike. Nonlinearly shrinking the sheet offers one method of controlling such temperature spikes. To see how the influence of the power-law stretching parameter, the wall temperature parameter and the suction parameter interact to modify the Nusselt number, see Tab.3.

Table 1. Values of skin friction $f''(0)$ for different values of m and f_w.

m	$f_w = 4$	$f_w = 4.5$	$f_w = 5$
1	-1.038378	-2.228009	-3.798063
5	-0.822047	-1.710771	-2.899753
10	-0.612710	-1.377775	-2.396709
10^2	-0.417008	-1.077373	-1.947628
10^3	-0.396991	-1.047454	-1.903221
10^5	-0.394984	-1.044463	-1.898786

Table 2. Values of wall temperature gradient $\theta'(0)$ for different values of m and f_w with $n = -0.2$, $\Pr = 1.5$.

m	$f_w = 4$	$f_w = 4.5$	$f_w = 5$
1	-28.38288	-34.753307	-41.363628
5	-8.790844	-11.587538	-14.120668
10	-7.191096	-9.891197	-12.384751
10^2	-5.756381	-8.305325	-10.740927
10^3	-5.614634	-8.143814	-10.571367
10^5	-5.600486	-8.127637	-10.554356

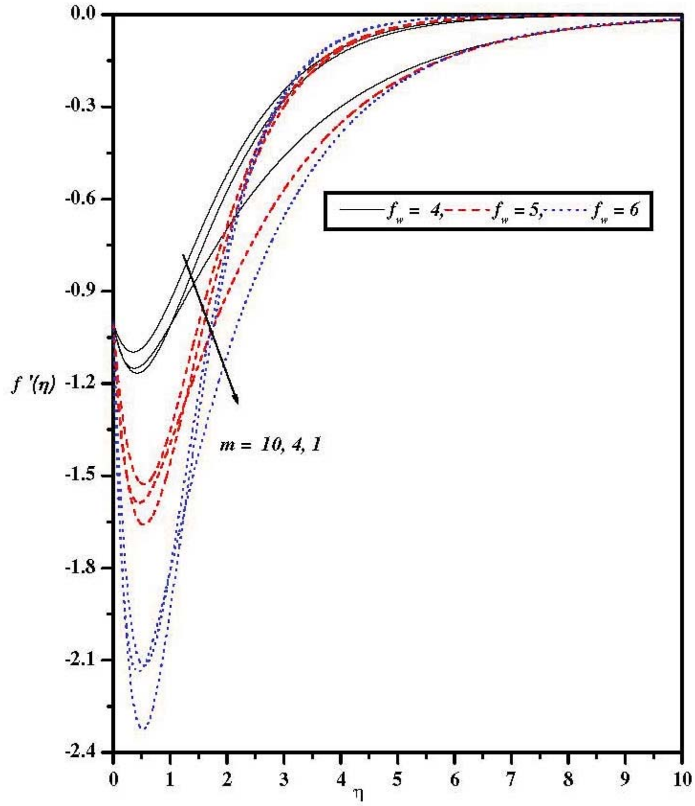

Fig.1. Horizontal velocity profiles for different values of *m* and f_w.

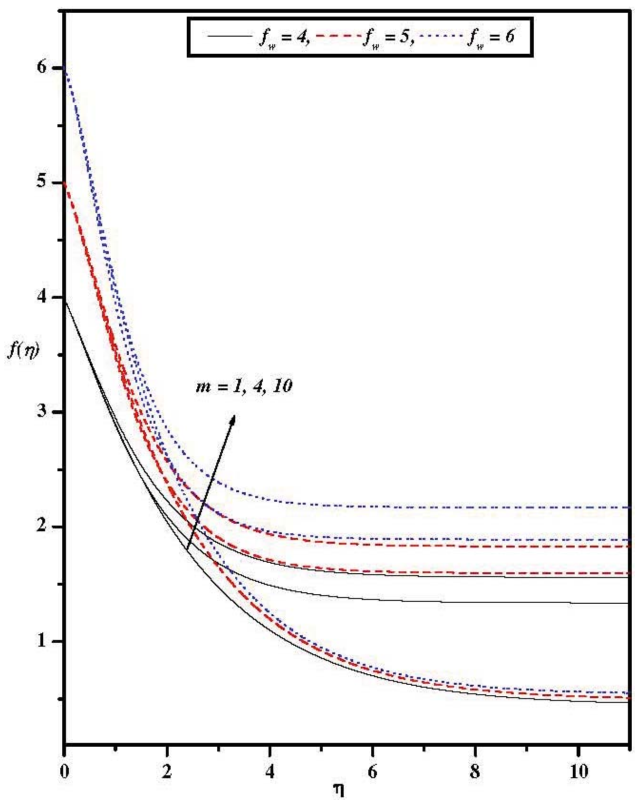

Fig.2. Transverse velocity profiles for different values of *m* and f_w.

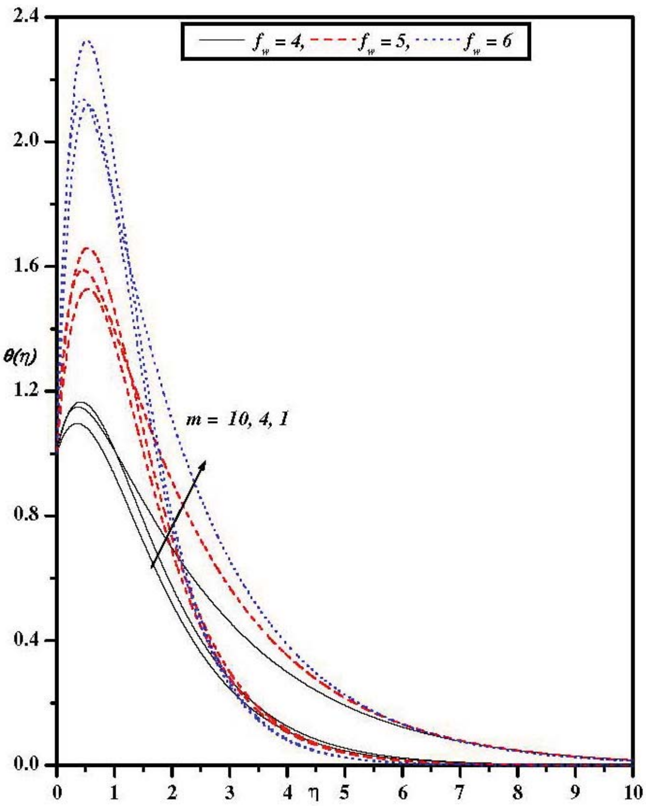

Fig.3. Temperature profiles for different values of m and f_w with $\Pr = 1$ and $n = 0$.

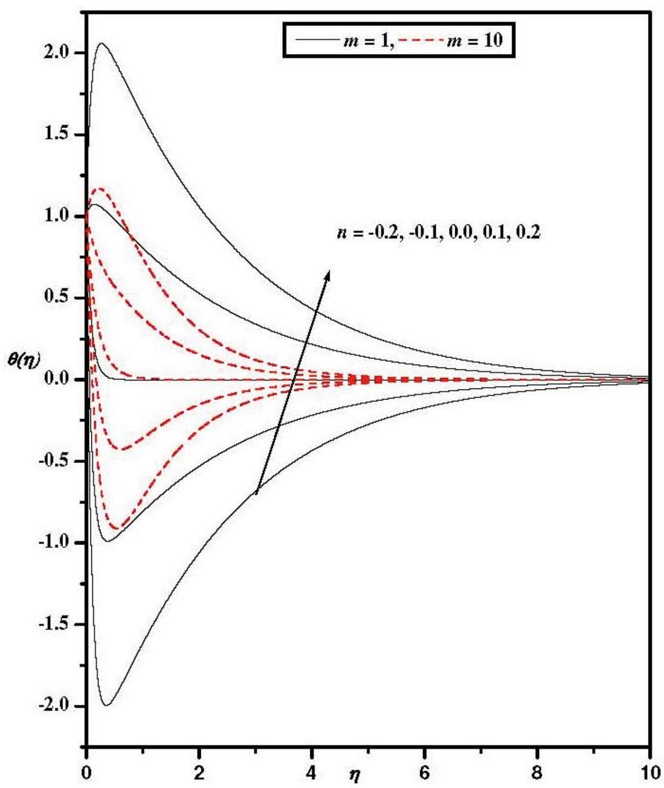

Fig.4. Temperature profiles for different values of m and n with $\Pr = 3$ and $f_w = 4$.

Table 3. Values of wall temperature gradient $\theta'(0)$ for different values of the parameters.

f_w	n	Pr =1.5			Pr = 2		
		$m=1$	$m=10$	$m=10^5$	$m=1$	$m=10$	$m=10^5$
4	-0.2	-28.382881	-7.191096	-5.600486	-29.106968	-9.289488	-7.696641
	-0.1	-16.856670	-4.455944	-3.488916	-18.244694	-6.257577	-5.221037
	0.0	-5.330457	-1.720793	-1.377345	-7.382421	-3.225667	-2.745433
	0.1	6.195755	1.014359	0.734226	3.479852	-0.193757	-0.269829
	0.2	17.721968	3.749511	2.845796	14.342126	2.838153	2.205775
4.5	-0.2	-34.753307	-9.891190	-8.127637	-35.242165	-11.85285	-10.230711
	-0.1	-20.434395	-6.007672	-4.909124	-21.830320	-7.845269	-6.765775
	0.0	-6.115480	-2.124155	-1.690612	-8.418475	-3.837679	-3.300839
	0.1	8.203434	1.759362	1.527900	4.993371	0.169912	0.164097
	0.2	22.522348	5.642879	4.746412	18.405216	4.177502	3.629033
5	-0.2	-41.363628	-12.384751	-10.554356	-41.606392	-14.142026	-12.496558
	-0.1	-24.124619	-7.456148	-6.291972	-25.523958	-9.283492	-8.168881
	0.0	-6.885610	-2.527544	-2.029589	-9.441525	-4.424958	-3.841204
	0.1	10.353399	2.401059	2.232795	6.640907	0.433576	0.403647
	0.2	27.592407	7.329663	6.495178	22.723341	5.292110	4.814149

The influence of suction on the velocity profiles is likewise more severe as the suction parameter increases in value. For large values of the suction parameters, the change in velocity profiles over the domain is much more drastic (see Fig.1). However, we again note that nonlinearly shrinking the sheet offers a way to control the magnitude of such velocity changes due to suction. We remark again that injection will in general not permit the existence of solutions to the self-similar problem considered here, as the introduction of mass will destroy similarity for this particular flow problem for a shrinking sheet. This is in contrast to the corresponding problem for the stretching sheet, where such mass injection still can be accounted for in the self-similar formulation.

While the solutions presented here are numerical in nature, note that it may be possible to construct analytical perturbation solutions for the problem in the asymptotic regime $m >> 1$. One would still need to consider perturbation about a nonlinear, rather than a linear, operator, but methods of inverting nonlinear operators exist in the literature (see, e. g., Van Gorder and Sweet, 2010; Van Gorder, 2011). Understanding the analytical behavior at intermediate to large values of *m* would be interesting and could lead to a better understanding of the requirements for the existence and uniqueness of solutions, which we do not address here.

Nomenclature

f – similarity stream function
f' – the first derivative of f with respect to η
f'' – the second derivative of f with respect to η
f''' – the third derivative of f with respect to η

f_w – mass transfer parameter at the sheet
M, N – coefficients defined in Eq.(2.5)
m – power law exponent parameter
n – wall temperature parameter
Nu – Nusselt number
Pr – Prandtl number
q_w – heat flux from the surface
Re_ξ – the local Reynolds number
T – temperature
T_c – constant defined in Eq.(2.5)
T_w – temperature of the plate
T_∞ – ambient temperature
U_w – a constant
u – fluid velocity in the x-direction
v – fluid velocity in the y-direction
x – coordinate along the shrinking sheet
y – coordinate perpendicular to the x-direction
α – effective thermal diffusivity of the fluid
ΔT – characteristic temperature difference
$\Delta\eta$ – grid size in the η direction
η – similarity variable
ξ – transformed x-co ordinate $Mx + N$
θ – nondimensional temperature
θ' – the first derivative of θ with respect to η
ν – kinematic viscosity
ρ – density
ψ – stream function
τ_w – shear stress at the surface

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