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IDENTIFICATION BASED ON A FINITE ELEMENT MODEL OF AN IMPACT FORCE OCCURRING ON A COMPOSITE STRUCTURE

Abdelali EL-BAKARI* and Abdellatif KHAMLICHI
Department STIC, National School of Applied Sciences, Abdelmalek Essaadi University
Tetouan, MOROCCO
MASGC, National School of Applied Sciences, Abdelmalek Essaadi University, Tetouan, MOROCCO
E-mail: aelbakari@uae.ac.ma

Issam HANAFI
National School of Applied Sciences, Abdelmalek Essaadi University, Alhoceima, MOROCCO

Based on the inverse approach and the finite element method, the quality of a reconstructed signal is discussed in this work. The responses caused by a distributed impact on a portion of a composite structure can be recovered using dynamic analysis. The structure is thought to be complex and made up of two different-sized plates made of two different materials. The robustness of the inversion method was studied, as well as the sensitivity of the numerical method compared to modal truncation and sampling of the frequency response function (FRF). Once the FRF had been identified, regularized deconvolution as per generalized singular value decomposition was used to reconstruct the impact signal characteristics. It was revealed that only one mode is required to reconstruct the impact signal.

Keywords: inverse problem, modal analysis, identification, impact force, composite structure.

1. Introduction

Many dynamic structural applications necessitate understanding of the force acting on a mechanical system. However, measuring it with sensors can be difficult in many cases, particularly if the measurement is burdensome or the force is scattered. In contrast, vibrational responses are easily measured. This is why indirect methods, such as rebuilding forces based on the inverse of a mechanical system's model, are recurrently preferred. Various models based on force reconstruction approaches have been proposed in latest years, particularly in the context of specific applications [1-2].

Zhao and Ye [3] extracted the characteristics of the inversion system matrix using the Singular Value Decomposition (SVD) using the matrix size change technique. They proved that the extraction effect of a single frequency is better than the transformation into wavelets. Lim and Pilkey [4] used the modal method for resolving the inverse problem, assuming that the position of the force is known. For computational efficiency, the pseudoinverse solution is then replaced with a dynamic programming solution [5]. Chinkaa *et al.* [6] used the modal analysis to assess and detect damage to a cantilever beam structure, identifying crack damage using Frequency Based Damage Detection Techniques (FBDDT). El-Bakari *et al.* [7] investigated the identification of a cantilever beam structure's impact force. They presented the deconvolution system problem and used modal analysis and General Singular Value Decomposition to reconstruct the force characteristics (GSVD). Liu *et al.* [8] investigated the identification of impact forces on plate structures, and the identification of impact forces was discovered using the nonconvex overlapping group sparsity (NOGS) regularization.

The impact force reconstruction can be used to better diagnose the safety of the structure, ensuring that the extent of the damage is well treated. When determining the impact force, a quadratic error minimization technique reduces the experimental effort required [9-10]. Varghese and Shankar [11] investigated the

* To whom correspondence should be addressed

identification of structural parameters in the time domain using the Multi-objective Optimization of combined conventional acceleration matching and power flow balance. In general, the recognition of impact characteristics for elastic linear structures can be constructed using various structural models. The structural models found in the literature can be found analytically, using the finite element method, or experimentally [12-14].

An impulse response function connects the pulse of the impact signal and the measurement points of the responses when the impact position is presumed to be known and the impact has the form of a point force. In this case, the signal can be reconstructed by inverting the transfer matrix and performing a regularized deconvolution [15-17]. When the impact position is assumed to be unknown, a quadratic error minimization technique between the calculated and measured responses identifies the impact signal, which includes historical force evolution and location.

Liu *et al.* [18] propose a hybrid support vector regression with multi-domain features to improve localization accuracy. The suggested research involves three stages: impact localization, multi-domain feature extraction, and signal pre-processing. It could improve the accuracy of predicting low-velocity impact locations. The impact force model serves as a guide in the design of the structure's impact resistance. It can also be used by other programs to identify both shock load and position. This method can be used to monitor the structural health of shock forces [19].

In this work, an error minimization formulation was considered to solve the localization problem, and then the resolution of the deconvolution problem allows the identification of the pressure generated by a non-punctual impact. The considered structure is a composite with a rectangular section made up of two isotropic and homogeneous plates. A pressure pulse applied to a given rectangular domain will excite it. The displacement along the plate's transverse direction at one point will be calculated in two ways, using a modal model and a transient dynamic model, respectively. This displacement will be used to simulate the measured response and will pose the inverse problem associated with impact pressure reconstruction. The response function in the time domain is found using the inverse Fourier transform (IFT). The frequency response functions between the arbitrary excitation points and the DOF that provide displacement measurement by some sensors are calculated using the modal analysis-based finite element method. The generalized Toeplitz matrix is then used to ensure the link between the pressure applied and the movements on a rectangular part of the composite structure. Regularization is applied to the reconstruction of the repetitive impact signal, which is described by a half-sinus function, using the SVD and the filtering truncation technique. For the calculation of the frequency response function, the reactivity of this technique used in this work is treated in terms of mesh size, sampling frequencies, and modal truncation order. Finally, we will discuss about the quality of the reconstructed signal based on the location of the sensors.

2. Direct problem formulation

Consider a structure made up of two assembled plates, each made of an isotropic and homogeneous linear elastic material. The thickness of both plates is the same. Figure 1 depicts the geometric area of the

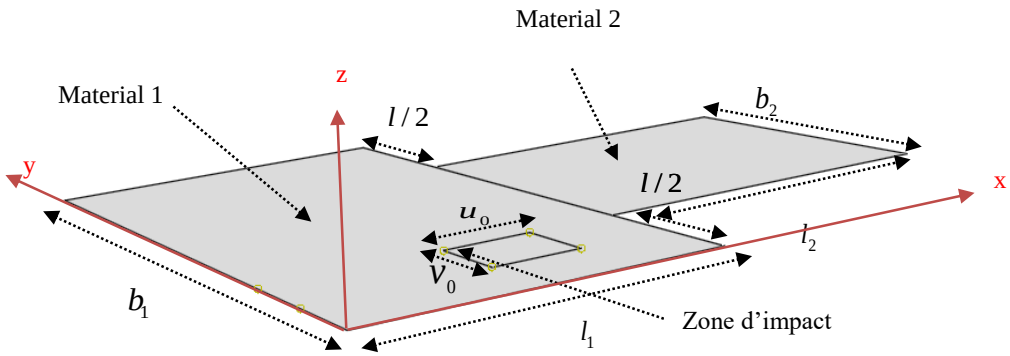


Fig.1. Geometry of the plate considered, it is a composite consisting of two isotropic and homogeneous linear elastic layers, the pressure is applied to the impact area.

composite plate, which includes two rectangles of differing dimensions, each of which represents a plate among the two assembled plates that form the structure. The plate is recessed along the *x=0* edge and free along all other edges.

Given the fact a time interval of duration d_c that is sampled by $\Delta d = d_c / (N-1)$ data of N time moments that are regularly spaced, the discrete response $y(k)$ for any time $k = 1,...,N$ is expressed as a function of the discrete input pressure $p(k)$ in the form of the discrete convolution product defined by Eq.(2.1). The function $t(k)$ in this equation is the linear system's discrete impulse response function.

$$y(k) = \sum_{j=1}^{k} t(j)\, p(k-j) \quad k = 1,2,...,N \,. \tag{2.1}$$

To solve the discrete problem given by Eq.(2.1) results in the following set of algebraic equation [7, 20]

$$Y = TP \tag{2.2}$$

where T is the Toeplitz matrix generated by the discrete impulse response function vector. This matrix organizes information about the modelled structure, but it also depends on the time discretization step used.

The matrix of the composite structure, which takes the form of two joined plates, is determined by solving Eq.(2.2). A FE model is used to handle these. The FRF is calculated from the finite element modes. Let us indicate the measurement of the DOF displacement, as well as any DOF that are normal to the pressure area applied

$$F_{ip}(w) = \sum_{j=1}^{N_{impact}} F_{ij}(w)\,, \tag{2.3}$$

with

$$F_{ij}(w) = \sum_{j=1}^{N_{\text{modes}}} \frac{\Theta_{ik}\Theta_{jk}}{w^2 - w_k^2 + 2i\beta_k w_k w} \tag{2.4}$$

where N_{impact} is the total number of degrees of normal freedom of movement belonging to the impact area, N_{modes} is the number of modes selected after modal truncation, w is the pulsation for which the response is considered, Θ_{ik} is the i-th component of the kth mode normalized with respect to mass, w_k is the kth mode angular frequency and β_k is the damping coefficient for the kth mode.

The temporal FRT which we noted t in Eq.(2.1) is obtained from the FRF which is defined by Eqs (2.3) and (2.4) by means of the inverse Fourier transform. This transform will be calculated by the *ifft* command in Matlab. Hence

$$t_{ip} = ifft(F_{ip})\,. \tag{2.5}$$

The matrix T is generated from the discrete vector t. The Matlab command to calculate it is written in the form

$$T_{ip} = tril\left(toeplitz(t_{ip})\right). \tag{2.6}$$

The matrix T is determined by the FE model used, the FRT sampling step Δf and the order of modal truncation used. All of these parameters will influence the dimension of the Toeplitz matrix as well as the values of its terms.

In general, the sampling pitch of the frequency interval must allow enough resolution to obtain a sufficiently accurate discrete FRT, especially when passing through the resonant frequencies defined by the system's own frequencies. However, due to Matlab limitations, the total number of sampling points cannot exceed *4096*. When the frequency interval is small, the matrix construction T is simplified because the sampling step can be taken in small increments. When the excitation spectrum is relatively high in frequency, a conflict arises between the resolution requirement and the maximum frequency to cover. In any case, the minimum frequency step must allow the solution to be calculated over a sufficient time interval by referring to the Shannon condition.

As a result, if d_c is the time span over which we want the signal to be covered, the step Δf must satisfy condition $\Delta f \leq 1/2d_c$. In general, this condition is not worse than the one associated with the discrete FRF resolution problem.

3. Regularization of the inverse problem

After obtaining the Toeplitz system, the inversion of Eq.(2.2) can be used to tackle the inverse problem of reconstructing the pressure signal. Because the matrix T is generally unconstrained, its inversion must be regularized. We employ a regularization technique based on generalized decomposition in singular values, followed by a truncation filter. The transformation associated with the problem defined in Eq.(2.2) that employs GSVD regularization is as follows

$$P = \Psi\Phi\Delta^{-1}U^t \times W = T^* \times W \tag{3.1}$$

where Ψ , U , Δ are the singular elements of T matrix, $T^* = \Psi\Phi\Delta^{-1}U^t$ is the regularized pseudo-inverse of T and Φ is the filter parameter.

Without filter factors, a low problem amplitude value defined by Eq.(3.1) makes the problem ill-conditioned. Filtering factors must be used to reduce the effect of low amplitude. The most commonly used regularization techniques are described in [16-17]. The regularization technique based on GSVD truncation will be used in the remainder of this work. Truncation is the process of eliminating the first terms of low indexes up to row k. This indicator is known as the modification variables. To eliminate oscillating singular vectors and small generalised singular values, the k-index must be chosen. The filter defined in Eq.(3.1) has the following form:

$$\Phi_{ij} = \lambda_i \delta_{ij} \quad i, j = 1,...,n \tag{3.2}$$

where $\lambda_i = 0$ if $i < k$ and $\lambda_i = 1$ otherwise.

To construct the Φ filter for the truncation technique, rank k must be identified. A range of criteria can be used for this purpose. Here, working with semi sinusoidal pressure signals, the digital experiment showed that $k \in [N-10, N]$ with $N = 1024$.

4. Case study

4.1. Problem analysis

The structure we model in the next section accepts the configuration presented in Fig.1. It is supposed to have the following dimensions: a thickness $e = 2h$ with $h = 0.01m$ and a length $l = 0.25m$, the thickness is common to the two homogeneous plates of the structure. The two plates will be designated by index *1* for the plate on the left, relative to the given orientation of the x-axis, and index *2* for the plate on the right. The left

plate has a length $l_1 = l = 0.25\,m$ and a width $b_1 = 2l = 0.5\,m$. The right plate has a square geometry for which the two dimensions shown in Fig.1 are equal: $l_2 = b_2 = l = 0.25\,m$.

The structure composed by assembly of the two plates is assumed to be embedded at its left edge $x = 0\,m$ and free on all other edges. The material of plate *1* is aluminium while steel is taken for plate *2*. The Young modules of the materials are $E_1 = 70\,GPa$ and $E_2 = 210\,GPa$. The Poisson coefficients are respectively $\upsilon_1 = 0.33$ and $\upsilon_2 = 0.3$, and the densities are $\rho_1 = 2700\,kg \cdot m^{-3}$ and $\rho_1 = 7800\,kg \cdot m^{-3}$.

A FE model of the structure was created using Abaqus software. The S4R shell element, which allows four nodes and five DOF per node and for which the reduced integration option is used, was used to mesh the structure. It is very important to use as option finite membrane deformations in order to avoid the appearance of parasitic modes like Hourglass blocking modes that represent modes of numerical insatiability associated with formulation with small membrane deformations.

In order to examine modal convergence, three different meshes were studied. These mesh sizes correspond to the following steps: *0.02 m* for coarse mesh; *0.01 m* for intermediate mesh size and *0.005 m* for refined mesh size. We use the Lanczos procedure by specifying that the own modes must be dimensioned in relation to the mass distribution. Figure 2 shows the first *100* frequencies calculated. Table 1 shows the convergence of the first five frequencies.

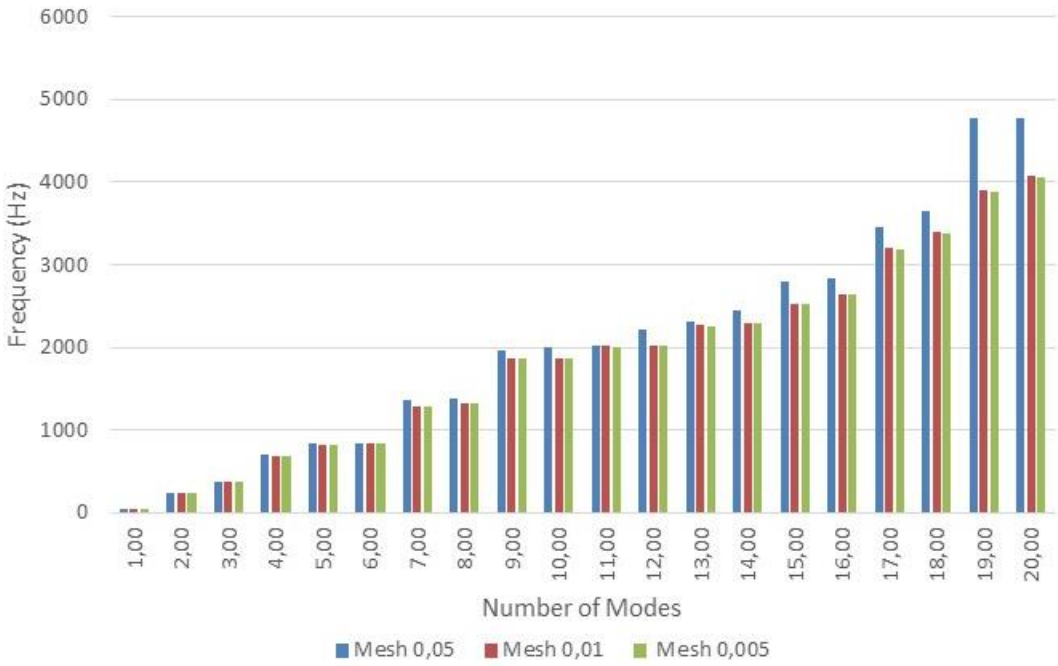


Fig.2. Convergence of the first *20* modes according to the pitch of the mesh used.

Table 1. The impact of mesh size on the structure's five specific frequencies.

Mesh size (*m*)	Mode 1 (*Hz*)	Mode 2 (*Hz*)	Mode 3 (*Hz*)	Mode 4 (*Hz*)	Mode 5 (*Hz*)
0.02	*55.219*	*245.282*	*376.348*	*687.685*	*817.825*
0.01	*55.161*	*244.877*	*374.972*	*683.963*	*813.839*
0.005	*55.1374*	*244.708*	*374.506*	*682.591*	*812.339*

Figure 2 and Tab.1 show that the modal convergence of the FE model is achieved for the first 65 modes using the intermediate mesh associated with the 0.01m value step. Indeed, there is no significant difference between the intermediate and the fine meshes for all these frequencies. On the other hand, for the last *35* modes

between the *66th* and *100th* a small difference appears. Note also that for the first *20* modes the coarse mesh already allows prediction of the clean frequencies with a good precision. We can therefore estimate that the convergence for the first *100* modes takes place for the fine mesh for which the mesh pitch is set at *0.005 m*. The finite element model in this case accepts the following characteristics:

- number of total nodes: *5928*,
- number of total elements: *3750*,
- element types: C3D8R.

4.2. Frequency response function

Using Eqs (2.3)and (2.4) we can obtain the FRF between any measurement point chosen on the structure and the impact zone. It is important to note that the terms that represent modal participations in the truncated series of the second member of the Eq.(2.4) are expressed as a function of modal depreciation. Its renders if possible allows in particular to avoid the singularity that manifests at the passage of resonant frequencies $w = w_k$. The coefficient β_k which depends on the mode considered has a considerable influence on the speed of the FRF and on the response of the structure. In practice, it must be measured for each mode. However, this operation is tricky and can be fraught with errors. Techniques for identifying modal damping exist in this field.

In the framework of this study, we will assume that the modal depreciation β_k is constant for all modes selected in the development before truncation.

We consider an impact occurring on a rectangular area of the structure for which the characteristics of the impact area correspond to a coordinate center: $s_0 = -0.15625\ m$ and $r_0 = -0.09375\ m$ and an extent that is given by $u_0 = 0.0625\ m$ and $v_0 = 0.0625\ m$.

The displacement sensors are assumed to be located on the structure at the points indicated in Fig.3. Their coordinates for the marker shown in this figure are given in Tab.2.

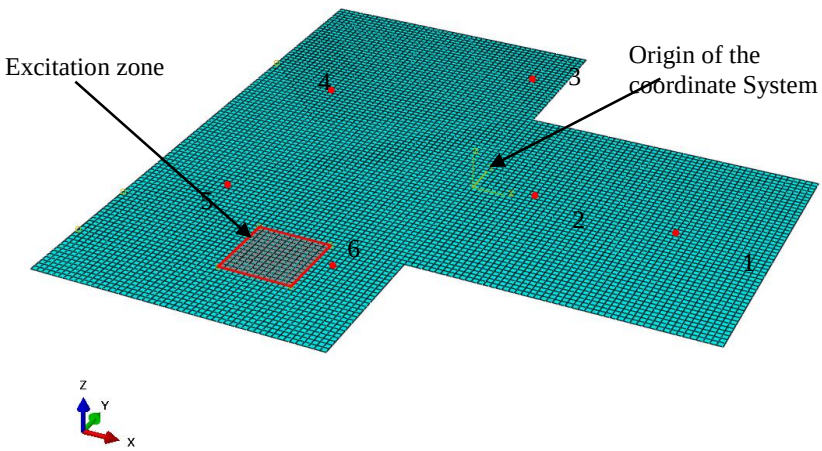
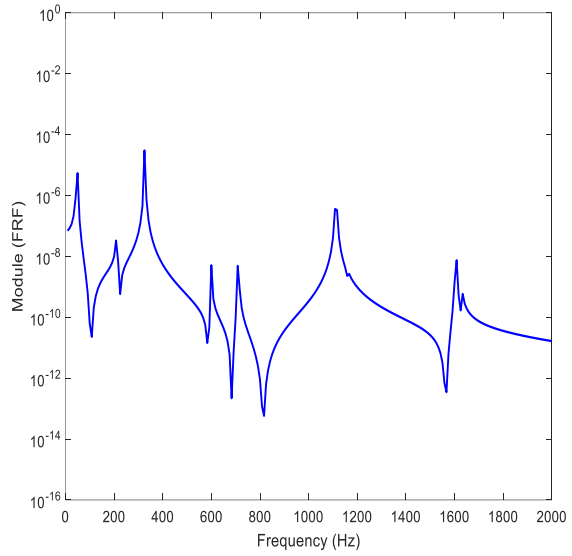


Fig.3. Fine mesh used for the structure showing the excitation area (red colour) and the positions of the transverse displacement sensors (depending on the *z* direction).

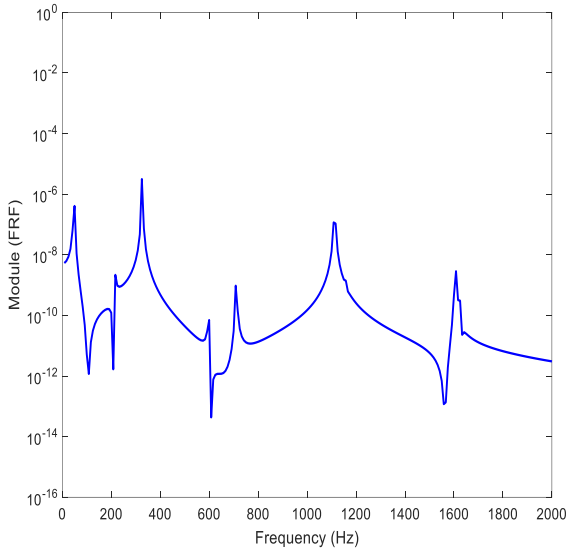
Figure 3 shows the fine mesh constructed using the CAE interface of the Abaqus software and used to develop the modal model. The impact area is represented in this figure by the red rectangle. In the construction of the FRF between the excitation zone and the response points, the numbers of the mesh nodes associated with the impact zone and the positions of the six sensors are used. For the sensors the numbers are given in Tab.2. Those of the impact area are obtained by defining a set in the form of a geometric surface containing them.

Table 2. Sensor positions relative to the mark shown in Fig.3.

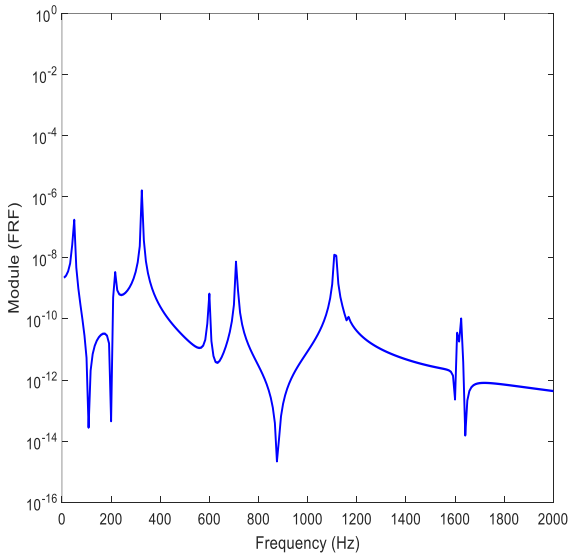
Node number on Fig.3	Mesh node number	*x*	*y*	*z*
1	*3412*	*0.17*	*0.005*	*0.02*
2	*3422*	*0.07*	*0.005*	*0.02*
3	*2603*	*-0.0416667*	*0.210864*	*0.02*
4	*1851*	*-0.173077*	*0.121711*	*0.02*
5	*1793*	*-0.182692*	*-0.0855263*	*0.02*
6	*154*	*-0.0520833*	*-0.15625*	*0.02*



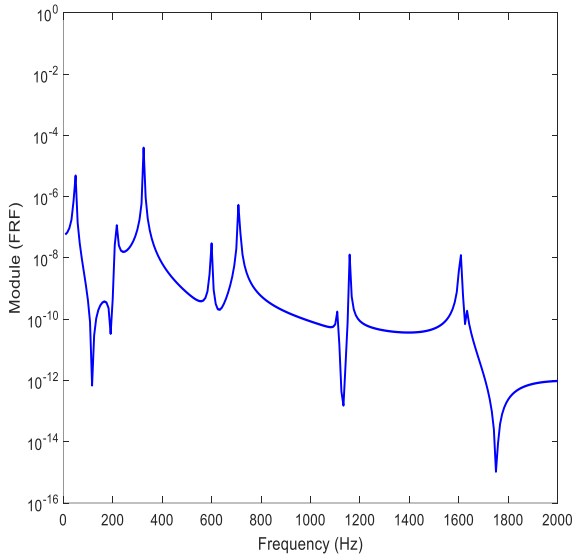
(a) Sensor 1



(b) Sensor 2

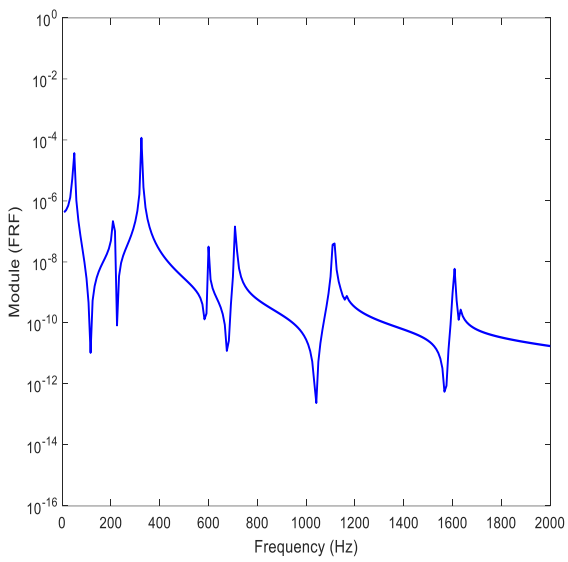


(c) Sensor 3

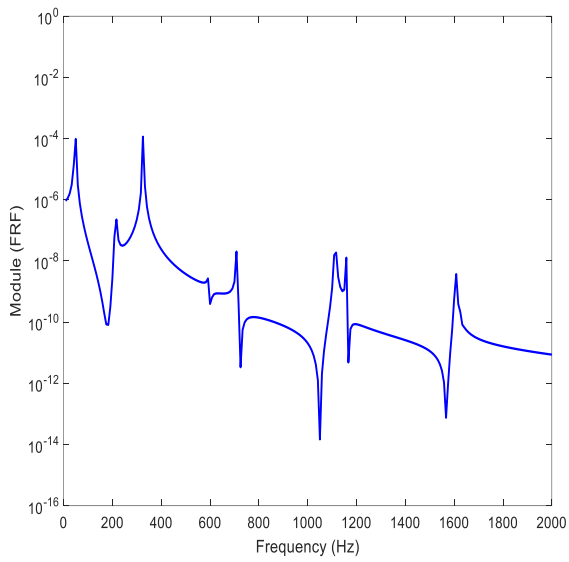


(d) Sensor 4

Fig.4. FRF module between impact area under uniform pressure and 6 sensors located according to Fig.3.



(e) Sensor 5



(f) Sensor 6

cont.Fig.4. FRF module between impact area under uniform pressure and 6 sensors located according to Fig.3.

By selecting the first *10* modes of the structure for which the maximum frequency is $f_{\max} = 1627.2\ Hz$, we have shown in Fig.4 the FRF module for all the sensors shown in Fig.3. The damping coefficient used is $\beta_k = 8.10^{-4}$.

4.3. Calculation of the response by the modal model

We plan to calculate the response over a time interval $d_c = 0.12\ s$. The time signal of the excitation pressure is taken as a double half-sinus over the time period $d = 0.01\ s$ and zero over the rest of the interval $[0, 0.12]\ s$. This signal is shown in Fig.5.

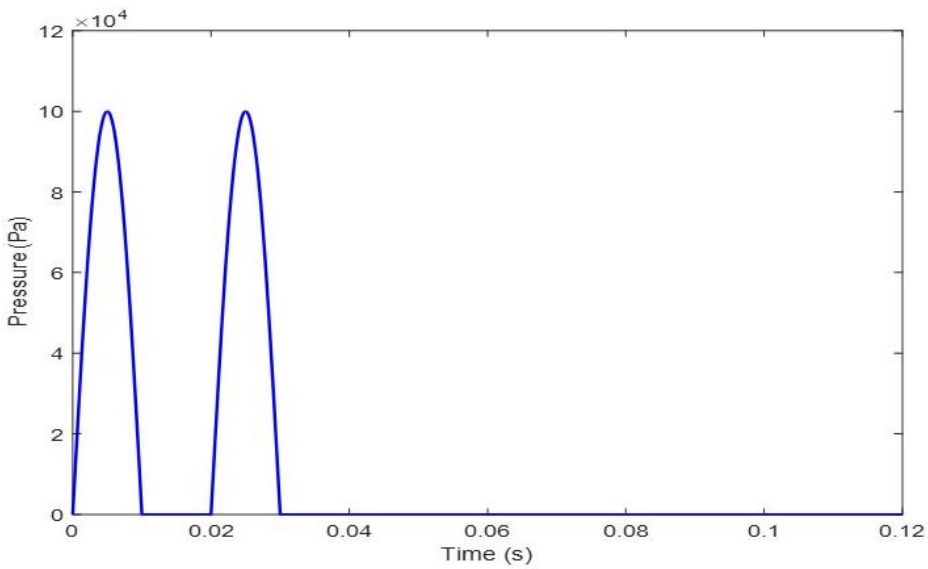


Fig.5. Time profile of the impact pressure signal.

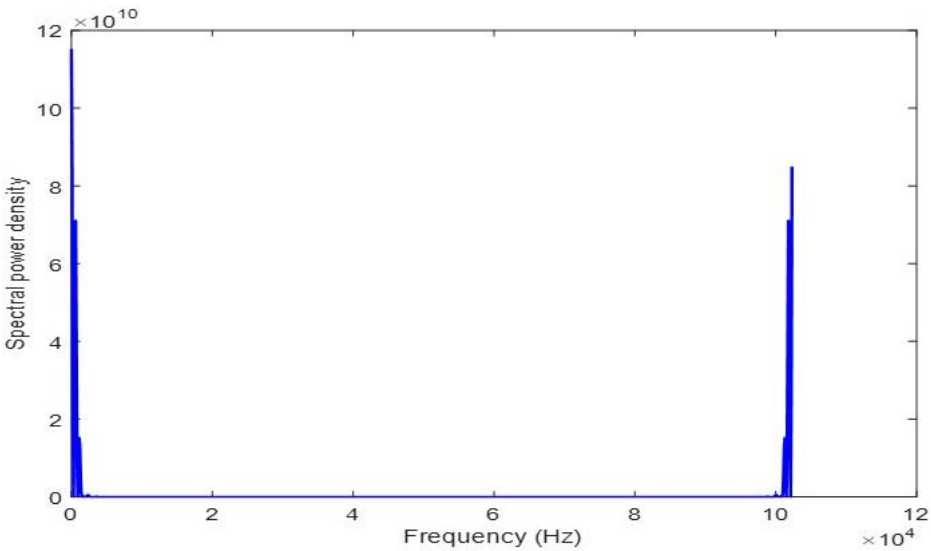


Fig.6. Impact pressure signal power spectral density.

The frequency spectrum associated with the excitation pressure determines which modes are likely to be dynamically excited. This makes it possible to distinguish them from those associated with high frequencies which will only intervene statically in the response of the structure. The spectrum can be obtained by using Matlab's *fft* command to calculate the fast Fourier transform.

Figure 6 shows the spectral power density of the pressure signal shown in Fig.5. We used the time step $\Delta d = 1.171875 \times 10^{-4}\, s$ for the calculation of this spectrum, which corresponds to using a sample size *1024* on [*0, 0.12*] *s*. Figure 6 shows that the maximum frequency is around *220 Hz*. Here we choose $f_{\max} = 1627.2\ Hz$ which corresponds to the first *5* modes. We can check that $\Delta d < 1/(2 f_{\max})$ which reflects the fact that Shannon's sampling condition is met.

With the FRF in Section 4.2, it is possible to construct a modal model using Eqs (2.5) and (2.6). We have chosen a modal model comprising the first *5* modes of the structure and we consider the impact problem outlined in Fig.3.

To find the FRF between the impact area and the mounting point of a sensor, we multiply the pressure applied by the surface of an element *i.e.* $S_e = 2.5 \times 10^{-5}\, m^2$, and we add together the contributions of all the nodes in the impact zone which are *196* in Eq.(2.5).

We then calculate the Toeplitz matrix using Eq.(2.6) with the choice of $\Delta d = 5 \times 10^{-4}\, s$ and $\Delta f = 1/(Nf \times \Delta t) = 0.48828125\, Hz$ where the number of points to operate the inverse Fourier transform is set to $Nf = 4096$.

We classified the sensors into three families according to the level of transverse displacement amplitude obtained:

- Sensors 1 and 2 which give strong amplitudes;
- Sensors 3 and 6 which give average amplitudes;
- Sensors 4 and 5 which give low amplitudes because they are placed near the underrun.

Figures 7, 8 and 9 give the *z* transverse displacement as calculated by the dynamic implicit model for these three sensor families.

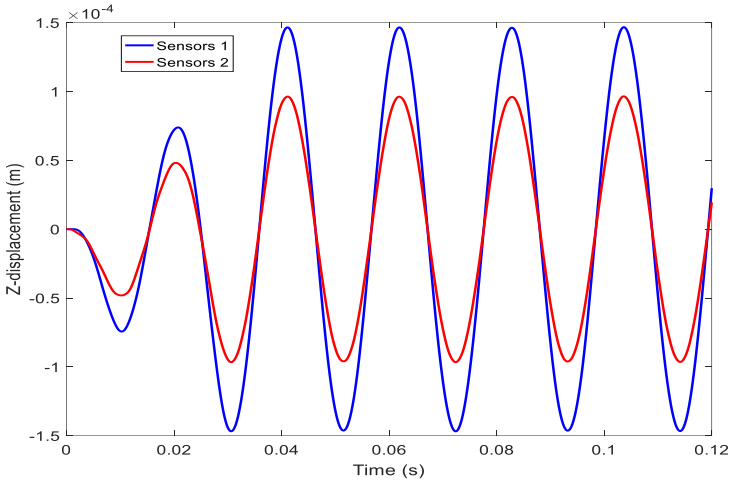


Fig.7. *Z*-axis displacement associated with sensors *1* and *2* as calculated by the transitional dynamic model.

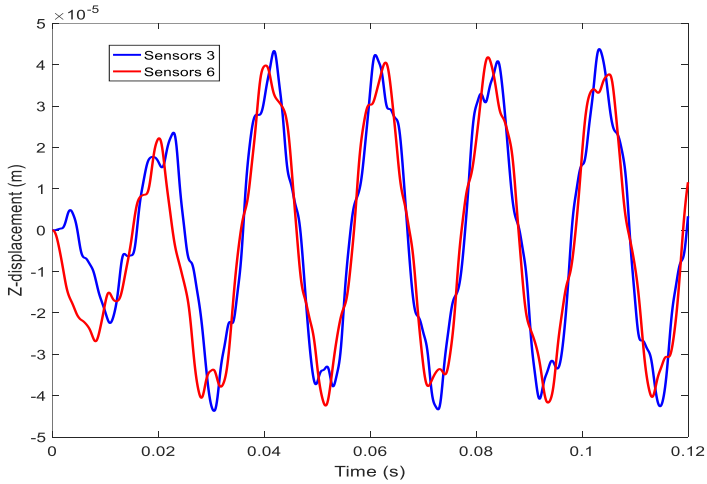


Fig.8. *Z*-axis displacement associated with sensors *3* and *6* as calculated by the transitional dynamic model.

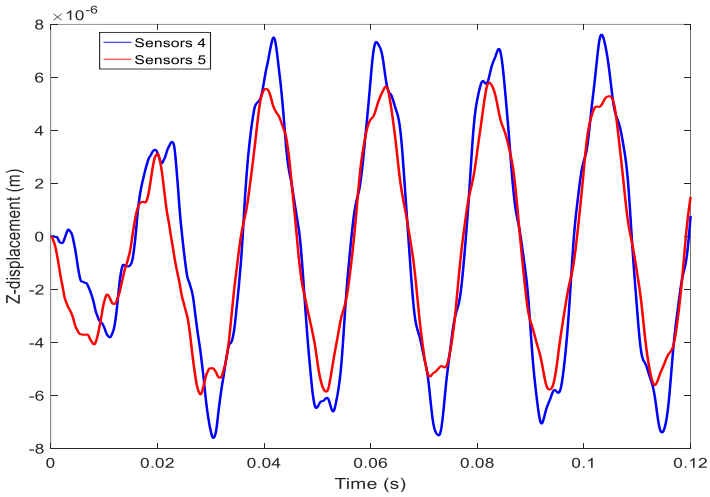


Fig.9. *Z*-axis displacement associated with sensors *4* and *5* as calculated by the transitional dynamic model.

4.5. Impact pressure reconstruction

With the data specified in Sections 4.1, 4.2 ,4.3 and 4.4 used to define the direct problem, we consider the reconstruction of the discrete impact pressure P in the case where it is supposed that the impact zone will be obviously identified in this problem. Figure 10 illustrate that if the regularization technique is not used the reconstruction of the impact force is very bad. Figure 11 shows in the case of sensor *1* the superposition of the rebuilt force with the actual force using *5* modes and the inversion method regularized by the GSVD.

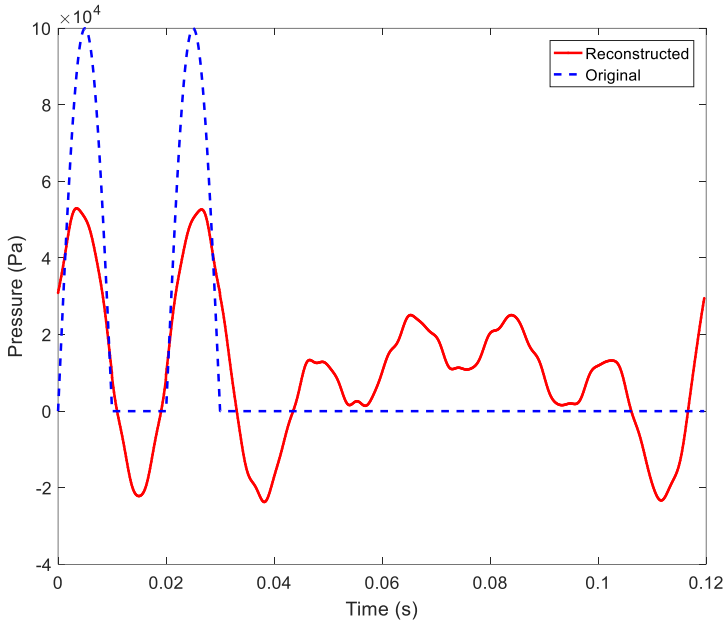


Fig.10. Evaluation of the input force without regularization with the reconstructed force profile.

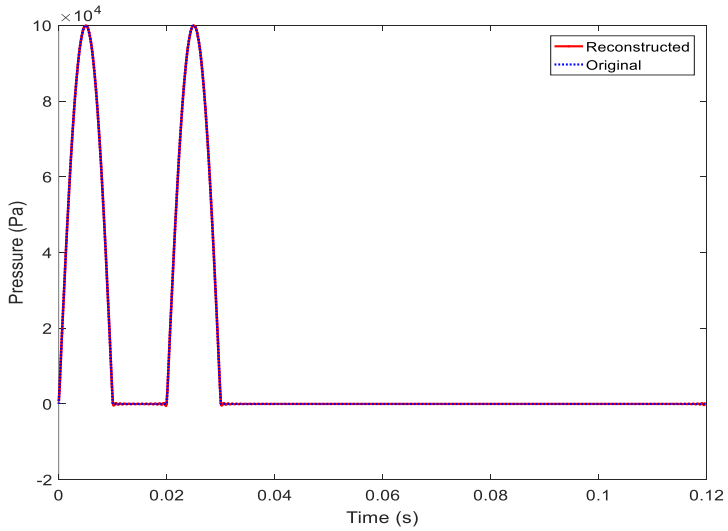


Fig.11. Comparison of the reconstructed force profile with the input force.

In addition, in order to describe the effectiveness of the TGSVD-based identification method, the relative error is calculated to determine the effect of the position of the measurement sensors. Figure 12 shows

that the reconstruction depends on the position of the sensors. This result shows that the sensor that gives important displacement values gives a better reconstruction.

The results obtained in this section show the possibility of reconstructing the signal of an impact force in the case of any complex structure using a modal model based on the first modes of the structure. The position of the sensors vis-à-vis the excitation point is very important because the quality of the reconstruction depends crucially on it. But theoretically the limitation is due to the quality of the modal model which is affected by the following two weaknesses: the modal depreciation that must be identified in practice and the calculation of the inverse Fourier transform which does not tolerate a large number of discretization points in the frequency domain.

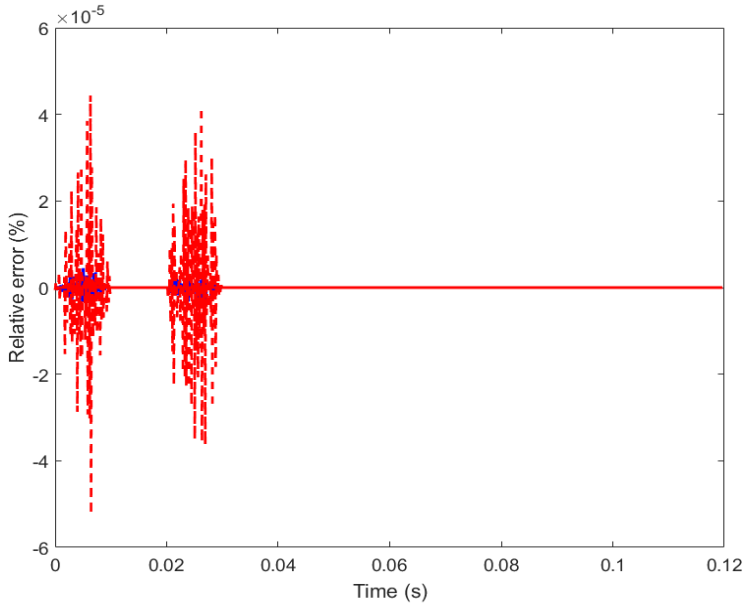


Fig.11. Comparison of the relative error between two sensors (sensor 1 blue colour and sensor 3 red colour).

5. Conclusions

The problem of reconstruction under impact pressure was addressed in this paper. The presented reconstruction method is based on the discretization of the convolution product, which describes the problem's transient dynamics, and for which the generalized decomposition in singular values of the obtained Toeplitz matrix may be required. The implementation of a generalist numerical approach based on the finite element method has shown how to reconstruct an impact force occurring on any structure. In practice, the impact spectrum is of low frequency, which simplifies the development of the modal model because only a few modes are required. So, once the mesh allowing the finite element method's convergence for the first modes is determined, these can be used to build the system's FRF, whose inversion then allows the FRT to be generated. The Toeplitz matrix convolution product can then be obtained in discrete form. The time signal of the impact force was reconstructed using inversion and TGSVD-based regulation.

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Nomenclature

FEM – Finite Element Method

FRF – Frequency Response Function

FRT – Time Response Function

GSVD – General Singular Value Decomposition

IFT – Inverse Fourier Transform

DOF – Degrees of Freedom

$y(k)$ – discrete dynamic response

$P(k)$ – discrete input pressure

$t(k)$ – temporal FRT

T – Toeplitz matrix

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