NUMERICAL STUDY OF A TRANSIENT MHD FLOW ACROSS AN OSCILLATING VERTICAL PLATE WITH THERMAL RADIATION AND VISCOUS DISSIPATION

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The flow of an electrically conducting fluid across a vertically positioned oscillating semi-boundless plate with uniform mass diffusion and temperature is examined in this study in terms of the effects of thermal radiation and viscous dissipation. The dimensionless governing equations were solved using an effective and unconditionally stable implicit finite-difference approach known as the Crank-Nicolson method. Based on the numerical results, the impacts of various physical parameter values on concentration, temperature; velocity; Sherwood numbers, Nusselt numbers and skin-friction profiles are displayed graphically and their consequences thoroughly analyzed. We observed that when the magnetic field, radiation and phase angle parameters are increased, the velocity is reduced. This shows that plate oscillation, radiation and magnetic fields affect the flow pattern significantly.

Keywords: MHD, finite-difference, thermal radiation, viscous dissipation, oscillating.

1. Introduction

Due to its numerous applications in astrophysics, fusion and the cooling of fission reactors, magna-hydrodynamics (MHD) has drawn the attention of many research scholars. Magnetohydrodynamic flows occur when an electrically conducting fluid (such as plasma or an ionized gas) passes through a magnetic field. The magnetic field is crucial in determining the structure of the corona, initiating mass ejections and triggering solar flares. The study of ionized gases as well as liquid metals in the existence of magnetic fields relies heavily on a free convection flow. Thermal radiation as well as viscous dissipation have a significant impact on geophysical flows, aerospace, chemical, solar and mechanical engineering. Using the Rosseland transition model, the radiation effects of heterogeneous convection on an upward plate with an unvarying surface temperature were deliberated by Hossain and Takhar [1]. According to Raptis *et al*. [2], thermal radiation has a strong effect on a vertically placed infinite plate with an induced magnetic field in an optically thin gray gas. Soundalgekar *et al*. [3] obtained an accurate result for a magnetic free convection flow across a swaying plate with uniform heat flux. Chaudhary and Arpita Jain [4] explored the magnetohydrodynamic fluid flowing through a vertically placed boundless swaying plate with a permeable medium. Rajput and Kumar [5] examined the MHD flow through a vertical plate subject to impulsive heating and varying temperature and mass diffusion. Seth *et al*. [6] employed an impulsively moving limitless upward plate with a ramping temperature to investigate the influence of rotation and radiation on the free convection flow of a viscous, electrically conducting fluid in a permeable medium. Anand Kumar and Singh [7] demonstrated that an induced magnetic field affects a fluid that is hydromagnetic and electrically conducting. Sharidan Shafie *et al*.

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[8] examined the instability of the boundary layer flow of Casson fluid on a vertically placed oscillating plate with a constant temperature of the wall. Using the Laplace transform method, they solved the dimensionless governing equations. Using ramping wall temperatures and isothermal temperatures, Kataria and Patel [9] investigated heat and mass transmission through an upward oscillating plate immersed in an unsteady natural convection MHD Casson fluid flow.

Azzam [10] investigated the semi-boundless upward plate containing a thick optically and electrically conducting viscous fluid moving at high temperature differences for radiation effects on magnetohydrodynamic convection. The author applied the Runge-Kutta method as well as the shooting technique for evaluating the boundary layer ordinary differential equations. The radiation effect on a transient magnetohydrodynamic natural convection flow at varying temperatures along an upward plate was examined by Abd-El-Naby *et al*. [11]. Muthucumaraswamy and Janakiraman [12] looked into how thermal radiation affected an unsteady natural convection flow over an ascending moving plate with mass transfer when there was a magnetic field present. Deka and Neog [13] investigated the radiation effect on a viscous, incompressible, hydromagnetic fluid flowing across a swaying upward plate with varying mass diffusion. Chamkha *et al*. [14] used numerical simulations to investigate the effects of thermal radiation and rotation on the transient free magnetohydrodynamic flow through an infinitely tall, movable absorbent plate. Vijaya Kumar and Vijaya Kumar Verma [15] investigated an unsteady magnetohydrodynamic flow through a vertically placed swaying plate with a mass diffusion, temperature gradient and transversely applied magnetic field. In their analysis of an MHD fluid flow through a permeable, vertically stretched sheet, Rashidi *et al*. [16] looked at free convective mass and heat transfer, as well as buoyancy and radiation effects. They used the Homotopy Analysis Method (HAM) to solve the governing boundary layer equations. Sahin Ahmed *et al*. [17] looked into the characteristics of heat and mass transfer across an oscillating vertical plate embedded in a Darcian permeable medium, as well as the effects of thermal radiation on a non-linear MHD viscous, electrically conducting, incompressible and Boussinesq's fluid flow. Muthucumaraswamy and Saravanan [18] performed a numerical analysis of the impact of a transient MHD flow through a vertically positioned oscillating plate subjected to uniform mass flux and heat radiation. Raptis [19] analysed heat radiation effects on the MHD flow of an optically thin, gray, electrically conducting, viscous and incompressible fluid over a vertical porous plate. In a porous, saturated medium with uniform permeability, varying temperatures and varying concentrations, Pattnaik *et al*. [20] examined the phenomenon of a transient natural convection MHD flow as well as the transfer of mass and heat on an exponentially slanted accelerating plate. In addition to this, they investigated how the existence of a destructive reaction and a heat source/sink influences flow phenomena when seen from a variety of perspectives. Rajput and Gaurav Kumar [21] investigated the flow of an electrically conducting, viscous and incompressible fluid through an inclined plate in a consistent transverse magnetic field while subjected to radiation and Hall currents. Prabhakar Reddy and Makinde [22] investigated the flow of an electrically conducting fluid through a slippery, permeable upward plate encased in a porous medium with an irregular hydro-magnetic boundary layer. The authors employed the Galerkin finite element method to resolve the governing partial differential equations. Yanala Dharmendar Reddy *et al*. [23] investigated the erratic MHD laminar boundary layer flow across a movable, accelerating upward plate subjected to radiation, heat absorption and chemical reactions. Lakshmi *et al*. [24] demonstrate the influence of thermal radiation influences the erratic MHD flow of an electrically conducting viscous fluid passing through an upward oscillating plate when an impulsively initiated oscillation is triggered by changing temperature and constant mass diffusion. The dimensionless, governing partial differential equations of the flow were numerically solved using the Galerkin finite element method.

Nevertheless, the effects of viscous dissipation on flow are disregarded in all of the works referenced above. When considering an unsteady magnetohydrodynamic convective flow, energy dissipation is significant. The importance of viscous dissipation in a natural convection flow was first demonstrated by Gebhart [25] using a semi-boundless isothermal vertical plate. Israel-Cookey *et al*. [26] simulated the impacts of an unsteady MHD free convective flow over a heated plate that was placed vertically in a porous medium with viscous dissipation and radiation. On a sloped, porous plane with wavering wall concentration and temperature, Chen [27] investigated the viscous dissipation as well as the impacts of Ohmic heating on heat and mass transfer and the momentum specifications of MHD natural convection flow. Under a uniform

transverse magnetic field, Prasad and Reddy [28] investigated the transient two-dimensional laminar natural convection flow of a viscous, electrically conducting fluid past a vertically heated, suction-producing porous plate embedded in a porous medium. Zueco Jordan [29] investigated an MHD natural convection flow on a vertical porous plate under the influence of radiation and viscous dissipation using a numerical simulation technique. Suneetha *et al.* [30] investigated how viscous dissipation and thermal radiation affected natural convective heat and mass transfer in an electrically conducting, viscous, incompressible fluid moving through an abruptly moveable vertical plate. Kishore *et al.* [31] studied the impacts of radiation and viscous dissipation on an MHD flow across an oscillating upward plate in a permeable medium with varying heat and mass diffusion. They implemented DuFort-Frankel's type for solving the dimensionless governing equations. Siva Reddy Sheri and Srinivasa Raju [32] investigated the Soret impacts on unstable magnetohydrodynamic natural convection flow using a semi-boundless upward plate. The authors employed the Finite Element Method (FEM) for solving the nonlinear partial differential equations. An attempt was made by Sreenivasulu *et al.* [33] to investigate how thermal radiation affects the slip flow of MHD boundary layers adjacent to a permeable exponential stretching surface with Joule heating and viscous dissipation. To solve the governing boundary layer equation, they used both the Runge-Kutta and shooting methods. Hasan *et al.* [34] analysed the MHD natural convective flow via an inclined stretched sheet. In their research, the authors considered the impacts of radiation and viscous dissipation as well. They solved the governing equations using the six-order Runge-Kutta iteration method as well as the Nachtsheim-Swigert shot iteration technique. The effects of thermal and viscous dissipation on an impulsively started inclined oscillating plate with changing temperature and mass diffusion were investigated by Shankar Goud and Rajashekar [35]. To solve the dimensionless governing equations, they employed the Finite Element Method. Jithender Reddy *et al.* [36] investigated an unstable magnetohydrodynamic natural convection, heat transfer and electrically conductive non-Newtonian Casson fluid along an oscillating upward porous plate with viscous dissipation. In a porous medium with different surface conditions, Prabhakar Reddy and Muthucumaraswamy [37] investigated the effect of heat radiation on the flow of an incompressible, electrically conducting fluid through an oscillating upward plate. They employed Ritz Finite Element Method for solving the equations and obtaining flow results. The objective of this paper is to look at how thermal radiation and viscous dissipation affect the transient MHD convection flow across a semi-boundless, oscillating upward plate with constant mass and surface temperature. Crank-Nicolson's finite-difference approach is used to get numerical solutions for dimensionless coupled nonlinear partial differential equations.

2. Problem formulation

Consider the transient MHD flow of a viscous, incompressible, electrically conducting fluid in two dimensions along a semi-boundless oscillating vertical plate. Here, the $x-$axis follows the plate vertically upward and the $y-$axis follows the plate normally. Initially, both the fluid and plate have the same concentration C'_∞ and temperature T'_∞. The temperature and concentration level of the plate are uplifted to T'_w and C'_w respectively, when the plate begins oscillating in its own plane with velocity $u_0 cos\omega't'$ at time $t' > 0$. Perpendicular to the plate, a magnetic field of uniform strength B_0 was applied. The induced magnetic field is negligible in comparison to the applied magnetic field because of the magnetic Reynolds number and viscous dissipation is taken into account when calculating energy. The fluid under consideration here is gray, absorbs and emits radiation, but not scattering medium. The boundary layer equations for free convection flow with the standard Boussinesq's approximation are provided with these constraints and assumptions:

$$u_x + u_y = 0 , \tag{2.1}$$

$$u_{t'} + uu_x + vu_y = g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) + \upsilon u_{yy} - \frac{\sigma B_0^2 u}{\rho} , \tag{2.2}$$

$$T'_{t'} + uT'_x + vT'_y = \frac{k}{\rho C_p} T'_{yy} - \frac{1}{\rho C_p}(q_r)_y + \frac{\upsilon}{\rho C_p}(u_y)^2,$$ (2.3)

$$C'_{t'} + uC'_x + vC'_y = DC'_{yy},$$ (2.4)

subject to

$$
\left.
\begin{array}{l}
I.C: t' \leq 0 : u = 0, \ v = 0, \ T' = T'_\infty, \ C' = C'_\infty, \\[2mm]
B.C: t' > 0 : u = u_0 \cos \omega' t', \ v = 0, \ T' = T'_w, \ C' = C'_w \ \text{at} \ y = 0, \\[2mm]
\qquad u = 0, \ T' = T'_\infty, \ C' = C'_\infty \qquad\qquad \text{at} \ x = 0, \\[2mm]
\qquad u \to 0, \ T' \to T'_\infty, \ C' \to C'_\infty \qquad\quad \text{as} \ y \to \infty,
\end{array}
\right\}
$$ (2.5)

The local radiant for a gray gas that is optically thin is given by

$$(q_r)_y = -4a^*\sigma\left(T'^4_\infty - T'^4\right).$$ (2.6)

Assuming that the temperature differences within the flow are small enough, so that Eq.(2.6) can be linearized by extending T'^4 in a Taylor series near T'_∞ and ignoring the terms of higher order, it has the following form:

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty.$$ (2.7)

Applying Eqs (2.6) and (2.7) in Eq.(2.3), we get:

$$T'_{t'} + uT'_x + vT'_y = \frac{k}{\rho C_p}\left(T'_{yy}\right) + \frac{16a^*\sigma T'^3_\infty\left(T'_\infty - T'\right)}{\rho C_p} + \frac{\upsilon}{\rho C_p}\left(u_y\right)^2.$$ (2.8)

The following boundless quantities are established by using the boundary layer equations and conditions:

$$
\left.
\begin{array}{l}
X = \dfrac{xu_0}{\upsilon}, \quad Y = \dfrac{yu_0}{\upsilon}, \quad U = \dfrac{u}{u_0}, \quad V = \dfrac{v}{u_0}, \quad t = \dfrac{t'u_0^2}{\upsilon}, \quad \omega = \dfrac{\omega'\upsilon}{u_0^2}, \\[4mm]
T = \dfrac{T' - T'_\infty}{T'_w - T'_\infty}, \quad C = \dfrac{C' - C'_\infty}{C'_w - C'_\infty}, \quad Gr = \dfrac{\upsilon g\beta(T'_w - T'_\infty)}{u_0^3}, \quad Gc = \dfrac{\upsilon g\beta^*(C'_w - C'_\infty)}{u_0^3}, \\[4mm]
Pr = \dfrac{\upsilon}{\alpha}, \quad Sc = \dfrac{\upsilon}{D}, \quad Ec = \dfrac{u_0^2}{C_p(T'_w - T'_\infty)}, \quad R = \dfrac{16a^*\upsilon^2\sigma T'^3_\infty}{ku_0^2}, \quad M = \dfrac{\sigma B_0^2\upsilon}{\rho u_0^2},
\end{array}
\right\}.
$$ (2.9)

Substituting Eq.(2.9) in Eqs (2.1)-(2.4), we get:

$$U_X + V_Y = 0,$$ (2.10)

$$U_t + UU_X + VU_Y = Gr\mathcal{T} + Gc\mathcal{C} + U_{YY} - MU \,, \tag{2.11}$$

$$\mathcal{T}_t + U\mathcal{T}_X + V\mathcal{T}_Y = \frac{1}{\Pr}(\mathcal{T}_{YY}) - \frac{R\mathcal{T}}{\Pr} + Ec(U_Y)^2 \,, \tag{2.12}$$

$$\mathcal{C}_t + U\mathcal{C}_X + V\mathcal{C}_Y = \frac{1}{Sc}\mathcal{C}_{YY} \,, \tag{2.13}$$

subject to

$$\left.\begin{array}{l}
t \le 0 : U = 0, \ V = 0, \ \mathcal{T} = 0, \ \mathcal{C} = 0, \\[2mm]
t > 0 : U = \cos\omega t, \ V = 0, \ \mathcal{T} = 1, \ \mathcal{C} = 1 \quad \text{at} \ Y = 0, \\[2mm]
\qquad U = 0, \ \mathcal{T} = 0, \ \mathcal{C} = 0 \qquad\qquad \text{at} \ X = 0, \\[2mm]
\qquad U \to 0, \ \mathcal{T} \to 0, \ \mathcal{C} \to 0, \qquad\quad \text{as} \ Y \to \infty,
\end{array}\right\}. \tag{2.14}$$

The boundless form of local and average skin-friction are:

$$\tau_X = -\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \quad \text{and} \quad \overline{\tau} = -\int_0^1 \left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX \,. \tag{2.15}$$

The boundless form of local and average Nusselt number are:

$$Nu_X = -X\left[\left(\frac{\partial \mathcal{T}}{\partial Y}\right)_{Y=0} \middle/ \mathcal{T}_{Y=0}\right] \quad \text{and} \quad \overline{Nu} = -\int_0^1 \left[\left(\frac{\partial \mathcal{T}}{\partial Y}\right)_{Y=0} \middle/ \mathcal{T}_{Y=0}\right] dX \,. \tag{2.16}$$

The boundless form of local and average Sherwood number are:

$$Sh_X = -X\left[\left(\frac{\partial \mathcal{C}}{\partial Y}\right)_{Y=0} \middle/ \mathcal{C}_{Y=0}\right] \quad \text{and} \quad \overline{Sh} = -\int_0^1 \left[\left(\frac{\partial \mathcal{C}}{\partial Y}\right)_{Y=0} \middle/ \mathcal{C}_{Y=0}\right] dX \,. \tag{2.17}$$

3. Numerical strategy

The nonlinear coupled partial differential equations (2.10)-(2.13) under the conditions given in Eq. (2.14) are evaluated by an unconditionally stable implicit finite difference scheme called Crank-Nicolson methodology. The following are the finite difference equations that correspond to Eqs (2.10)-(2.13):

$$\frac{\left[U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^n - U_{i-1,j}^n + U_{i,j-1}^{n+1} - U_{i-1,j-1}^{n+1} + U_{i,j-1}^n - U_{i-1,j-1}^n\right]}{4\Delta X} +$$

$$+\frac{\left[V_{i,j}^{n+1} - V_{i,j-1}^{n+1} + V_{i,j}^n - V_{i,j-1}^n\right]}{2\Delta Y} = 0, \tag{3.1}$$

$$\frac{\left[U_{i,j}^{n+1} - U_{i,j}^{n}\right]}{\Delta t} + U_{i,j}^{n} \frac{\left[U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^{n} - U_{i-1,j}^{n}\right]}{2\Delta X} +$$

$$+ V_{i,j}^{n} \frac{\left[U_{i,j+1}^{n+1} - U_{i,j-1}^{n+1} + U_{i,j+1}^{n} - U_{i,j-1}^{n}\right]}{4\Delta Y} =$$

$$= \frac{Gr}{2}\left[T_{i,j}^{n+1} + T_{i,j}^{n}\right] + \frac{Gc}{2}\left[C_{i,j}^{n+1} + C_{i,j}^{n}\right] - \frac{M}{2}\left[U_{i,j}^{n+1} + U_{i,j}^{n}\right] +$$

$$+ \frac{\left[U_{i,j-1}^{n+1} - 2U_{i,j}^{n+1} + U_{i,j+1}^{n+1} + U_{i,j-1}^{n} - 2U_{i,j}^{n} + U_{i,j+1}^{n}\right]}{2(\Delta Y)^2},$$

(3.2)

$$\frac{\left[T_{i,j}^{n+1} - T_{i,j}^{n}\right]}{\Delta t} + U_{i,j}^{n} \frac{\left[T_{i,j}^{n+1} - T_{i-1,j}^{n+1} + T_{i,j}^{n} - T_{i-1,j}^{n}\right]}{2\Delta X} +$$

$$+ V_{i,j}^{n} \frac{\left[T_{i,j+1}^{n+1} - T_{i,j-1}^{n+1} + T_{i,j+1}^{n} - T_{i,j-1}^{n}\right]}{4\Delta Y} =$$

$$= \frac{1}{Pr} \frac{\left[T_{i,j-1}^{n+1} - 2T_{i,j}^{n+1} + T_{i,j+1}^{n+1} + T_{i,j-1}^{n} - 2T_{i,j}^{n} + T_{i,j+1}^{n}\right]}{2(\Delta Y)^2} +$$

$$- \frac{R}{2Pr}\left[T_{i,j}^{n+1} + T_{i,j}^{n}\right] + Ec\left[\frac{U_{i,j+1}^{n} - U_{i,j}^{n}}{\Delta Y}\right]^2,$$

(3.3)

$$\frac{\left[C_{i,j}^{n+1} - C_{i,j}^{n}\right]}{\Delta t} + U_{i,j}^{n} \frac{\left[C_{i,j}^{n+1} - C_{i-1,j}^{n+1} + C_{i,j}^{n} - C_{i-1,j}^{n}\right]}{2\Delta X} + V_{i,j}^{n} \frac{\left[C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1} + C_{i,j+1}^{n} - C_{i,j-1}^{n}\right]}{4\Delta Y}$$

$$= \frac{1}{Sc} \frac{\left[C_{i,j-1}^{n+1} - 2C_{i,j}^{n+1} + C_{i,j+1}^{n+1} + C_{i,j-1}^{n} - 2C_{i,j}^{n} + C_{i,j+1}^{n}\right]}{2(\Delta Y)^2}.$$

(3.4)

Here, i denotes the x – coordinate of the grid point, j the y – coordinate and n the time variable t. A rectangular mesh with sides $X_{max} = 1$ and $Y_{max} = 10$ is assumed. The sizes of mesh are fixed as $\Delta X = 0.05$, $\Delta Y = 0.125$ and $\Delta t = 0.01$ as time step. The finite-difference equations (3.1)-(3.4) represent a tridiagonal system of equations that can be solved using the Thomas algorithm, as explained in Carnahan *et al.* [38].

4. Outcomes and discussion

Numerous simulations have been employed to determine the effect of thermofluidic and hydrodynamic parameters on dimensionless velocities, temperatures, concentrations, skin-friction, Nusselt numbers and Sherwood numbers. In the current analysis, the following physical parameters are used by default: phase angle $\omega t = \dfrac{\pi}{6}$, radiation parameter $R = 2$ (strong thermal radiation), thermal Grashof number $Gr = 5$, mass Grashof number $Gc = 5$, magnetic parameter $M = 2$, Eckert number $Ec = 0.5$, Prandtl number $Pr = 0.71$ (air) and Schmidt number $Sc = 0.6$ (water vapour). All graphs, unless otherwise specified, correspond to these values.

Figure 1 depicts the consistent state velocity patterns for various values of the phase angle. As the phase angle is increased, it is seen that the velocity decreases. This demonstrates that high levels of oscillation
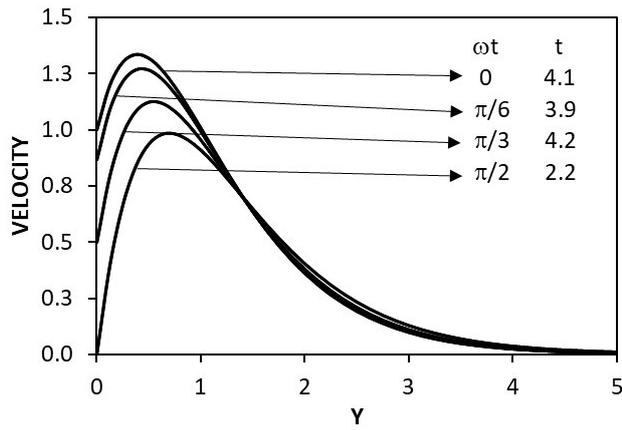
Fig.1. Velocity profiles for different values of ω*t*.
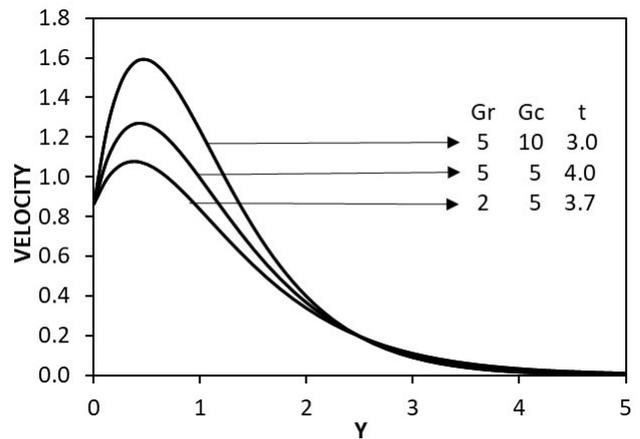
Fig.2. Velocity profiles for different values of *Gr* and *Gc*.
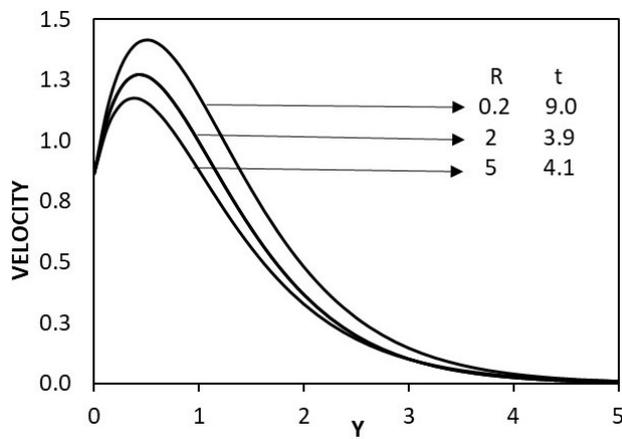
Fig.3. Velocity profiles for different values of *R*.
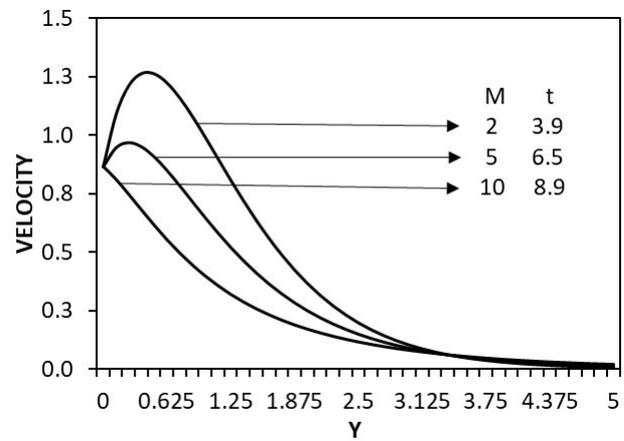
Fig.4. Velocity profiles for different values of *M*.
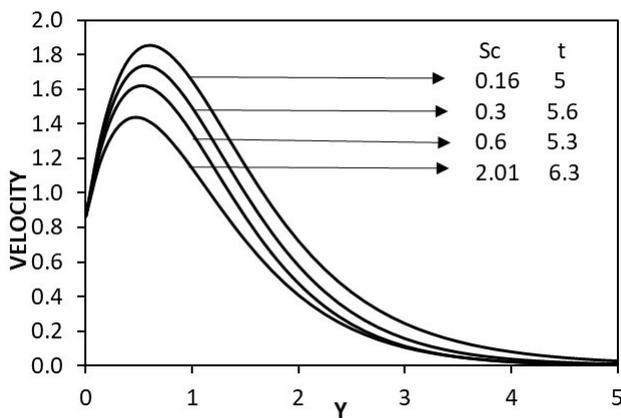
Fig.5. Velocity profiles for different values of *Sc*.
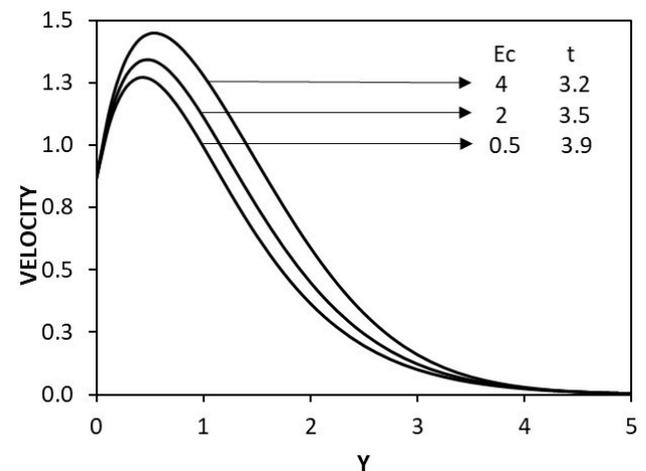
Fig.6. Velocity profiles for different values of *Ec*.

result in a reduction in velocity. Furthermore, it was found that the vertical plate $\omega t = 0$ requires a longer period to reach a consistent state than the flat plate $\omega t = \dfrac{\pi}{2}$. Figure 2 represents the impacts of thermal and mass Grashof numbers over velocity. It has been observed that a rise in the thermal or mass Grashof number results in a rise in air velocity. It comes as a result of the gradual increase in the mass buoyancy effect caused by a rise in the thermal as well as mass Grashof numbers.

The impacts of the radiation parameter R over velocity are presented in Fig.3. It is identified that as the radiation parameter increases, velocity decreases. This demonstrates that the velocity decreases when there is more heat radiation present. Figure 4 demonstrates the influence of the magnetic field parameter M on velocity. The velocity attains extremity very close to the plate and it falls eventually beyond $y > 0.5$ and goes to zero as $y \to \infty$. The velocity profile falls off steeply with increasing magnetic parameters. When a uniform transverse magnetic field is applied normal to the plate, resistance is generated in the opposite direction of flow. This is known as the "Lorentz force". The momentum barrier layer thins out as a result of this resistance and the fluid moves more slowly. The magnetic field effects cause a decrease in maximum velocity and boundary layer thickness.

Figure 5 depicts the velocity profiles resulting from variations in the Schmidt number Sc. It is demonstrated that increasing the Schmidt number reduces velocity. As the Schmidt number rises, so does the steady state value. However, the Schmidt number determines how long it takes to attain the steady-state. Furthermore, a boundary layer forms alongside a semi-boundless upward moving plate through a fluid moving in that direction. This demonstrates that the buoyancy force generated by mass diffusion significantly ascends the maximum velocity. Figure 6 exhibits the consistent state velocity profiles for various values of the Eckert number Ec values. The graph clearly shows that as the Eckert number increases, so does the velocity.

Figure 7 shows the effect of the Schmidt number Sc on concentration. The graph illustrates that when the Schmidt number climbed, the plate concentration decreased dramatically. Physically, an increase in Sc greatly suppresses molecular diffusivity. Figure 8 illustrates the temperature profile variation for various thermal radiation parameter R values. It has been revealed that raising the radiation parameter lowers the boundary layer's velocity and temperature. Temperature is shown to decrease dramatically from its peak at the plate $y = 0$ to the free stream $y \to \infty$, where temperature is infinitesimally small for any value of R. Figure 9 exhibits the impact of the Eckert number Ec on temperature. It was discovered that increasing the dissipation parameter enhances the temperature. Viscous dissipation affects flow fields by increasing energy levels, which causes higher fluid temperatures and as a result, higher buoyancy forces.
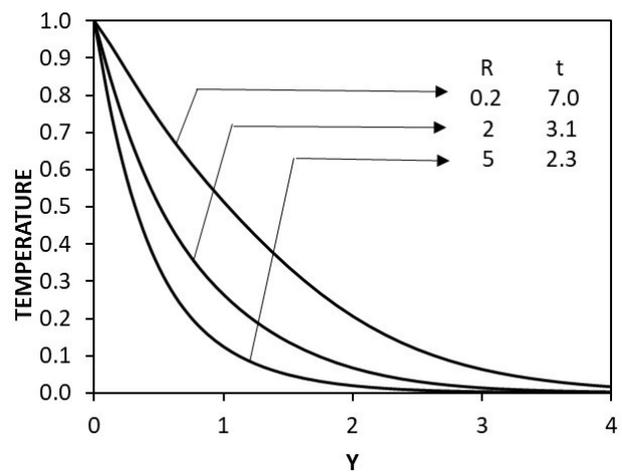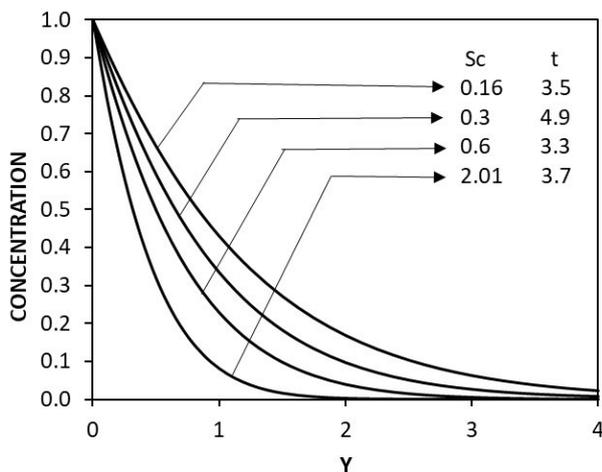
Fig.7. Concentration profiles for different values of *Sc*.   Fig.8. Temperature profiles for different values of *R*.
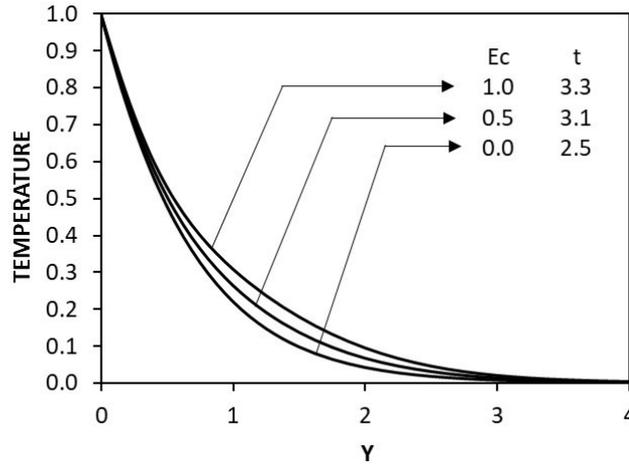
Fig.9. Temperature Profiles for different values of *Ec*.

Figure 10 shows the local skin friction as an axial coordinate function X for diverse phase angle ωt values. As phase angle values drop, the local wall shear stress rises. This demonstrates that the shear stress on a vertical plate at time $\omega t = 0$ is greater than that on a horizontal plate at $\omega t = \dfrac{\pi}{2}$. In Figure 11, we see how the average skin-friction changes as a function of time t over a range of phase angles, at $X = 1.0$. In general, when the phase angle increases, the average skin friction value drops. A comparison of local Nusselt numbers with Eckert numbers Ec is illustrated in Fig.12. Reducing the dissipative parameter boosts the heat transfer rate. Figure 13 depicts the average Nusselt number as a time function t for various values of the Eckert number Ec at $X = 1.0$. As the dissipative parameter is decreased, the average Nusselt number rises. Figure 14 shows how the Schmidt number Sc affects the local Sherwood number. Generally, mass transfer rates increase as Schmidt numbers increase. Nevertheless, as illustrated in Fig.7, this pattern is inverted in the concentration profile. The correlation between the Schmidt number and the typical Sherwood number is shown in Fig.15. It has been ascertained that as the Schmidt number values rise, so does the average Sherwood number.
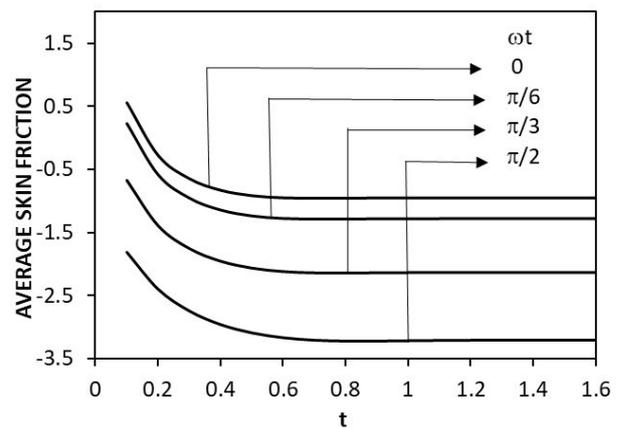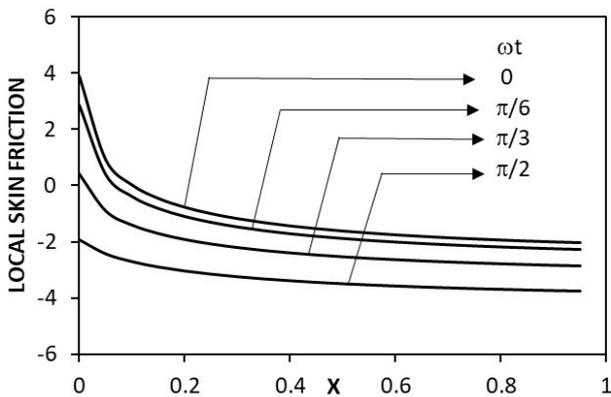
Fig.10. Local skin friction for different values of ωt.   Fig.11. Average skin friction for different values of ωt.
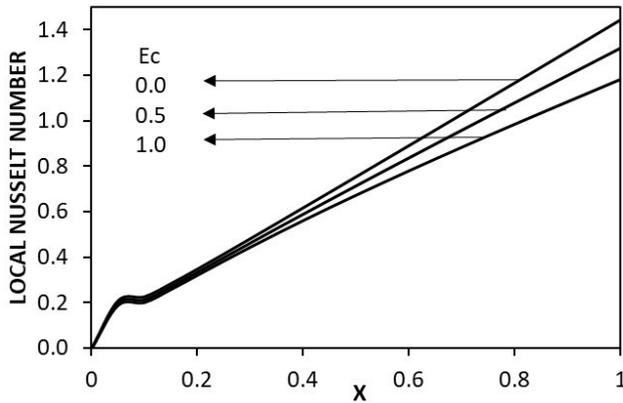
Fig.12. Local Nusselt number for different values of *Ec*.
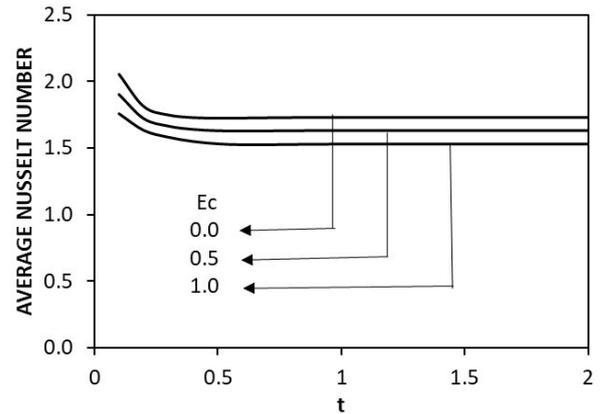
Fig.13. Average Nusselt number for different values of *Ec*.
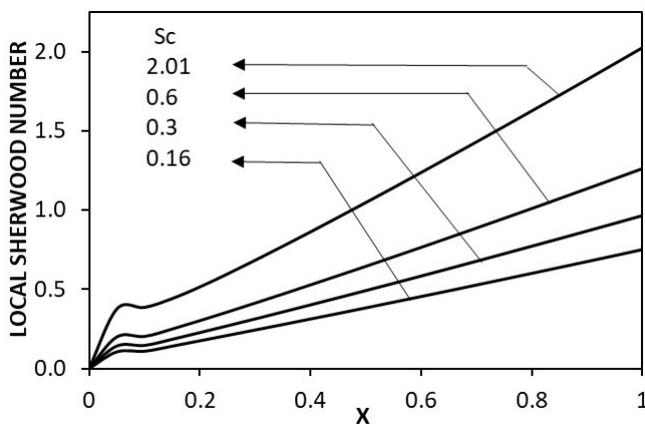
Fig.14. Local Sherwood number for different values of *Sc*.
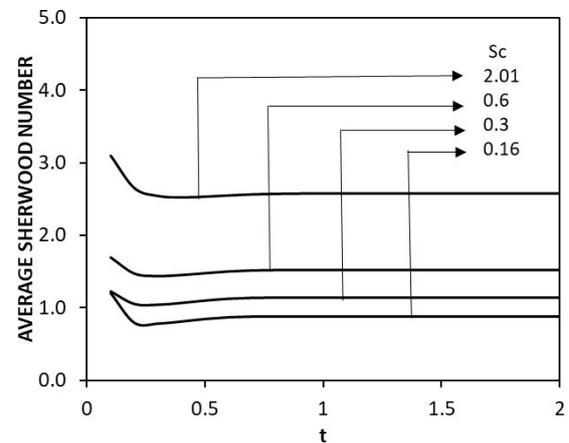
Fig.15. Average Sherwood number for different values of *Sc*.

5. Conclusions

The influence of viscous dissipation as well as thermal radiation on an unsteady MHD flow across a semi-boundless vertically placed oscillating plate with constant mass as well as surface temperature was studied. An implicit finite difference strategy is employed for solving non-linear coupled partial differential equations. The results for different parameters are shown graphically. The present study brings out the following significant findings:

- Velocity drops as the phase angle and the magnetic parameter increases (air).
- As the radiation parameter declines, velocity and temperature profiles rise.
- A rise in Eckert number leads to a rise in temperature, while a rise in the radiation parameter leads to a fall in temperature.
- The velocity and temperature profiles are enhanced by the Eckert number, but this trend is reversible for average and local Nusselt numbers.
- Concentration drops as the Schmidt number rises, whereas the local and average Sherwood numbers show the opposite tendency.
- The research reveals that the amount of time steps needed to reach the steady state is significantly influenced by the phase angle, magnetic field and radiation parameters.

Acknowledgements

Nomenclature

a^* – absorption constant

B_0 – magnetic induction $\left(kg \cdot s^{-2} A^{-1}\right)$

$\mathcal{C}$ – dimensionless concentration

C' – concentration $\left(mol \cdot m^{-3}\right)$

C_p – specific heat at constant pressure $\left(J \cdot kg^{-1} K^{-1}\right)$

D – mass diffusion coefficient

Ec – Eckert number

Gc – mass Grashof number

Gr – thermal Grashof number

g – acceleration due to gravity $\left(m \cdot s^{-2}\right)$

k – thermal conductivity $\left(J \cdot m^{-1} K^{-1}\right)$

M – magnetic parameter

Pr – Prandtl number

q_r – radiation heat flux density $\left(W \cdot m^{-2}\right)$

R – radiation parameter

Sc – Schmidt number

$\mathcal{T}$ – dimensionless temperature

$\mathcal{T}'$ – temperature of the fluid near the plate (K)

t – dimensionless time

t' – time (s)

u_0 – velocity of the plate

u, v – velocity components of the fluid in x, y-directions respectively $\left(m \cdot s^{-1}\right)$

U, V – dimensionless velocity components in X, Y-direction respectively

α – thermal diffusivity $\left(m^2 \cdot s^{-1}\right)$

β – volumetric coefficient of thermal expansion $\left(K^{-1}\right)$

β^* – volumetric coefficient of expansion with concentration $\left(K^{-1}\right)$

μ – coefficient of viscosity $\left(Pa \cdot s\right)$

ν – kinematic viscosity $\left(m^2 \cdot s^{-1}\right)$

ρ – density of the fluid $\left(kg \cdot m^{-3}\right)$

σ – electrical conductivity $\left(kg^{-1} m \cdot s^3 A^2\right)$

ωt – dimensionless phase angle

$\omega' t'$ – phase angle $\left(rad\right)$

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