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STATIC, VIBRATION AND BUCKLING ANALYSIS OF SKEW COMPOSITE AND SANDWICH PLATES UNDER THERMO MECHANICAL LOADING

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Static, vibration and buckling behavior of laminated composite and sandwich skew plates is studied using an efficient C^0 FE model developed based on refined higher order zigzag theory. The C^0 FE model satisfies the inter-laminar shear stress continuity at the interfaces and zero transverse shear stress conditions at plate top and bottom. In this model, the first derivatives of transverse displacement have been treated as independent variables to overcome the problem of C^1 continuity associated with the plate theory. The C^0 continuity of the present element is compensated in the stiffness matrix formulation by adding a suitable term. In order to avoid stress oscillations observed in the displacement based finite element, the stress field derived from temperature is made consistent with the total strain field by using field consistent approach. Numerical results are presented for different static, vibration and buckling problems by applying the FE model under thermo mechanical loading, where a nine noded C^0 continuous isoparametric element is used. It is observed that there are very few results available in the literature on laminated composite and sandwich skew plates based on refined theories. As such many new results are also generated for future reference.

Key words: skew plate, finite element, refined theory, thermal load.

1. Introduction

Composite materials are widely used in many engineering applications due to their high stiffness/strength to weight ratio. Laminated composite structures are weak in shear due to their low shear modulus compared to extensional rigidity. Considerable research (Reddy and Palaninathan, 1995; Jaunky *et al*. 1995; Wang, 1997; Babu and Kant, 1999) has been published on buckling response of skew composite laminates. In these investigations, numerical methods such as the finite element method, the Rayleigh-Ritz method, etc. are used. Reddy and Palaninathan (1995) used an triangular finite element based on the classical laminated plate theory. Jaunky *et al*. (1995) and Wang (1997) employed the Rayleigh-Ritz method incorporating first-order shear deformation effects. Babu and Kant (1999) presented two C^0 shear deformable finite element formulations for the buckling analysis of skew laminated composite and sandwich panels. A 16-node bi-cubic Lagrange element is used in all the formulations. Haldar (2002) and Sheikh *et al*. (2002) used a high precision triangular element based on the first order shear deformation theory for both cross ply and angle ply skew laminate. In all these investigations skew laminates subjected to only mechanical loads are considered. In the case of skew laminated composite and sandwich plates subjected to thermal loads, there are few studies (Prabhu and Durvasula, 1974a; 1974b; 1976) available on thermal buckling of isotropic skew plates. Kant and Babu (2000) employed higher order shear deformation theory based on finite element models for the thermal buckling analysis of skew laminated composite and sandwich plates subjected to thermal loading. Prakash *et al*. (2008) investigated thermal postbuckling behavior of functionally graded material of skew plates. Vosoughi *et al*. (2011) analyzed postbuckling behavior of laminated composite skew

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plates subjected to thermal loading. There are virtually no papers available in the open literature for the skew laminated composite and sandwich plates based on the refined higher order shear deformation theory. Keeping all the aspects in view an attempt has been made to analyze skew composite and sandwich plates by using a nine noded C^0 finite element model based on the refined higher order shear deformation theory.

2. Formulation

The in-plane displacement fields for the refined higher order zigzag theory are taken as below

$$u_\alpha = u^0{}_\alpha + \sum_{k=0}^{nu-1} S_\alpha^k \left(Z - Z_k\right) H\left(Z - Z_k\right) + \\ + \sum_{k=0}^{nl-1} T_\alpha^k \left(Z - \rho_k\right) H\left(-Z + \rho_k\right) + \xi_\alpha Z^2 + \varphi_\alpha Z^3 \tag{2.1}$$

where u_α^0 denotes the in-plane displacement of any point on the mid surface, n_u and n_l represent the number of upper and lower layers, respectively S_α^k, T_α^k , are the slopes of k-th layer corresponding to upper and lower layers, respectively, $\xi_{\alpha,} \varphi_\alpha$ are the higher order unknown terms, $H\left(Z - Z_k\right), \left(Z - \rho_k\right)$ are unit step functions and the subscript α represents the co-ordinate directions [$\alpha = 1, 2$ i.e., x, y in this case], respectively and

$$u_3 = w\left(x, y\right). \tag{2.2}$$

Also, we have the stress strain relationship of a lamina, say k^{th} lamina, which may be expressed in the structural axes system (x-y) as

$$\bar{\sigma} = \left[\bar{Q}_k\right]\left\{\bar{\varepsilon}\right\}. \tag{2.3}$$

Now by utilizing the transverse shear stress free condition at the top and bottom of the plate $\sigma_{3\alpha|z=\pm h/2} = 0$ the components ξ_α and φ_α could be expressed as

$$\Phi_\alpha = -\frac{4}{3h^2}\left\{w_{,\alpha} + \frac{1}{2}\left(\sum_{k=0}^{nu-1} S_\alpha^K + \sum_{k=0}^{nl-1} T_\alpha^k\right)\right\}, \tag{2.4}$$

and

$$\xi_\alpha = -\frac{1}{2}\left(\sum_{k=0}^{nu-1} S_\alpha^k + \sum_{k=0}^{nl-1} T_\alpha^k\right). \tag{2.5}$$

Similarly, by imposing the transverse shear stress continuity conditions at the layer interfaces the following expressions for S_α and T_α are obtained as below

$$S_\alpha^k = a_{\alpha\gamma}^k \left(w_{,\gamma} + \psi_\gamma\right) + b_{\alpha\gamma}^k w_{,\gamma}, \tag{2.6}$$

$$T_\alpha^k = c_{\alpha\gamma}^k \left(w_{,\gamma} + \psi_\gamma\right) + d_{\alpha\gamma}^k w_{,\gamma} \tag{2.7}$$

where $a_{\alpha\gamma}^{k}$, $b_{\alpha\gamma}^{k}$, $c_{\alpha\gamma}^{k}$, $d_{\alpha\gamma}^{k}$ are constants depending on material and geometric properties of individual layers, $w_{,\gamma}$ is the derivatives of transverse displacement while $\gamma = 1, 2$ and $S_{\alpha}^{0} = \psi_{\alpha}$ is the rotation of normal at the mid surface about co-ordinate directions [$\alpha = 1, 2$, i.e., x, y in this case].

By using Eqs (2.2)-(2.6) the strain vector can be evaluated by

$$\{\bar{\varepsilon}\} = [H]\{\varepsilon\}. \tag{2.8}$$

$\{\bar{\varepsilon}\}$ is the strain field vector (5×1) and $\{\varepsilon\}$ is the strain vector (17×1) at the reference plane (i.e., at the mid plane) where the $[H]$ is the matrix (5×17) consists of terms containing z and some term related to material properties

$$w_{,1} = \frac{\partial w}{\partial x} = w_1 \qquad \text{and} \qquad w_{,2} = \frac{\partial w}{\partial y} = w_2, \tag{2.9}$$

$$\{\varepsilon\}^T = \left\{ \begin{array}{c} \dfrac{\partial u_1^{\,0}}{\partial x} \dfrac{\partial u_2^{\,0}}{\partial y} \dfrac{\partial u_2^{\,0}}{\partial x} + \dfrac{\partial u_1^{\,0}}{\partial y} \dfrac{\partial w_1}{\partial x} \dfrac{\partial w_2}{\partial y} \dfrac{\partial w_2}{\partial x} \dfrac{\partial w_1}{\partial y} \dfrac{\partial \Psi_1}{\partial x} \dfrac{\partial \Psi_2}{\partial y} \\ \dfrac{\partial \Psi_2}{\partial x} \dfrac{\partial \Psi_1}{\partial y} \Psi_1 \Psi_2 \dfrac{\partial w}{\partial x} \dfrac{\partial w}{\partial y} w_1 w_2 \end{array} \right\}. \tag{2.10}$$

The strain displacement relation may be written as below

$$\{\varepsilon\} = [B]\{\delta\} \tag{2.11}$$

where $[B]$ is the strain-displacement matrix and $\{\delta\}$ is the element nodal displacement vector.

Thermal strain due to temperature change is given by

$$\{\varepsilon_{th}\} = \begin{Bmatrix} \alpha_x \Delta T \\ \alpha_x \Delta T \\ \alpha_{xy} \Delta T \\ 0 \\ 0 \end{Bmatrix}, \tag{2.12}$$

in which ΔT is the change of temperature with respect to reference temperature, $\alpha_x, \alpha_y, \alpha_{xy}$ are the thermal expansion coefficients in the structural axis (x-y-z) system therefore, the net strain may be written as

$$\{\bar{\varepsilon}_n\} = \{\bar{\varepsilon}\} - \{\varepsilon_{th}\}, \tag{2.13}$$

in which $\{\bar{\varepsilon}_n\}$ is the total strain and $\{\varepsilon_{th}\}$ is the thermal strain, respectively.

For the present study, a nine nodedisoparametric element with seven unknowns has been usedin the proposed finite element model

$$\{P\} = \iint [N]^T q\,dx\,dy \tag{2.14}$$

where $\{P\}$ is the nodal force, $[N]$ is the shape functions matrix and q is the intensity of transverse load respectively.

Thermal loading may be obtained as below

$$\{F\} = \iint [B]^T [H]^T \left\{F^N\right\} dxdy \tag{2.15}$$

where $\left\{F^N\right\}^T = \left[N_x^N, N_y^N, N_{xy}^N, \text{etc.}\right]$ and $\{F\}$ is the thermal load respectively,

and $$\left\{N_x^N,\ N_y^N, N_{xy}^N\right\}^T = \sum_k^n \int_{zk-1}^{zk} \left\{\bar{Q}_{ij}\right\}_k \{\varepsilon_{th}\} dz, \tag{2.16}$$

here $i, j = 1, 2, 6$ and $\{\varepsilon_{th}\}$ is the thermal strain components.

It can be observed that the total strain field is always interpolated to a lower order when compared to the thermal strain fields. Hence thermal strain fields should be consistently reconstituted to the order of in-plane normal strain field to get accurate strains and stresses over the element domain. Therefore, this is accordingly taken care to make them field consistent (Naganarayana *et al*., 1997).

The following thermal case has been considered:
uniform temperature across the depth

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} \cdot \Delta T, \tag{2.17}$$

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & -2cs \\ s^2 & c^2 & 2cs \\ cs & -cs & c^2 - s^2 \end{bmatrix} [Q]_k \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_{12} \end{Bmatrix}_k \tag{2.18}$$

where $\alpha_1, \alpha_2, \alpha_{12}$ are thermal expansion coefficients in the material axis system and $c = \cos\theta$, $s = \sin\theta$ and θ is the angle between the principal material axis and structural axis.

In this case, $\Delta T = T$, therefore, the thermal force

$$\{F\} = \iiint [B]^T [H]^T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} \cdot T dv \tag{2.19}$$

where, T_u = temperature at top surface, T_L = temperature at bottom surface.

By applying the virtual work method, we get

$$[k]\{\delta\} = \{P\} \tag{2.20}$$

where $[k]$ is the element stiffness matrix and $\{P\}$ is the element nodal load vector as written below.

The element mass matrix may be written as

$$\left[m^e\right] = \sum_{k=1}^{nu+nl} \int \rho_k \left[C\right]^T \left[F\right]^T \left[F\right]\left[C\right] dxdydz \tag{2.21}$$

where ρ_i is the mass density of the k-th layer and [C] is the shape function matrix the geometric stiffness matrix $\left[k_g\right]$ of an element may be written as

$$\left[k_g\right] = \sum_{i=1}^{nu+nl} \iiint \left[G\right]^T \left[S^k\right]\left[G\right] dxdydz \tag{2.22}$$

where $\left[S_k\right]$ is the in-plane stress components of the k-th layer.

For the linear thermal buckling problems, the stability equation can be expressed as

$$\left(\left[K\right] - \lambda\left[K_G\right]\right)\{\partial\} = 0, \tag{2.23}$$

in which $\left[K\right]$ and $\left[K_g\right]$ are the global elastic and geometric stiffness matrix and λ is the critical temperature parameter respectively.

In the first step, a static problem is solved to calculate thermal stresses at the Gauss points of different elements for the assumed temperature rise. These thermal stresses are then used to form the matrix $\left[S^k\right]$ of the geometric stiffness matrix and the linear thermal buckling problem is solved to calculate the critical buckling temperature. Finally, the thermal vibration problem is solved an eigen value problem by taking different temperatures just below the calculated critical buckling temperature.

The equation of thermal vibration may be written as

$$\left(\left[K'\right] - \omega^2\left[M\right]\right)\{\partial\} = 0 \tag{2.24}$$

where $\left[K'\right] = \left[K\right] - \lambda\left[K_G\right]$,

in which $\left[K'\right]$ is the reduced stiffness matrix, $\left[M\right]$ is mass matrix, λ is a fraction of critical buckling temperature and ω is the frequency of thermal vibration, respectively.

A computer program has been written as per the above formulation. The boundary conditions used in different cases are as follows:

1. Simply supported boundary conditions on all sides (SSSS)

$u_1 = u_2 = w = \Psi_1 = w_1 = 0$ at $x = 0, a,$

$u_1 = u_2 = w = \Psi_2 = w_2 = 0$ at $y = 0, b.$

2. Clamped boundary conditions on all sides (CCCC)

$u_1 = u_2 = w = \Psi_1 = \Psi_2 = w_1 = w_2 = 0$ at $x = 0, a$ and $y = 0, b.$

3. Numerical results and discussion

Various numerical examples of skew isotropic and laminated composite plates (Fig.1) having different features are solved by the proposed C^0 finite element and the results obtained are presented with some published results for necessary comparison. The whole plate (Fig.1) is modeled with different mesh arrangements shown in different tables. As the sides BC and AD (Fig.1) are inclined to the global axis system (x-y), the degrees of freedom of the nodes on these two sides are transformed in the local axis system (x'-y') for incorporation of boundary conditions for edges other than clamped. Since very few results are available in the open literature for skew plates under thermo- mechanical loading conditions, a number of numerical problems of skew plate have been solved considering different boundary conditions, ply orientations, thickness ratio and aspect ratio. The results obtained by using the proposed finite element method is first validated with the published results and many new results are generated for future reference as there is no result available in the literature based on the refined theories for skew plates according to the author's best knowledge.

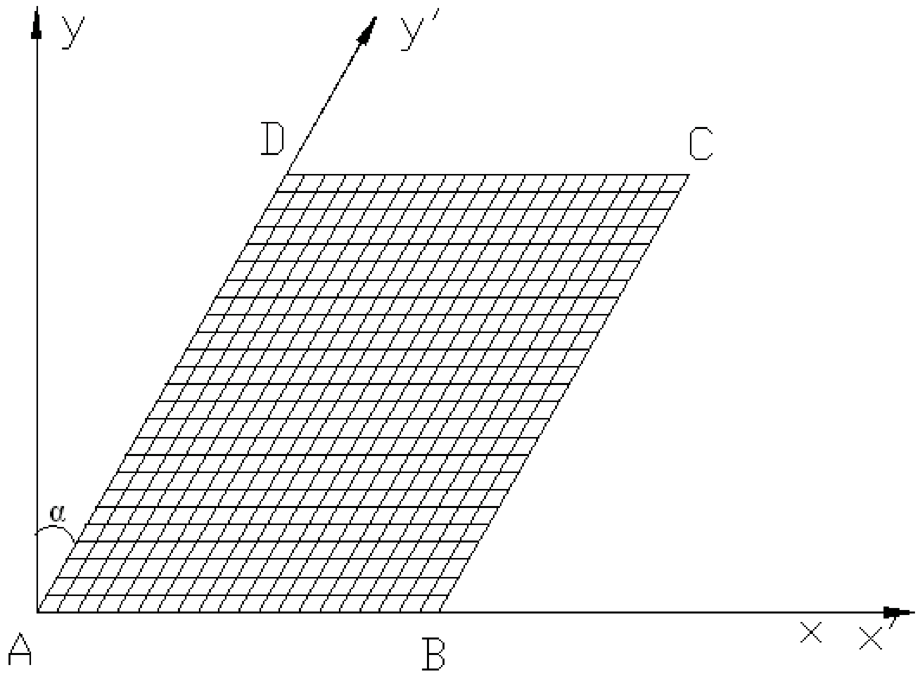

Fig.1. Skew plate.

3.1. Simply supported skew laminate ($0^0/90^0/0^0$) under uniform loading

The present problem of simply supported square cross-ply ($0^0/90^0/0^0$) skew laminate under uniform loading has been analyzed for different skew angles for the thickness ratio (a/h) *100* and *10*. The non-dimensional central deflection $\bar{w} = \dfrac{10^3\, wE_2 h^3}{qa^4}$ and in-plane normal stresses are given by $\bar{\sigma}_1 = \dfrac{h^2 \sigma_1}{qa^2}$, $\bar{\sigma}_2 = \dfrac{h^2 \sigma_2}{qa^2}$. The material properties of individual layers are given by: $E_1/E_2 = 25$, $G_{12} = G_{13} = 0.5\, E_2$, $G_{23} = 0.2\, E_2$, $\nu_{12} = \nu_{13} = 0.25$.

The present results obtained using the proposed model based on the refined theory are presented in Tab.1 for different skew angles. The obtained results are compared with the results of Halder (2002) and Kabir (1995), respectively. The present results are in well agreement with the results of Halder (2002) and Kabir (1995), respectively.

Table 1. Non dimensional central deflection and in-plane normal stresses of a simply supported skew laminate ($0^0/90^0/0^0$) subjected to uniformly distributed load.

Thickness ratio (*a*/*h*)	Skew angle	References	$\overline{w}$	$\overline{\sigma}_1$	$\overline{\sigma}_2$
100	30^0	Present (*8×8*)	*0.5446*	*0.6618*	*0.2898*
		Present (*12×12*)	*0.5436*	*0.6579*	*0.2799*
		Present (*16×16*)	*0.5436*	*0.6579*	*0.2798*
		Chakrabbarti (2003)	*0.5452*	*0.6444*	*0.2629*
		Haldar (2002)	*0.5458*	*0.6348*	-
	45^0	Present (*8×8*)	*0.3577*	*0.4491*	*0.3279*
		Present (*12×12*)	*0.3599*	*0.4458*	*0.3181*
		Present (*16×16*)	*0.3599*	*0.4458*	*0.3181*
		Chakrabbarti (2003)	*0.3631*		*0.3007*
		Haldar (2002)	*0.3621*		-
	60^0	Present (*8×8*)	*0.1380*	*0.1971*	*0.2663*
		Present (*12×12*)	*0.1432*	*0.1984*	*0.2673*
		Present (*16×16*)	*0.1432*	*0.1989*	*0.2673*
		Chakrabbarti (2003)	*0.1455*		*0.2572*
		Haldar (2002)	*0.1455*		-
		Kabir (1995)	*0.1520*		-
10	30^0	Present (*8×8*)	*0.8752*	*0.7057*	*0.4064*
		Present (*12×12*)	*0.8774*	*0.7043*	*0.4073*
		Present (*16×16*)	*0.8774*	*0.7040*	*0.4078*
		Chakrabbarti (2003)	*0.8621*	*0.6617*	*0.3709*
		Haldar (2002)	*0.8193*	*0.6005*	-
	45^0	Present (*8×8*)	*0.5625*	*0.4794*	*0.4141*
		Present (*12×12*)	*0.5667*	*0.4795*	*0.4167*
		Present (*16×16*)	*0.5667*	*0.4795*	*0.4167*
		Chakrabbarti (2003)	*0.5707*	*0.4543*	*0.3786*
		Haldar (2002)	*0.5507*	*0.4056*	-
	60^0	Present (*8×8*)	*0.2343*	*0.2089*	*0.2663*
		Present (*12×12*)	*0.2384*	*0.2104*	*0.2683*
		Present (*16×16*)	*0.2384*	*0.2104*	*0.2682*
		Chakrabbarti (2003)	*0.2505*	*0.2058*	*0.3023*
		Haldar (2002)	*0.2455*	*0.1758*	-
		Kabir (1995)	*0.2600*	*0.0852*	-

3.2. Clamped isotropic skew plates

In this example, clamped isotropic skew plates with the thickness ratio ($a/h=100$) have been analyzed for different skew angles subjected to a uniform temperature rise throughout the thickness. The material properties for the isotropic plate are given by

$$E_1 = E_2 = E_3 = 1GPa,\ v_{12} = v_{21} = 0.3,\ G_{12} = G_{13} = G_{23} = 0.3846GPa,\ \alpha_1 = \alpha_2 = 10^{-6}/^0C.$$

The normalized critical buckling temperature is defined as follows: $\lambda_{cr} = \alpha_0\ T$ where T is the critical temperature, α_0 is the normalization factor which is taken as $= 10^{-6}$. The critical temperature (λ_{cr}) obtained for the thickness ratio ($a/h=100$) by using the proposed FE model is shown in Tab.2. It is observed that the present results are close to the results of Prakash (2008) and Prabhu and Durvasula (1974).

Table 2. Normalized critical buckling temperature $(\lambda_{cr} = \alpha_0 T)$ for a clamped isotropic skew plate ($a/h=100$).

Skew angle	Present	Prakash (2008)	Prabhu (1974)
0^0	3.7509	3.7374	3.7100
15^0	3.9895	3.9782	3.9500
30^0	4.9080	4.8417	4.800
45^0	7.3124	7.0110	6.9200

3.3. Symmetric square cross-ply ($0^0/90^0/90^0/0^0$) skew laminate

3.3.1.

The present problem of a symmetric cross-ply ($0^0/90^0/90^0/0^0$) skew laminate for different skew angles subjected to a uniform temperature rise throughout the thickness is considered. The material properties are as follows $E_1/E_2=15, G_{12}=G_{13}=0.5\ E_2$, $G_{23}=0.3356E_2$, $v_{12}=0.3$, $v_{23}=0.49$, $\alpha_1/\alpha_0=0.015$, $\alpha_2/\alpha_0=1.0$.

The critical buckling temperature $\left(\lambda'_{cr} = 10^2 \times \alpha_0 T\right)$ obtained for the thickness ratio ($a/h=100$) by using the proposed FE model is shown in Tab.3. It is found that the results obtained using the present FE model are quite close to Kant *et al.* (2000) and Vosoughi *et al.* (2011), respectively.

Table 3. Normalized critical buckling temperature $\left(\lambda'_{cr} = 10^2 \times \alpha_0 T\right)$ for a skew laminate ($0^0/90^0/90^0/0^0$) ($a/h=100$).

Boundary Conditions	References	Skew angle			
		0^0	15^0	30^0	45^0
SSSS	Present	0.0998	0.1019	0.1123	0.1443
	Kant (2000)	0.0996	0.1017	0.1116	0.1427
	Vosoughi (2011)	-	0.1018	0.1118	0.1427
CCCC	Present	0.3385	0.3464	0.3592	0.4206
	Kant (2000)	0.3348	0.3441	0.3572	0.4169
	Vosoughi (2011)		0.3446	0.3578	0.4179

3.3.2.

In this example, a symmetric cross-ply ($0^0/90^0/90^0/0^0$) skew laminate for different skew angles subjected to uniform temperature rise throughout the thickness is considered. The material properties are same as in section 3.3.1. The critical buckling temperature $\left(\lambda'_{cr} = 10^2 \times \alpha_0 T\right)$ obtained for the thickness ratio ($a/h = 10$) by using the proposed FE model is shown in Tab.4. It is observed that the results obtained using the present FE model are quite close to Kant *et al.* (2000) and Vosoughi *et al.* (2011), respectively.

Table 4. Normalized critical buckling temperature $\left(\lambda_{cr} = \alpha_0 T\right)$ for a cross-ply ($0^0/90^0/90^0/0^0$) skew laminate ($a/h = 10$).

Boundary Conditions	References	Skew angle			
		0^0	15^0	30^0	45^0
SSSS	Present	*0.0756*	*0.0770*	*0.0829*	*0.0983*
	Kant (2000)	*0.0757*	*0.0767*	*0.0821*	*0.0985*
	Vosoughi (2011)	-	*0.0794*	*0.0860*	*0.1041*
CCCC	Present	*0.1638*	*0.1654*	*0.1725*	*0.1939*
	Kant (2000)	*0.1601*	*0.1618*	*0.1690*	*0.1893*
	Vosoughi (2011)	-	*0.1712*	*0.1799*	*0.2060*

4. Conclusions

An analysis of skew laminated composite and sandwich skew plates is mode using an efficient C^0 FE model developed on the basis of the refined higher order zigzag theory. The C^0 FE model satisfies the inter-laminar shear stress continuity at the interfaces and zero transverse shear stress conditions at the plate top and bottom. In this model, the first derivatives of transverse displacement have been treated as independent variables to overcome the problem of C^1 continuity associated with the plate theory. The C^0 continuity of the present element is compensated in the stiffness matrix formulation by adding a suitable term. It has been established through various numerical examples that the present higher order zigzag theory can accurately predict the deflection vibration and buckling of general laminated composite and sandwich plates.

Nomenclature

a – dimension of the plate along x - axis
$[B]$ – strain displacement matrix
$[B]$ – strain-displacement matrix (derivative of shape functions)
b – dimension of the plate along y - axis
$[D]$ – rigidity matrix
E_1 – Young's modulus in the major principal material direction of the lamina
E_2 – Young's modulus in the transverse material direction of the lamina
E_3 – Young's modulus in the transverse material direction (Per. to plane) of the lamina
G_{12} – in-plane shear modulus
G_{13}, G_{23} – out of plane shear moduli
$H(z-z_i)$, $H(-z+z_j)$ – heavy side unit step functions
h – overall thickness of the structure
h_c – total thickness of the core in a sandwich structure
h_f – total thickness of face sheets in a sandwich structure
$[K]$ – global stiffness matrix

$[K']$ – reduced stiffness matrix
$[K_G]$ – global geometric stiffness matrix
$\left[k^e\right]$ – element stiffness matrix
$\left[k_g\right]$ – element geometric stiffness matrix
$[M]$ – global mass matrix
$\left[m^e\right]$ – element mass matrix
$[N]$ – shape function matrix
n_l – number of layers below the mid plane of the laminated structure
n_u – number of layers above the mid plane of the laminated structure
$\{P\}$ – global load vector
$\{P_e\}$ – element load vector
$\{p\}$ – element nodal load vector
$\left\{\bar{Q}^k\right\}$ – rigidity matrix
q – intensity of transverse loading
q – intensity of transverse load
$\left[S^k\right]$ – in plane stress components of the *k*-th layer
S_α^k – *k*-th layer corresponding to upper layers
$S_\alpha^0 = \Psi_\alpha$ – rotation of normal at the mid surface
T_L – temperature at bottom surface
T_U – temperature at top surface
T_α^k – *k*-th layer corresponding to lower layers
u_α^0 – in-plane displacement of any point on mid surface
x, y – Cartesian co-ordinates/plane
z – thickness co-ordinate/transverse direction
$\alpha=1, 2$ – x, y direction respectively
α_0 – normalization factor
$\alpha_1, \alpha_2, \alpha_{12}$ – thermal expansion coefficients in the material axis system
$\alpha_x, \alpha_y, \alpha_{xy}$ – thermal expansion coefficients in the structural axis (x-y-z) system
$\beta_1, \beta_2, \beta_{12}$ – moisture expansion coefficients in the material axis system
$\beta_x, \beta_y, \beta_{xy}$ – moisture expansion coefficients in the structural axis (x-y-z) system
ΔT – change of temperature
$\{\delta\}$ – nodal displacement vector
$\{\delta\}$ – nodal unknown vector
$\{\varepsilon_{th}\}$ – thermal strain
$\{\bar{\varepsilon}\}$ – strain field vector
$\{\bar{\varepsilon}_n\}$ – total strain
θ – fiber orientation angle with respect to the principal material axis
λ – buckling load parameter
λ_{cr} – critical buckling temperature
ν_{12}, ν_{21} – in-plane major and minor Poision's ratios
ν_{13}, ν_{31} – out of plane (x-z) major and minor Poisson's ratios
$\xi_\alpha, \varphi_\alpha$ – higher order unknowns
ρ_c – mass density of core layer in a sandwich plate
ρ_f – mass density of face layer in a sandwich plate

ρ_i – mass density of the *i*-th layer
σ_i – in-plane normal stresses
σ_{xx}, σ_{yy} – in-plane normal stresses
σ_{xy}, τ_{xy} – in-plane shear stress
$\{\overline{\sigma}\}$ – stress vector at any point of the plate
τ_{ij} – in-plane shear stress
τ_{xz}, τ_{yz} – transverse shear stresses
$\{\varphi\}$ – mode shape vector for free vibration
$\{\Psi\}$ – mode shape vector for buckling
ω – frequency of vibration

Subscript

i , j – counters
k – counter
l – lower
u – upper
x – *x*- direction
y – *y*- direction
12, 21 – plane directions
13, 31, 23, 32 – transverse directions

Superscript

i – layer/interface
j – layer/interface

Abbreviations

GPa – Gega Pascal

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