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ONE-DIMENSIONAL THERMAL SHOCK PROBLEM FOR A SEMI-INFINITE HYGROTHERMOELASTIC ROD

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The present research article deals with the study of a boundary value problem of a one-dimensional semi-infinite hygro-thermoelastic rod of length *l*. The deformation of the rod is under consideration when the left boundary of the hygro-thermoelastic rod is subjected to a sudden heat source. The solutions of the considered variables are decomposed in terms of normal modes. Analytical expressions of displacement, moisture concentration, temperature field, and stresses are obtained and presented graphically for different periods. By studying the one-dimensional thermal shock problem for a semi-infinite hygrothermoelastic rod, the authors aim to gain insights into the fundamental behavior of materials subjected to rapid temperature changes and moisture effects.

Key words: one-dimensional, heat source, temperature distribution, hygrothermoelastic rod, moisture concentration.

1. Introduction

The theory of dynamic thermoelasticity finds applications in different fields of science has been explored by researchers. The classical coupled theory of thermoelasticity was developed which is parabolic in nature and predicts infinite velocity of heat propagation thereby making it impossible to accept. To overcome this paradox, different theories of generalized thermoelasticity which involve a hyperbolic heat equation have been developed in the past few decades. This type of equation admits a finite speed of thermal signals. Chandrasekharaiah and Srinath [1] considered the linear theory of thermoelasticity without energy dissipation to discuss the effect of continuous point heat source in homogeneous and isotropic thermoelastic media. Sharma *et al.* [2] investigated a homogeneous isotropic thermoelastic half-space under the effect of a mechanical and thermal source. Baksi *et al.* [3] investigated a three-dimensional problem in a rotating magneto-thermoelastic medium with thermal relaxation. Mallik and Kanoria [4] discussed periodically varying heat source in a functionally graded isotropic thermoelastic medium. The problem of a moving heat source in a magneto-thermoelastic strip was discussed by He and Cao [5]. A problem of heat sources in a semi-infinite thermoelastic cylinder was discussed by Tripathi *et al.* [6]. The dynamic response of a semi-infinite thermoelastic plate by the finite element method was presented by Xia *et al.* [7]. Ailawalia and Budhiraja [8] studied deformation in a thermo-microstretch elastic medium underlying a non-viscous fluid layer subjected to an internal heat source. Abbas [9] applied the eigen-value approach in a fractional-order thermoelastic medium. Ailawalia and Sachdeva [10] presented the effect of an internal heat source in a thermoelastic solid with micro-temperatures. Sarkar and Mondal [11] inspected thermoelastic interactions in a slim strip under a dual-phase-lag model due to a moving heat source. They also discussed the transient responses in a two-temperature generalized thermoelastic infinite medium due to a time-dependent heat source [12]. Mondal *et*

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al. [13] derived a new thermoelastic mathematical model to describe the transient response under a moving heat source in a rod. Kaur and Lata [14] discussed deformation in a thin circular plate of transversely isotropic nature subjected to a periodically varying heat source. Marin [15-16] studied some problems in a thermoelastic medium with voids.

The interaction between heat, moisture, and deformation is considered an important issue in engineering science. Materials get damaged when moisture and temperature interact with mechanical stresses. These results suggest that more study is required for the interaction of moisture and temperature. In solids, atoms move randomly, and the average spacing between them remains constant as long as they vibrate about their equilibrium positions in a symmetric manner. Motion of particles from one part to another also occurs which is known as diffusion. Further exchange of kinetic energy occurs between adjacent particles in the form of heat which travels from high to low temperature. But it follows from the second law of thermodynamics that the exchange of heat can be reversible if heat flow occurs at an infinitely small rate in the presence of a temperature gradient. The principle of irreversible thermodynamics governs physiochemical processes such as heat conduction, diffusion, etc. Sih *et al.* [17] and Weitsman [18] developed coupled equations for a hygrothermoelastic medium keeping in view principles of irreversible thermodynamics and continuum mechanics. Basi *et al.* [19] analyzed anisotropic inhomogeneous and laminated plates under hygrothermoelastic effects. Fluid-saturated porous media subjected to finite deformation under hygrothermoelastic theory were examined by Advani *et al.* [20]. A coupled micro-macro mechanical approach was adopted by Aboudi and Williams [21] to examine hygrothermoelastic composites. Altay and Dokmeci [22] derived Hamiltonian-type variational principles for governing the behavior of a hygrothermoelastic medium. Rao and Sinha's [23] research analyzes how multidirectional composites behave in a hygrothermal medium. Vibration characteristics of hygrothermoelastic laminated composite doubly curved shells were studied by Kundu and Han [24]. Chiba and Sugano [25] discussed how layered plates behave under the influence of hygrothermal loading. Alsubari *et al.* [26] studied on the bending behavior of simply supported anisotropic cylindrical shells under the influence of moisture and temperature. The bending of simply supported orthotropic cylindrical shells under hygrothermoelasticity was demonstrated by Mohamed *et al.* [27]. Hosseini and Ghadiri [28] discussed two-dimensional problems in a coupled hygrothermoelastic medium. The potential theory method was adopted by Zhao *et al.* [29] to obtain a steady-state solution in a hygrothermoelastic medium. Zhang *et al.* [30] solved the time-fractional hygrothermoelastic problem of a centrally symmetric sphere. Lamba and Deshmukh [31] discussed the unsteady state responses of a finite long solid cylinder subjected to axisymmetric hygrothermal loading. Bhoyar *et al.* [32] employed a two-temperature model for hygrothermoelastic diffusion theory and discussed the bending of an elliptic plate. Recently Ailawalia *et al.* [33] discussed wave propagation in an initially stressed hygrothermoelastic medium. Anand *et al.* [34] investigated the reflection of hygrothermal waves in a nonlocal theory of coupled thermo-elasticity. In addition to the above-mentioned work, some other prominent work [35-41] has been done in the field of thermoelasticity.

This paper deals with the study of deformation in a hygrothermoelastic rod of length l. The rod is subjected to sudden heating at one end. The displacement components, moisture concentration, temperature distribution, and stress components are evaluated and presented graphically to show the effect of these quantities at different time intervals.

2. Basic equations

A thin semi-infinite hygrothermoelastic rod is considered. The rod occupies the region $x \geq 0$. Following Hosseini and Ghadiri [26], governing equations in the hygrothermoelastic medium without body forces and heat sources are given by:

$$\sigma_{ji,j} = \rho \ddot{u}_i, \tag{2.1}$$

$$D_T\theta_{,ii} + D_T^m m_{,ii} - \dot{\theta} - \frac{\alpha_{ij}^T T_0}{\rho c}\dot{u}_{j,j} = 0, \tag{2.2}$$

$$D_m m_{,ii} + D_m^T\theta_{,ii} - \dot{m} - \frac{\alpha_{ij}^m m_0 D_m}{k_m}\dot{u}_{j,j} = 0, \tag{2.3}$$

where

$$\beta_{ij}^T = \alpha_T\delta_{ij},\ \ \alpha_T = (3\lambda + 2\mu)\gamma_T, \tag{2.4}$$

$$\beta_{ij}^m = \alpha_m\delta_{ij},\ \ \alpha_m = (3\lambda + 2\mu)\gamma_m. \tag{2.5}$$

Further, constitutive stress-strain relations [26] are given by:

$$\sigma_{ij} = C_{ijkl}\epsilon_{ij} - \beta_{ij}^m m - \beta_{ij}^T\theta, \tag{2.6}$$

where

$$C_{ijkl} = \frac{2G\nu}{1-2\nu}\delta_{ij}\delta_{kl} + G\delta_{ik}\delta_{jl} + G\delta_{il}\delta_{jk}, \tag{2.7}$$

$$\epsilon_{ij} = \frac{u_{j.i} + u_{i,j}}{2}\epsilon. \tag{2.8}$$

The displacement components for the problem under consideration are assumed to be of the form $u_x = u(x,t),\ u_y = u_z = 0$.
For the considered one-dimensional problem Eqs (2.1), (2.2), and (2.3) reduce to,

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \beta_m\frac{\partial m}{\partial x} - \beta_T\frac{\partial\theta}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{2.9}$$

$$D_T\frac{\partial^2\theta}{\partial x^2} + D_T^m\frac{\partial^2 m}{\partial x^2} - \frac{\partial\theta}{\partial t} - \frac{\beta_T T_0}{\rho c}\frac{\partial}{\partial t}\frac{\partial u}{\partial x} = 0, \tag{2.10}$$

$$D_m\frac{\partial^2 m}{\partial x^2} + D_m^T\frac{\partial^2\theta}{\partial x^2} - \frac{\partial m}{\partial t} - \frac{\beta_m m_0 D_m}{k_m}\frac{\partial}{\partial t}\frac{\partial u}{\partial x} = 0. \tag{2.11}$$

The stress component σ_{xx} in one dimension reduces to,

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \beta_m m - \beta_T\theta. \tag{2.12}$$

Introducing the following dimensionless variables,

$$x' = \frac{1}{l}x,\ \ u' = \frac{1}{l}u,\ \ t' = \frac{D_m}{l^2}t,\ \ m' = m,\ \ \theta' = \frac{\theta}{T_0},\ \ \sigma_{xx}' = \frac{\sigma_{xx}}{\lambda}, \tag{2.13}$$

in Eqs (2.9)-(2.11), we get the following equations in dimensionless form,

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}-\beta_m\frac{\partial m}{\partial x}-\beta_T T_0\frac{\partial\theta}{\partial x}=\frac{\rho D_m^2}{l^2}\frac{\partial^2 u}{\partial t^2}, \tag{2.14}$$

$$D_T T_0\frac{\partial^2\theta}{\partial x^2}+D_T^m\frac{\partial^2 m}{\partial x^2}-D_m T_0\frac{\partial\theta}{\partial t}-\frac{\beta_T T_0 D_m}{\rho c}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right)=0, \tag{2.15}$$

$$D_m\frac{\partial^2 m}{\partial x^2}+D_m^T T_0\frac{\partial^2\theta}{\partial x^2}-D_m\frac{\partial m}{\partial t}-\frac{\beta_m m_0 D_m^2}{k_m}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right)=0. \tag{2.16}$$

The dimensionless stress component σ_{xx} is given by,

$$\sigma_{xx}=\frac{1}{\lambda}\left[(\lambda+2\mu)\frac{\partial u}{\partial x}-\beta_m m-\beta_T T_0\theta\right]. \tag{2.17}$$

3. Solution

The solution of the physical variables may be assumed in the form of normal modes as:

$$\{u,m,\theta\}=\left[\bar{u},\bar{m},\bar{\theta}\right](x)\exp(\omega t), \tag{3.1}$$

where ω is complex. frequency.

Using the above solution in (2.14)-(2.16), we obtain the following equations:

$$\left[a_1 D^2-a_2\right]\bar{u}-a_3 D\bar{m}-a_4 D\bar{\theta}=0, \tag{3.2}$$

$$b_1 D\bar{u}+b_2 D^2\bar{m}+\left[b_3 D^2-b_4\right]\bar{\theta}=0, \tag{3.3}$$

$$c_1 D\bar{u}+\left[c_2 D^2-c_3\right]\bar{m}+c_4 D^2\bar{\theta}=0, \tag{3.4}$$

where

$$a_1=\lambda+2\mu,\ a_2=\frac{\rho c_2^2\omega^2}{l^2},\ a_3=\beta_m,\ a_4=\beta_T T_0,$$

$$b_1=\frac{-a_4 c_2\omega}{\rho c},\quad b_2=D_T^m,\quad b_3=D_T T_0,\quad b_4=c_2 T_0\omega, \tag{3.5}$$

$$c_1=\frac{-\beta_m m_0 c_2^2\omega}{k_m},\quad c_2=D_m,\quad c_3=c_2\omega,\quad c_4=D_m^T T_0,\quad D=\frac{d}{dx}.$$

On solving Eqs (3.2)-(3.4), we obtain a sixth-order differential equation in terms of $\bar{u}$, $\bar{m}$, $\bar{T}$ as:

$$\left[D^6 + A_1 D^4 + A_2 D^2 + A_3\right]\left(\overline{u},\overline{m},\overline{\theta}\right) = 0, \tag{3.6}$$

where

$$A_1 = \frac{f_1 g_5 - f_2 g_4 + f_4 g_1 - f_3 g_2}{f_1 g_4 - f_3 g_1}, \quad A_2 = \frac{f_4 g_2 - f_3 g_3 - f_2 g_5}{f_1 g_4 - f_3 g_1}, \quad A_3 = \frac{f_4 g_3}{f_1 g_4 - f_3 g_1},$$

$$f_1 = a_1 c_4, \quad f_2 = a_2 c_4 - a_4 c_1, \quad f_3 = a_4 c_2 - a_3 c_4, \quad f_4 = a_4 c_3, \tag{3.7}$$

$$g_1 = a_1 b_3, \quad g_2 = a_4 b_1 - a_1 b_4 - a_2 b_3, \quad g_3 = a_2 b_4, \quad g_4 = a_4 b_2 - a_3 b_3, \quad g_5 = a_3 b_4.$$

The appropriate solution of Eq.(3.6) satisfying the radiation conditions may be expressed as:

$$\begin{aligned}\overline{u} &= B_1 \exp\{-k_1 x\} + B_2 \exp\{k_1 x\} + B_3 \exp\{-k_2 x\} + \\ &+ B_4 \exp\{k_2 x\} + B_5 \exp\{-k_3 x\} + B_6 \exp\{k_3 x\},\end{aligned} \tag{3.8}$$

$$\begin{aligned}\overline{m} &= B_1^* \exp\{-k_1 x\} + B_2^* \exp\{k_1 x\} + B_3^* \exp\{-k_2 x\} + \\ &+ B_4^* \exp\{k_2 x\} + B_5^* \exp\{-k_3 x\} + B_6^* \exp\{k_3 x\},\end{aligned} \tag{3.9}$$

$$\begin{aligned}\overline{\theta} &= B_1^{**} \exp\{-k_1 x\} + B_2^{**} \exp\{k_1 x\} + B_3^{**} \exp\{-k_2 x\} + \\ &+ B_4^{**} \exp\{k_2 x\} + B_5^{**} \exp\{-k_3 x\} + B_6^{**} \exp\{k_3 x\},\end{aligned} \tag{3.10}$$

where

$$\pm k_i^2 \left(i = 1,2,3\right)$$

are the roots of

$$\left(k^6 + A_1 k^4 + A_2 k^2 + A_3\right) = 0, \tag{3.11}$$

and the parameters B_j^* and B_j^{**} depend on $B_j, \left(j = 1,\ldots 6\right)$ and are given by:

$$\begin{aligned}&B_1^* = H_{11} B_1, \quad B_2^* = -H_{11} B_2, \quad B_3^* = H_{12} B_3, \quad B_4^* = -H_{12} B_4, \\ &B_5^* = H_{13} B_5, \quad B_6^* = -H_{13} B_6, \quad B_1^{**} = -H_{21} B_1, \quad B_2^{**} = H_{21} B_2, \\ &B_3^{**} = -H_{22} B_3, \quad B_4^{**} = H_{22} B_4, \quad B_5^{**} = -H_{23} B_5, \quad B_6^{**} = H_{23} B_6, \\ &H_{1i} = \frac{k_i \left(f_1 k_i^2 - f_2\right)}{f_3 k_i^2 - f_4}, \quad H_{2i} = \frac{a_1 k_i^2 - a_2 + a_3 k_i H_{1i}}{a_4 k_i}.\end{aligned} \tag{3.12}$$

4. Boundary conditions

We consider the stress-free hygrothermoelastic rod at a uniform temperature T_0 with its boundary $0 \le x \le l$. The boundary $x = 0$ is subjected to sudden heating. The appropriate boundary conditions are given by,

$$\begin{aligned}
&\text{(i) } u(0,t)=0\,,\\
&\text{(ii) } u(l,t)=0\,,\\
&\text{(iii) } \frac{\partial m}{\partial \mathrm{x}}(0,\mathrm{t})=0\,,\\
&\text{(iv) } \frac{\partial m}{\partial x}=0,\\
&\text{(v) } \theta(0,t)=\theta_0\exp(\omega t),\\
&\text{(vi) } \theta(l,t)=0.
\end{aligned} \tag{4.1}$$

Using (3.1), (3.8)-(3.10) in the boundary conditions (4.1), we get the following non-homogenous system of six equations:

$$B_1+B_2+B_3+B_4+B_5+B_6=0, \tag{4.2}$$

$$B_1\exp\{-k_1l\}+B_2\exp\{k_1l\}+B_3\exp\{-k_2l\}+B_4\exp\{k_2l\}+B_5\exp\{-k_3l\}+B_6\exp\{k_3l\}=0, \tag{4.3}$$

$$k_1H_{11}B_1+k_1H_{11}B_2+k_2H_{12}B_3+k_2H_{12}B_4+k_3H_{13}B_5+k_3H_{13}B_6=0, \tag{4.4}$$

$$\begin{aligned}
&k_1H_{11}\exp\{-k_1l\}B_1+k_1H_{11}\exp\{k_1l\}B_2+k_2H_{12}\exp\{-k_2l\}B_3+\\
&+k_2H_{12}\exp\{k_2l\}B_4+k_3H_{13}\exp\{-k_3l\}B_5+k_3H_{13}\exp\{k_3l\}B_6=0,
\end{aligned} \tag{4.5}$$

$$-H_{21}B_1+H_{21}B_2-H_{22}B_3+H_{22}B_4-H_{23}B_5+H_{23}B_6=\theta_0\exp(\omega t), \tag{4.6}$$

$$\begin{aligned}
&-H_{21}\exp\{-k_1l\}B_1+H_{21}\exp\{k_1l\}B_2-H_{22}\exp\{-k_2l\}B_3+\\
&+H_{22}\exp\{k_2l\}B_4-H_{23}\exp\{-k_3l\}B_5+H_{23}\exp\{k_3l\}B_6=0.
\end{aligned} \tag{4.7}$$

The above non-homogenous system of six equations is solved by developing codes in MATLAB and the values of constants $B_n\,(n=1,\ldots 6)$ are evaluated.

Using the expressions of $\bar{u}$, $\bar{m}$ and $\bar{\theta}$ given by (3.8)-(3.10) in expressions (3.1), the displacement component, moisture concentration, temperature field, and stress in hygrothermoelastic medium are obtained.

5. Numerical computation

The analytical results are verified in this section by taking a wood slab as a hygro-thermoelastic material. The elastic constants given by Chang and Weng [42] and Yang *et al.* [43] are shown in Tab.1.

Table 1. Values of material constants.

Symbol	Value	Symbol	Value
E	$64.3\times10^9\ N/m^2$	k	$0.65w/m\left({}^{\circ}K\right)$
ρ	$370\,Kg/m^3$	c	$2500\,J/Kg\left({}^{\circ}K\right)$
ν	0.33	K_m	$2.2\times10^{-8}\,Kg/msM$
m_0	10%	D_m	$2.16\times10^{-6}\ m^2/\sec$
α_m	$2.68\times10^{-3}\ cm/cm(\%H_2O)$.	D_m^T	$0.648\times10^{-6}\ m^2(\%H_2O)/s\left({}^{\circ}K\right)$
T_0	$283^{\circ}K$	D_T^m	$2.1\times10^{-7}\ m^2\left({}^{o}K\right)/s(\%H_2O)$
α^T	$31.3\times10^{-6}\ cm/cm\left({}^{\circ}K\right)$	D_T	$k/\rho c$

The numerical results are obtained for displacement, force stress, moisture concentration, and temperature distribution for $l=1.0,\ \theta_0=1.0$ against the horizontal distance x. The graphical results are shown for four values of non-dimensional time ($t=0.01,\ 0.1,\ 0.5$ and 1.5).

6. Discussions

The variations of displacement follow a linear trend in the region $0\le x<8.2$ and then increase sharply. This sharpness is maximum for $t=0.01$ and more interestingly, the sharpness decreases with an increase in time t, respectively. This behavior in the region $0\le x<8.2$ can be attributed to the material's elastic response, where the displacement is directly proportional to the applied stress. The sudden increase can be attributed to the occurrence of a stress concentration or a localized deformation mechanism. It is possible that the material reaches its limit of linear elasticity and undergoes plastic deformation or experiences a structural discontinuity. The variations of force stress are linear in nature with differences in slope. Also, the slope of the variation of force stress decreases with an increase in time t. The difference in slopes suggests that different regions of the material experience different levels of stress, which can be attributed to varying mechanical properties, geometric constraints, or external loading conditions.

Similar to the variations of force stress, the variations of moisture concentration also follow a linear path. It is to mention here that the slope of the variation of moisture concentration is zero, i.e., the variations are parallel to the horizontal distance. However, the magnitude of the values decreases with an increase in time. The parallel nature of the variations (i.e., zero slope) suggests that the moisture concentration does not change along the horizontal distance, implying a constant moisture gradient. This behavior could be influenced by factors such as moisture diffusion, capillary action, or moisture transport through the material. The variations in the temperature field are similar in nature to the variations of displacement. These variations of displacement, force stress, moisture concentration, and temperature field are shown in Figs 1-4, respectively.

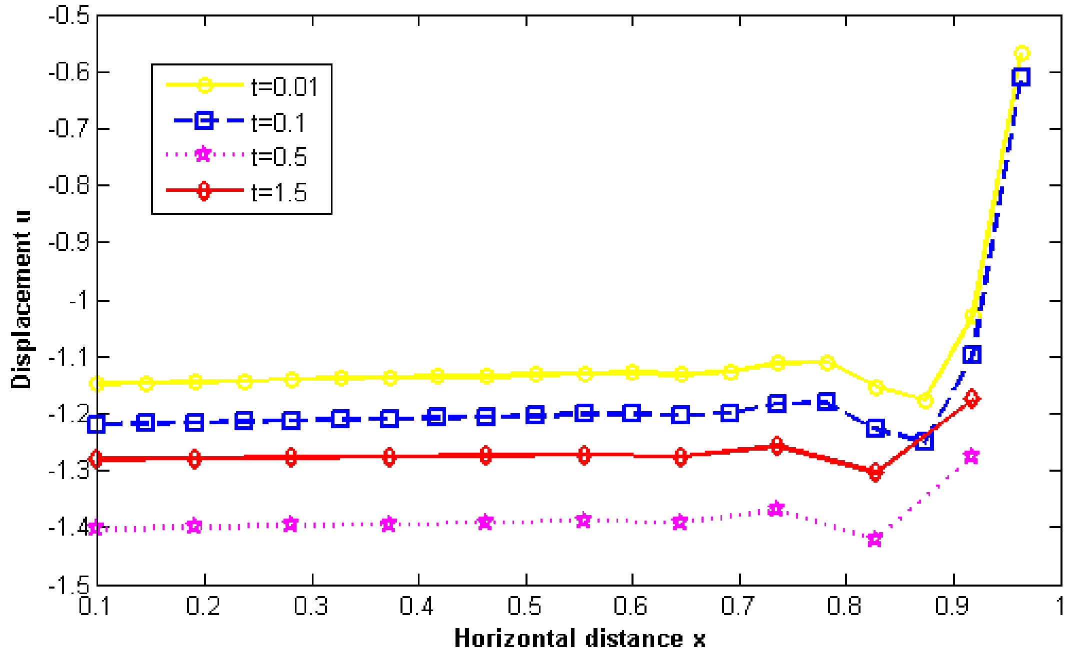

Fig.1. Displacement with distance x.

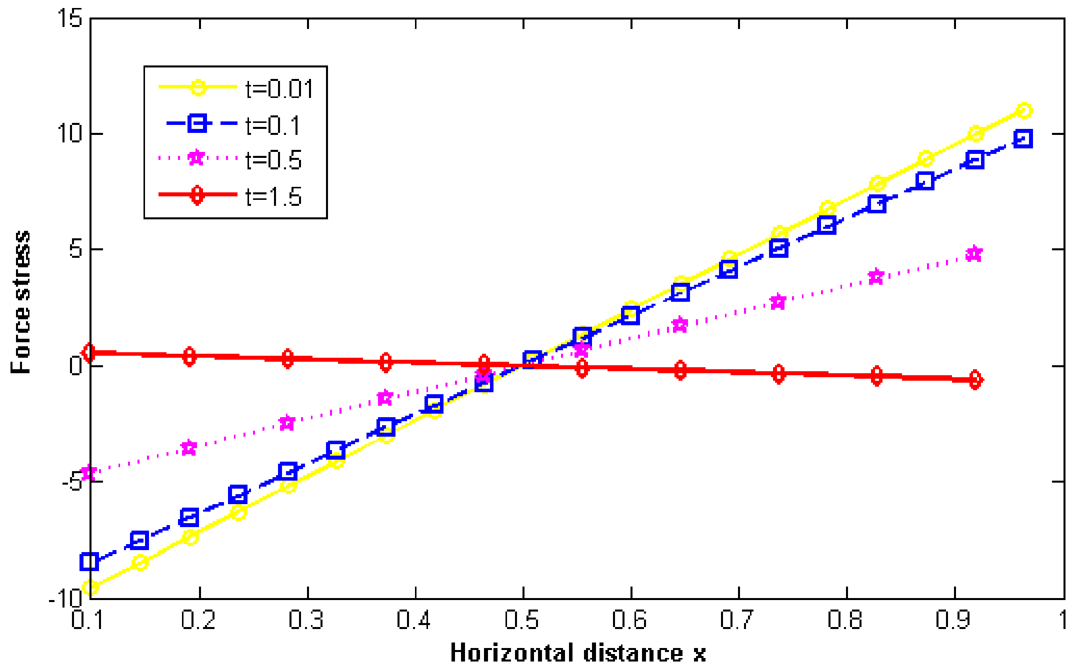

Fig.2. Force stress with distance x.

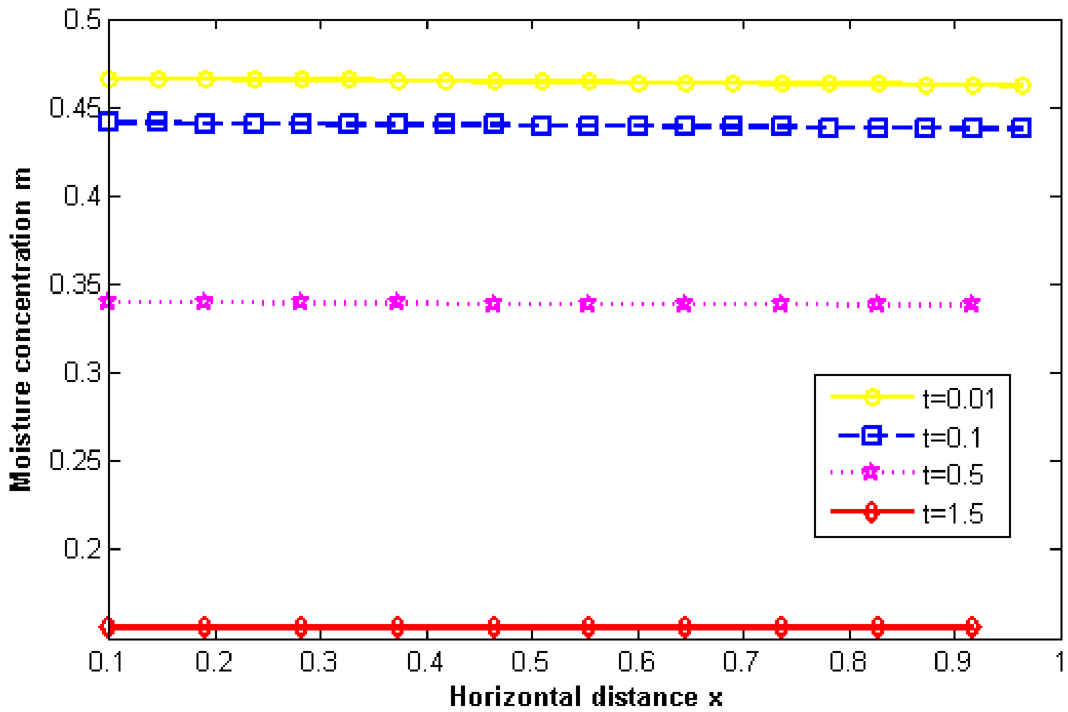

Fig.3. Moisture concentration with distance *x*.

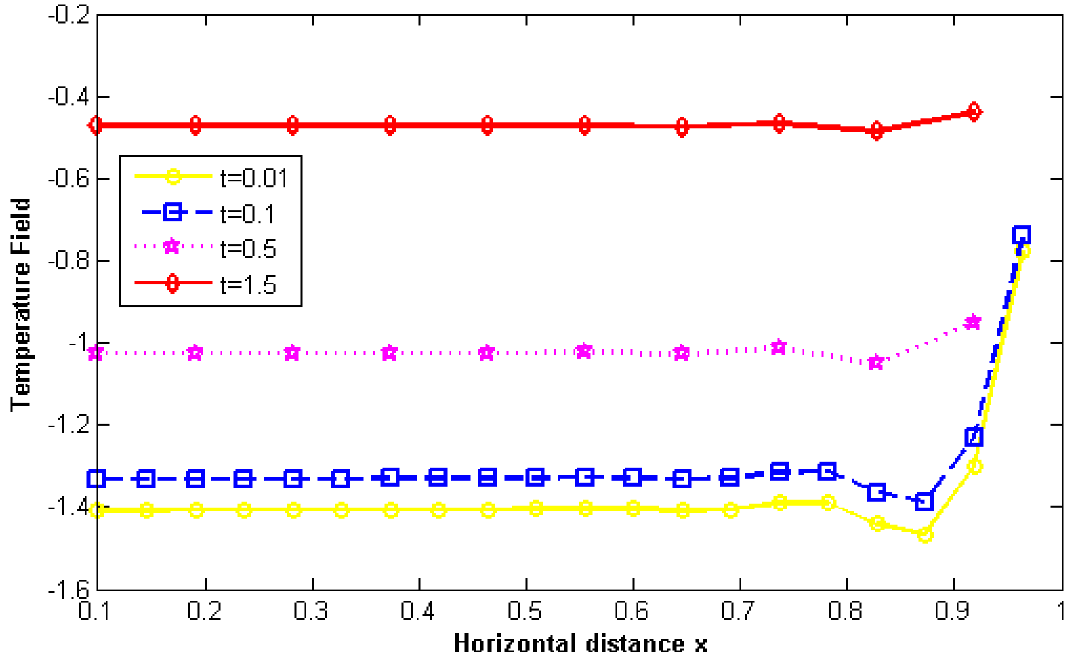

Fig.4. Temperature field with distance *x*.

7. Conclusion

The analytical and numerical findings of the research problem are as follows:

1 The results can help in the design and optimization of materials and structures for various engineering applications, where thermal and moisture-induced stresses can lead to failures or reduced performance.
2 The analytical results show that three waves propagate in the medium namely displacement, moisture, and thermal wave.
3 The variations of force stress and moisture concentration are linear in nature.
4 The variations of displacement and temperature field are similar in nature.
5 As expected, the moisture concentration remains constant with horizontal distance.

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Nomenclature

ϵ_{rs}, – components of strain
σ_{ij} – components of stress
u_i – components of displacement
ρ – density
D_T – temperature diffusivity
θ – temperature field
m – moisture concentration
T_0 – initial temperature
D_m – moisture diffusivity
D_T^m, D_m^T – diffusivities
c – heat capacity
m_0 – reference moisture
k_m – moisture diffusivity
α_{ij}^T and α_{ij}^m – material coefficients
λ,μ – Lame's constants such that $\lambda = \dfrac{\nu E}{(1+\nu)(1-2\nu)}$ and $\mu = \dfrac{E}{2(1+\nu)}$

References

[1] Chandrasekharaiah D.S. and Srinath K.S. (1998): *Thermoelastic interactions without energy dissipation due to a point heat source*.– Journal of Elasticity, vol.50, No.2, pp.97-108.
[2] Sharma J.N., Chauhan R.S., and Kumar R. (2000): *Time-harmonic sources in a generalized thermoelastic continuum*.– Journal of Thermal Stresses, vol.23, No.7, pp.657-674.
[3] Baksi A., Bera R.K., and Debnath L. (2005): *A study of magnetothermoelastic problems with thermal relaxation and heat sources in a three-dimensional infinite rotating elastic medium*.– International Journal of Engineering Science, vol.43, No.19-20, pp.1419-1434.
[4] Mallik S.H. and Kanoria M. (2007): *Generalized thermoelastic functionally graded solid with a periodically varying heat source*.– International Journal of Solids and Structures, vol.44, No.22-23, pp.7633-7645.
[5] He T. and Cao L. (2009): *A problem of generalized magneto-thermoelastic thin slim strip subjected to a moving heat source*.– Mathematical and Computer Modelling, vol.49, No.7-8, pp.1710-1720.
[6] Tripathi J.J., Kedar G.D. and Deshmukh K.C. (2014): *Dynamic problem of generalized thermoelasticity for a semi-infinite cylinder with heat sources*.– Journal of Thermoelasticity, vol.2, No.1, pp.1-8.
[7] Xia R., Tian X. and Shen Y. (2014): *Dynamic response of two-dimensional generalized thermoelastic coupling problem subjected to a moving heat source*.– Acta Mechanica Solida Sinica, vol.27, No.3, pp.300-305.

[8] Ailawalia P. and Budhiraja S. (2015): *Disturbance in thermo-microstretch elastic medium with internal heat source*.– Mechanics of Advanced Materials and Structures, vol.22, No.9, pp.776-783.

[9] Abbas I.A. (2015): *Eigenvalue approach to fractional order generalized magneto-thermoelastic medium subjected to moving heat source*.– Journal of Magnetism and Magnetic Materials, vol.377, pp.452-459.

[10] Ailawalia P. and Sachdeva S. (2018): *Internal heat source in a temperature dependent thermoelastic half space with microtemperatures*.– Journal of Computational Applied Mechanics, vol.49, No.2, pp.351-358.

[11] Sarkar N. and Mondal S. (2019): *Thermoelastic interactions in a slim strip due to a moving heat source under dual-phase-lag heat transfer*.– Journal of Heat Transfer, vol.141, No.12. 124501, https://doi.org/10.1115/1.4044920

[12] Sarkar N. and Mondal S. (2019): *Transient responses in a two temperature thermoelastic infinite medium having cylindrical cavity due to moving heat source with memory dependent derivative*.– ZAMM Journal of Applied Mathematics and Mechanics/Zeitschrift fr Angewandte Mathematik und Mechanik, vol.99, No.6, pp. 201800343, https://doi.org/10.1002/zamm.201800343

[13] Mondal S., Sur A., Bhattacharya D. and Kanoria M. (2020): *Thermoelastic interaction in a magneto-thermoelastic rod with memory-dependent derivative due to the presence of moving heat source*.– Indian Journal of Physics, vol.94, pp.1591-1602.

[14] Kaur I. and Lata P. (2019): *Transversely isotropic thermoelastic thin circular plate with constant and periodically varying load and heat source*.– International Journal of Mechanical and Materials Engineering, vol.14, Article No.10, pp.13

[15] Marin M. (1996): *Generalized solutions in elasticity of micropolar bodies with voids*.– Revista de la Academia Canaria de Ciencias, vol.8, No.1, pp.101-106.

[16] Marin M. (1997): *An uniqueness result for body with voids in linear thermoelasticity*.– Rendiconti di Matematica Applicata, Roma, vol.17, No.1, pp.103-113.

[17] Sih G.C., Michopoulos J.G and Chou S.C. (1986) *Hygrothermoelasticity*.– Dordrecht, The Netherlands: Martinus Nijhoff Publishing.

[18] Weitsman Y. (1987): *Stress-assisted diffusion in elastic and viscoelastic materials*.– Journal of the Mechanics and Physics of Solids, vol.35, No.1, pp.73-93.

[19] Basi S., Rogers T.G. and Spencer A.J.M. (1991): *Hygrothermoelastic analysis of anisotropic inhomogeneous and laminated plates*.– Journal of the Mechanics and Physics of Solids, vol.39, No.1, pp.1-22, https://doi.org/10.1016/00225096(91)90028-M.

[20] Advani S.H., Lee T.S., Lee J.K. and Kim C.S. (1993): *Hygrothermomechanical evaluation of porous media under finite deformation. Part Infinite element formulations*.– International Journal for numerical methods in Engineering, vol.36, No.1, pp.147-160, https://doi.org/10.1002/nme.1620360109.

[21] Aboudi J. and Williams T.O. (2000): *Coupled micro-macromechanical analysis of hygrothermoelastic composites*, International Journal of Solids and Structures, vol.37, pp.4149-4179, https://doi.org/10.1016/S0020-7683(99)00145-6.

[22] Altay G.A. and Dokmeci M.C. (2000): *Some Hamiltonian-type variational principles for motions of a hygrothermoelastic medium.* Journal of Thermal Stresses, vol.23, No.3, pp.273-284. https://doi.org/10.1080/014957300280443.

[23] Rao V.V.S. and. Sinha P.K. (2004): *Dynamic response of multidirectional composites in hygrothermal environments*.– Composite Structures, vol.64, pp.329-338, https://doi.org/10.1016/j.compstruct.2003.09.002.

[24] Kundu C.K. and Han J.H. (2009): *Vibration characteristics and snapping behavior of hygro-thermo-elastic composite doubly curved shells*.– Composite Structures, vol.91, No.3, pp.306-317. https://doi.org/10.1016/j.compstruct.2009.05.009.

[25] Chiba R. and Sugano Y. (2011): *Transient hygrothermoelastic analysis of layered plates with one-dimensional temperature and moisture variations through the thickness*.– Composite Structures, vol.93, pp.2260-2268, https://doi.org/10.1016/j.compstruct.2011.03.014.

[26] Alsubari S., Ali J.M., and Aminanda Y. (2015): *Hygrothermoelastic analysis of anisotropic cylindrical shells.*– Composite Structures, vol.131, pp.151-159, https://doi.org/10.1016/j.compstruct.2015.04.035.

[27] Mohamed Ali J.S., Alsubari S. and Aminanda Y. (2016): *Hygrothermoelastic analysis of orthotropic cylindrical shells.*– Latin American Journal of Solids and Structures, vol.13, No.3, pp.573-589, http://dx.doi.org/10.1590/1679-78252249.

[28] Hosseini S.M. and Ghadiri Rad M.H. (2017): *Application of meshless local integral equations for two-dimensional transient coupled hygrothermoelasticity analysis: Moisture and thermoelastic wave propagations under shock loading*.– Journal of Thermal Stresses, vol.40, No.1, p.4054, https://doi.org/10.1080/01495739.2016.1224134.

[29] Zhao M., Dang H., Fan C. and Chen Z. (2018): *Three-dimensional steady-state general solution for isotropic hygrothermoelastic media.*– Journal of Thermal Stresses, vol.41, No.8, 951-972, https://doi.org/10.1080/01495739.2018.1449040.

[30] Zhang X.Y., Peng Y. and Li X.F. (2018): *Time-fractional hygrothermoelastic problem for a sphere subjected to heat and moisture flux*.– Journal of Heat Transfer, vol.140, No.12, pp.122002, https://doi.org/10.1115/1.4041419.

[31] Lamba N.K. and Deshmukh K.C. (2019): *Hygrothermoelastic response of a finite solid circular cylinder.*– Multidiscipline Modeling in Materials and Structures, vol.16, No.1, pp.37-52, https://doi.org/10.1108/MMMS-12-2018-0207.

[32] Bhoyar S., Varghese V. and Khalsa L. (2020): *Hygrothermoelastic response in the bending analysis of elliptic plate due to hygrothermal loading.*– Journal of Thermal Stresses, vol.43, No.3, pp.372-400, https://doi.org/10.1080/01495739.2019.1711477.

[33] Ailawalia P., Gupta D. and Sharma V. (2022): *Surface waves in hygrothermoelastic half-space with hydrostatic initial stress.*– Mechanics of Advanced Materials and Structures, vol.29, No.16, pp.2380-2389, https://doi.org/10.1080/15376494.2020.1862942

[34] Yadav A.K., Carrera E., Marin M and Othman M.I.A. (2022): *Reflection of hygrothermal waves in a nonlocal theory of coupled thermoelasticity*.– Mechanics of Advanced Materials and Structures. (In press). https://doi.org/10.1080/15376494.2022.2130484

[35] Abbas I.A. (2007): *Finite element analysis of the thermoelastic interactions in an unbounded body with a cavity*.– Forsch Ingenieurwes, vol.71, pp.215-222, doi:10.1007/s10010-007-0060-x.

[36] Zenkour A.M. and Abbas I.A. (2014): *Nonlinear transient thermal stress analysis of temperature-dependent hollow cylinders using a finite element model*.– International Journal of Structural Stability and Dynamics, vol.14, No.7, p.1450025, doi:10.1142/S0219455414500254.

[37] Abbas I.A and Kumar R. (2016): *2D deformation in initially stressed thermoelastic half-space with voids*.– Steel and Composite Structures, vol.20, pp.1103-1117, doi:10.12989/scs.2016.20.5.1103.

[38] Hobiny A. and Abbas I.A. (2019): *A GN model on photothermal interactions in a two-dimensions semiconductor half space*.– Results in Physics, vol.15, 102588.

[39] Alzahrani F., Hobiny A., Abbas I.A and Marin M. (2020): *An eigenvalues approach for a two-dimensional porous medium based upon weak, normal and strong thermal conductivities*.– Symmetry, vol.12, No.5, 848, doi:10.3390/sym12050848.

[40] Abbas I.A. and Marin M. (2020): *Photo-thermal interactions in a semi-conductor material with cylindrical cavities and variable thermal conductivity*.– Journal of Taibah University for Science, vol.14, 1369-1376, doi:10.1080/16583655.2020.1824465.

[41] Marin M., Hobiny A. and Abbas I.A. (2021): *The effects of fractional time derivatives in porothermoelastic materials using finite element method*.– Mathematics, vol.9, No.14, p.1606. doi:10.3390/math9141606.

[42] Chang W.J. and Weng C.I. (2000): *An analytical solution to coupled heat and moisture diffusion transfer in porous materials.*– International Journal of Heat and Mass Transfer, vol.43, pp.3621-3632, https://doi.org/10.1016/S00179310(00)00003-X

[43] Yang Y.C., Chu S.S., Lee H.L. and Lin S.L. (2006): *Hybrid numerical method applied to transient hygrothermal analysis in an annular cylinder.*– International Communications in Heat and Mass Transfer, vol.33, pp.102-111, https://doi.org/10.1016/j.icheatmasstransfer.2005.08.007.

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