Repetitive process based indirect-type iterative learning control for batch processes with model uncertainty and input delay

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Abstract

This paper develops an indirect iterative learning control scheme for batch processes with time-varying uncertainties, input delay, and disturbances. In this paper, a predictor based on a state observer is designed to estimate the future state and to compensate for the input delay. Then a feedback controller based on the estimated state and the set-point error is used to track the specified reference trajectory, where, of the options available, a robust H_∞ controller is designed in the presence of time-varying uncertainties and load disturbances. Then a proportional plus derivative type iterative learning control law is designed. An injection molding process model demonstrates the new method's effectiveness, and a comparison with a direct-type design is given. In this alternative approach, all design objectives are simultaneously considered.

Keywords: Batch process, Iterative learning control, Input-delay, Repetitive process, Robust H_∞ control, Linear matrix inequality

1. Introduction

Iterative learning control (ILC) is a method of updating the input signal for systems that repeatedly complete the same finite duration task, with a stoppage time or resetting to the starting location after each one is complete. The pick and place task for robotics is one for which ILC is particularly suitable. In such an application, the robot collects the payload from a fixed location, transfers it over a finite duration, places it on a synchronized moving conveyor, returns to the starting location, collects the next payload, and repeats these steps as many times as required or until a halt is required for maintenance or other purposes. Each execution is termed a trial (pass and iteration are also used in the literature), and the duration of each trial is termed the trial length.

A prevalent form of ILC is where a reference trajectory describes the desired output from the system on each trial and is specified at the outset. Then the difference between this trajectory and the output of a trial can be used to form a sequence that describes the propagation of the error from trial to trial. Design can then proceed with forcing this sequence to converge in terms of a suitably chosen norm, either to zero (ideally) or within a specified bound. At the end of each trial, all information generated over the trial length is available for use in constructing the input to the subsequent trial.

Since the first work, widely credited to [1], ILC has been an active research area in algorithm development and applications. The survey papers [2, 3] are sources for the early literature. Recent results that combine algorithm development with experimental validation include broiler weight optimization [4], nano-positioning, e.g., [5], and there have also been applications in healthcare, e.g., ventricular assist devices [6] and robotic-assisted stroke rehabilitation, e.g., [7].

One approach to ILC design for discrete dynamics is to use the so-called lifted model. Consider the single-input single-output case for ease of presentation. Then since the trial length is finite, the values of a variable can be assembled as the entries in a vector (termed a super-vector), and the error dynamics updating from trial to trial are described by a standard linear systems difference equation. Hence trial to trial error convergence and control law design can proceed by standard methods. The trial length is finite, however; consequently, trial-to-trial error convergence can occur even if (for linear dynamics) the state matrix is unstable.

An alternative setting for ILC design is to use its 2D systems structure, i.e., from trial to trial and along the trial, respectively. Repetitive processes are a distinct class of 2D systems where the information in one of the two directions is of finite duration and hence a closer fit to ILC dynamics. Repetitive processes are a distinct class of 2D systems where the information in one of the two directions is of finite duration and hence a closer fit to ILC dynamics. Therefore in this paper, ILC design is based on the stability theory for repetitive processes.

Repetitive processes arose, see, e.g., [8], which also gives the original references to other 2D systems models from the modeling of industrial systems where repeated sweeps through dynamics defined over a finite duration occur, e.g., successive reduction of the thickness of a metal bar. In keeping with the ILC literature, a sweep is termed a trial, and the duration is termed the trial length. Once a trial is complete, the system

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resets to the starting location. During the subsequent trial, the output from the previous one acts as a forcing function, thereby contributing to its dynamics. The unique control problem is oscillations that increase in amplitude from trial to trial, which also cannot be controlled by the application of standard systems designs.

One possibility that arises from using the repetitive process setting for ILC design is the (sometimes termed) direct type design, where a control law is simultaneously designed to enforce trial-to-trial error convergence and regulate the dynamics produced on any trial. This setting also enables designs that use two loops, where each can be designed separately, and such a design is often referred to as an indirect type. The former class of designs has also been experimentally validated, e.g., [9].

Most often, indirect-type ILC designs have a two-loop structure [10]. In the inner of these loops, e.g., a three-term controller or one based on the internal model principle [11] or, as one further example, model predictive control [12], are designed to ensure stability along the trials. The outer loop controller is ILC based, and its function is to update the set point for the inner loop. Batch processing is a common feature in many process industries, such as injection molding [13], where each entry in the batch is subject to the same processing over a finite time interval. Hence an application area for ILC (and also repetitive control). One feature of batch processing is the presence of time delays and uncertainty in the model of the dynamics used for design. These topics are the subject of this paper, using descriptions of the dynamics to which repetitive process stability theory can be applied.

A starting point for the literature on the application of ILC and related approaches (repetitive control and run-to-run control) is the survey paper [14]. More recent results on direct-type ILC design using the repetitive process/2D systems setting include [15], where the Kalman-Yakubovich-Popov lemma is used to apply control action of finite frequency domains. One possible implementation-related issue with such a design is the amount of information to be stored.

In batch processing, time delays often arise because of the sensor response lag and signal transmission between the controllers or actuators etc. In turn, this has led to research on the stability analysis and controller design for batch processing in the presence of time delays. Early results, including using a 2D systems setting for analysis, are given in, e.g., [11, 16]. Previous research has used a 2D systems state estimator and then constructed a 'delay-free' 2D systems model, leading to linear matrix inequality (LMI) conditions to ensure the H_∞ performance [17]. Recently, by constructing an equivalently novel extended 2D model, predictive control has also been combined with ILC method to deal with the asynchronous switching problem in complex batch processes with time delay and uncertainties [18, 19].

This paper develops an indirect type ILC design for batch processes with time delays, time-varying uncertainties and disturbances. In the case of disturbances, both repetitive, i.e., the same on each trial, and non-repetitive are considered. With the aim of keeping the ILC law complexity to a minimum, a PD-type is considered, but the analysis extends to more complex laws if required by a particular application.

Section 2 gives the required background and details the design of the feedback control loop with H_∞ performance. In section 3, the stability theory for linear repetitive processes is used to develop an LMI-based robust ILC design for both repetitive and non-repetitive disturbances. Section 4 gives the results and associated discussion from applying the new results in this paper to an example, including comparisons with a direct ILC design. Finally, section 5 gives overall conclusions on the new results in this paper and section 6 discusses areas for possible future research.

Throughout this paper, 0 and I denote the null and identity matrices, respectively, with compatible dimensions. The notation $P \succ 0$ denotes a symmetric positive definite matrix, $(*)$ denotes block entries in symmetric matrices, and diag $[\cdot]$ denotes a diagonal matrix with compatibly dimensioned block entries.

The following lemma is used in the proofs of the main results.

Lemma 1. *[20] Given symmetric matrix Ω, as well as matrices X, Y with appropriate dimensions, and unknown matrix Δ satisfies $\Delta^T\Delta \prec I$, then*

$$\Omega + X\Delta Y + Y^T\Delta^T X^T \prec 0, \tag{1}$$

holds when and only when there exists constant $\varepsilon > 0$ such that

$$\Omega + \varepsilon XX^T + \varepsilon^{-1}YY^T \prec 0. \tag{2}$$

Lemma 2. *[21] (Schur's complement) Given matrices χ_1, χ_2, χ_3 with appropriate dimensions, and satisfying $\chi_2 \succ 0$, then the inequality $\chi_1 + \chi_3^T\chi_2^{-1}\chi_3 \prec 0$ can be written as*

$$\begin{bmatrix} \chi_1 & \chi_3^T \\ \chi_3 & -\chi_2 \end{bmatrix} \prec 0$$

or

$$\begin{bmatrix} -\chi_2 & \chi_3 \\ \chi_3^T & \chi_1 \end{bmatrix} \prec 0.$$

2. Background and Feedback Control Loop Tuning

This paper considers discrete-time batch processes subject to input delay, time-varying uncertainties, and disturbances described by the state-space model

$$\begin{aligned} x_{k+1}(t+1) &= (A + \Delta A(t))x_{k+1}(t) \\ &+ (B + \Delta B(t))u_{k+1}(t-\tau) + B_d d_{k+1}(t), \\ y_{k+1}(t) &= Cx_{k+1}(t), \end{aligned} \tag{3}$$

where t and k denote the time and trial indices. $x_{k+1}(t) \in \mathbb{R}^p$, $u_{k+1}(t) \in \mathbb{R}^m$ and $y_{k+1}(t) \in \mathbb{R}^n$ are, respectively, the state, input, and output vectors, $d_{k+1}(t) \in \mathbb{R}^m$ represents disturbances (if present), and τ is the known input delay. The initial state vector is $x_k(0)$ and is set as zero in each trail. $\Delta A(t)$ and $\Delta B(t)$ denote the uncertainties, which are norm-bounded and time-varying. Moreover,

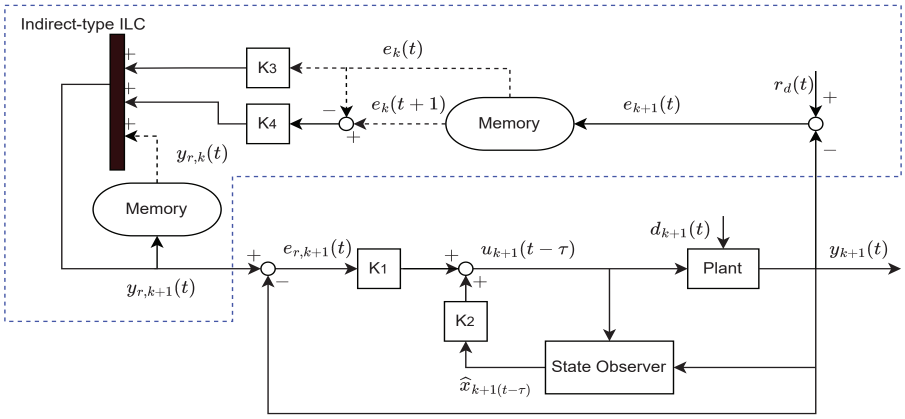

Figure 1: The indirect-type ILC structure

$$[\Delta A(t) \quad \Delta B(t)] = D\Delta(t)[F_a \quad F_b]. \tag{4}$$

where D, F_a and F_b are constant matrices that are known or can be estimated and $\Delta(t)$ is the uncertainty, which is assumed to satisfy $\Delta^T(t)\Delta(t) \leq I$. For ease of notation, $\Delta(t)$, $\Delta A(t)$, $\Delta B(t)$ are written as Δ, ΔA, and ΔB, respectively, in the rest of this paper.

Fig 1 shows a PD-type indirect ILC structure, consisting of a feedback control loop and the ILC law. The feedback loop, sometimes termed the inner loop regulates along the time behavior, and its structure and design is detailed in this section. The ILC forms the so-called outer loop, based on a PD-type law in this paper, and is specified and designed in the next section.

Let $r_d(t)$ denote the specified reference trajectory, and hence the error on trial k is

$$e_{k+1}(t) = r_d(t) - y_{k+1}(t). \tag{5}$$

i.e., the difference between the reference trajectory and the output on this trial. For analysis, the sequence $\{e_k\}_k$ is of central importance. The ILC design problem is to find a control law that, when applied, will force this sequence to converge from trial to trial (k) and regulate the dynamics along the trials (t). Convergence to zero in k of $\{e_k\}_k$ is the objective, and in this case the desired reference is equal to the system output.(In some cases, this may have to be relaxed to convergence in some neighborhood of zero, as measured by the norm on the underlying function space). The interaction between the two loops is that the ILC law adjusts the set point for the inner loop.

The set-point error of the inner loop is

$$e_{r,k+1}(t) = y_{r,k+1}(t) - y_{k+1}(t), \tag{6}$$

where $e_{r,k+1}(t)$ is the set-point error, $y_{r,k+1}(t)$ is the set-point updated by the indirect-type ILC law, which differs from the reference trajectory $r_d(t)$. Moreover, $y_{r,k+1}(t)$ is set as zero for the initial trial.

The presence of disturbance and uncertainties makes obtaining accurate information on the current state vector challenging. In addition, all entries in this vector may not be available for measurement, and also there will be a phase lag due to the input time delay. For these reasons, an observer is used to estimate the future state vector, which has the structure

$$\begin{aligned} \hat{x}_{k+1}(t+1) &= A\hat{x}_{k+1}(t) + Bu_{k+1}(t) \\ &+ L[y_{k+1}(t+\tau) - \hat{y}_{k+1}(t)], \\ \hat{y}_{k+1}(t) &= C\hat{x}_{k+1}(t), \end{aligned} \tag{7}$$

where $\hat{x}_{k+1}(t)$ denotes the τ-step ahead prediction of the state vector $x_{k+1}(t)$, i.e, the estimate of the state vector $x_{k+1}(t+\tau)$, $\hat{y}_{k+1}(t)$ denotes the prediction of the output vector $y_{k+1}(t+\tau)$, and L is the observer gain matrix to be determined. Also, the prediction error is defined as

$$\hat{e}_{k+1}(t) = x_{k+1}(t+\tau) - \hat{x}_{k+1}(t). \tag{8}$$

where $\hat{e}_{k+1}(t)$ denotes the prediction error at $t+\tau$.

To achieve no steady-state tracking error along the time (trial) direction in the inner loop, a local controller combined with observer-based state feedback plus set-point tracking error information is used, where

$$u_{k+1}(t-\tau) = K_1 e_{r,k+1}(t) + K_2\hat{x}_{k+1}(t-\tau), \tag{9}$$

and K_1 and K_2 are matrices to be designed.

Combining (6) - (9), gives

$$\begin{aligned} \hat{x}_{k+1}(t+1) &= (A + BK_2 - BK_1C)\hat{x}_{k+1}(t) \\ &+ (-BK_1C + LC)\hat{e}_{k+1}(t) \\ &+ BK_1 y_{r,k+1}(t+\tau), \\ \hat{e}_{k+1}(t+1) &= (\Delta A + \Delta BK_2 - \Delta BK_1C)\hat{x}_{k+1}(t) \\ &+ (A + \Delta A - \Delta BK_1C - LC)\hat{e}_{k+1}(t) \\ &+ \Delta BK_1 y_{r,k+1}(t+\tau) + B_d d_{k+1}(t+\tau), \end{aligned} \tag{10}$$

Introduce the augmented vector $\xi_{k+1}(t) = \left[\hat{x}^T_{k+1}(t-\tau) \quad \hat{e}^T_{k+1}(t-\tau)\right]^T$, Then the state-space model (3) can be rewritten in delay-free formulation as

$$\begin{aligned} \xi_{k+1}(t+1) &= \hat{A}\xi_{k+1}(t) + \hat{B}y_{r,k+1}(t) + \hat{B}_d d_{k+1}(t), \\ y_{k+1}(t) &= \widetilde{C}\xi_{k+1}(t), \end{aligned} \tag{11}$$

where

$$\hat{A} = \begin{bmatrix} A + BK_2 - BK_1C & -BK_1C + LC \\ \Delta A + \Delta BK_2 - \Delta BK_1C & A + \Delta A - \Delta BK_1C - LC \end{bmatrix},$$
$$\hat{B} = \begin{bmatrix} BK_1 \\ \Delta BK_1 \end{bmatrix}, \hat{B}_d = \begin{bmatrix} 0 \\ B_d \end{bmatrix}, \widetilde{C} = \begin{bmatrix} C & C \end{bmatrix}.$$

The controller given above only involves signals on the same trial; hence, the trial subscript (k) for variables is omitted in the remainder of this section. The inner loop-controlled system is described by

$$\begin{aligned} \xi_{k+1}(t+1) &= \hat{A}\xi_{k+1}(t) + \hat{B}_d d_{k+1}(t), \\ e_{r,k+1}(t) &= -\widetilde{C}\xi_{k+1}(t), \end{aligned} \tag{12}$$

where $\hat{A} = \widetilde{A} + \Delta\widetilde{A}$ and

$$\begin{aligned} &\widetilde{A} = A_1 + B_1\widetilde{K} + \widetilde{L}, \Delta\widetilde{A} = D_0\Delta F, \\ &A_1 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, B_1 = \begin{bmatrix} B \\ 0 \end{bmatrix}, \widetilde{L} = \begin{bmatrix} 0 & LC \\ 0 & -LC \end{bmatrix}, D_0 = \begin{bmatrix} 0 \\ D \end{bmatrix}, \\ &\widetilde{K} = \begin{bmatrix} K_2 - K_1C & -K_1C \end{bmatrix}, F = F_1 + F_b\widetilde{K}, F_1 = \begin{bmatrix} Fa & Fa \end{bmatrix}. \end{aligned} \tag{13}$$

The following robust H_∞ design is the first new result of this paper.

Theorem 1. *The controlled dynamics (12) is robustly stable with a H_∞ performance level γ_c if there exist compatibly dimensioned matrices $W > 0, P, R$ and a positive scalar ε_c such that the following LMI is feasible*

$$\begin{bmatrix} -W & \widetilde{A}W & \hat{B}_d & 0 & 0 & \varepsilon_c D_0 \\ (*) & -W & 0 & \widetilde{C}^T & (FW)^T & 0 \\ (*) & (*) & -\rho_c I & 0 & 0 & 0 \\ (*) & (*) & (*) & -I & 0 & 0 \\ (*) & (*) & (*) & (*) & -\varepsilon_c I & 0 \\ (*) & (*) & (*) & (*) & (*) & -\varepsilon_c I \end{bmatrix} < 0, \tag{14}$$

where

$$\begin{aligned} \widetilde{A}W &= A_1W + B_1P + R, \\ FW &= F_1W + F_bP, \\ \rho_c &= \gamma_c^2. \end{aligned}$$

Moreover, if (14) is feasible, then the observer gain and local feedback controller are obtained from the following parametrization

$$\begin{bmatrix} L \\ -L \end{bmatrix}\begin{bmatrix} 0 & C \end{bmatrix} = RW^{-1}, \begin{bmatrix} K_2 - K_1C & -K_1C \end{bmatrix} = PW^{-1}. \tag{15}$$

Proof. Stability and robust H_∞ performance is equivalent to

$$\left\|e_{r,k+1}(t)\right\|_2 < \gamma_c \left\|d_{k+1}(t)\right\|_2, \tag{16}$$

where

$$\begin{aligned} \left\|e_{r,k+1}(t)\right\|_2 &= \sqrt{\sum_{t=0}^{\alpha} e_{r,k+1}^T(t)e_{r,k+1}(t)}, \\ \left\|d_{k+1}(t)\right\|_2 &= \sqrt{\sum_{t=0}^{\alpha} d_{k+1}^T(t)d_{k+1}(t)}, \quad 0 \le t \le \alpha, \ \gamma_c > 0. \end{aligned}$$

Define the H_∞ control objective function as

$$J_c = \left\|e_{r,k+1}(t)\right\|_2^2 - \gamma_c^2 \left\|d_{k+1}(t)\right\|_2^2, \tag{17}$$

and consider the Lyapunov function

$$V(t) = \xi_{k+1}^T(t)S\xi_{k+1}(t), S = \text{diag}\begin{bmatrix} S_1 & S_2 \end{bmatrix} > 0.$$

Then, since zero initial state is assumed, $x_k(0) = 0, e_k(0) = 0, \xi_{k+1}(0) = 0$, (17) can be rewritten as

$$J_c < \sum_{t=0}^{\alpha} G_c^T(t)\Lambda_c G_c(t),$$

where

$$\begin{aligned} G_c(t) &= \begin{bmatrix} \xi_{k+1}(t) \\ d_{k+1}(t) \end{bmatrix}, \\ \Lambda_c &= \begin{bmatrix} \hat{A} & \hat{B}_d \end{bmatrix}^T S \begin{bmatrix} \hat{A} & \hat{B}_d \end{bmatrix} - \begin{bmatrix} S - \widetilde{C}^T\widetilde{C} & 0 \\ 0 & \gamma_c^2 \end{bmatrix}. \end{aligned}$$

By the robust stability theorem, for any $G_c \neq 0$, the requirement that (12) is stable with prescribed H_∞ performance is equivalent to $\Lambda_c < 0$. Applying the Schur's complement formula twice, this last condition can be written as

$$\begin{bmatrix} -S^{-1} & \hat{A} & \hat{B}_d & 0 \\ (*) & -S & 0 & \widetilde{C}^T \\ (*) & (*) & -\gamma_c^2 I & 0 \\ (*) & (*) & (*) & -I \end{bmatrix} < 0, \tag{18}$$

Left and right-multiplying this last expression by $\text{diag}\begin{bmatrix} I & S^{-1} & I & I \end{bmatrix}$ and then setting $W = S^{-1}, P = \widetilde{K}S^{-1}, R = \widetilde{L}S^{-1}$ and $\rho_c = \gamma_c^2$, results in

$$\Omega_c + X_c\Delta Y_c + Y_c^T\Delta^T X_c^T < 0, \tag{19}$$

where

$$\begin{aligned} \Omega_c &= \begin{bmatrix} -W & \widetilde{A}W & \hat{B}_d & 0 \\ (*) & -W & 0 & \widetilde{C}^T \\ (*) & (*) & -\rho_c I & 0 \\ (*) & (*) & (*) & -I \end{bmatrix}, \\ X_c &= \begin{bmatrix} D_0^T & 0 & 0 & 0 \end{bmatrix}^T, \\ Y_c &= \begin{bmatrix} 0 & F_1W + F_bP & 0 & 0 \end{bmatrix}. \end{aligned}$$

Applying Lemma 1, (19) can be rewritten as

$$\Omega_c + \varepsilon_c X_c X_c^T + \varepsilon_c^{-1} Y_c^T Y_c < 0, \tag{20}$$

Next, by the Schur's complement formula and pre- and post-multiplying by $\text{diag}\begin{bmatrix} I & \varepsilon_c^{1/2} & \varepsilon_c^{1/2} \end{bmatrix}$, gives

$$\begin{bmatrix} \Omega_c & Y_c^T & \varepsilon_c X_c \\ (*) & -\varepsilon_c I & 0 \\ (*) & (*) & -\varepsilon_c I \end{bmatrix} < 0. \tag{21}$$

and the proof is complete.

To reduce the effect caused by disturbance and obtain the optimal H_∞ performance, the following optimization program can be implemented to Theorem 1.

$$\min_{W>0,\varepsilon_c>0} \rho_c \quad s.t.\ (14)$$

In the nominal model case, i.e., $\Delta = 0, d_{k+1}(t) = 0$, the LMI of (14) reduces to

$$\begin{bmatrix} -W & -\widetilde{A}W & 0 \\ (*) & -W & \widetilde{C}^T \\ (*) & (*) & -I \end{bmatrix} \prec 0. \tag{22}$$

and the design again uses the parameterization (15). Design based on (22) and (15) is used in the first case considered in Section 5.

3. Indirect-type ILC design

The ILC law in Fig 1 is chosen to have the PD-type structure, i.e.,

The set-point related PD-type ILC law has the structure

$$y_{r,k+1}(t) = y_{r,k}(t) + K_3 e_k(t) + K_4[e_k(t+1) - e_k(t)], \tag{23}$$

where K_3 and K_4 are to be determined. The ILC law is designed to update the set-point, and forcing the set-point to follow the desired trajectory along the trial gradually. Note that the ILC law updates the set-point along the trial without modifying the structure of the inner closed-loop feedback system. For ease of presentation set $K = K_3 - K_4$. This particular law has been chosen because it is widely used in applications and its structure is relatively simple, often a critical aspect in terms of applications. Other ILC laws can also be considered.

In the new design developed in the remainder of this paper, the first stage is to use Theorem 1 to select the observer gain matrix L and the control law matrices K_1 and K_2. The analysis that follows then enables design of K_3 and K_4 in (23). Also, the case study section includes comparisons with a direct type ILC law.

In order to transform the system (3) into an equivalent repetitive process model, define a trial-direction error function as

$$\delta f_{k+1}(t+1) = f_{k+1}(t) - f_k(t), \tag{24}$$

where f denotes, as appropriate, $\hat{x}, d, \hat{e}, \xi$ and y. Then it follows from (11) and (24) that

$$\begin{aligned} \delta\xi_{k+1}(t+1) &= \hat{A}\delta\xi_{k+1}(t) + \hat{B}\delta y_{r,k+1}(t) + \hat{B}_d\delta d_{k+1}(t), \\ \delta y_{k+1}(t) &= \widetilde{C}\delta\xi_{k+1}(t), \end{aligned} \tag{25}$$

where $\delta y_{r,k+1}(t) = Ke_k(t-1) + K_4 e_k(t)$. Furthermore, using the definition of the tracking error in (5), gives

$$\begin{aligned} e_{k+1}(t+1) &= e_k(t) + y_k(t) - y_{k+1}(t) \\ &= -\delta y_{k+1}(t+1) + e_k(t) \\ &= e_k(t) - \widetilde{C}\delta\xi_{k+1}(t+1). \end{aligned} \tag{26}$$

Combing (25) and (26), the controlled dynamics are described by the following state space model

$$\begin{aligned} \delta\xi_{k+1}(t+1) &= \hat{A}\delta\xi_{k+1}(t) + \hat{B}_1 e_k(t-1) \\ &+ \hat{B}_0 e_k(t) + \hat{B}_d\delta d_{k+1}(t), \\ e_{k+1}(t) &= \hat{C}\delta\xi_{k+1}(t) + \hat{D}_1 e_k(t-1) \\ &+ \hat{D}_0 e_k(t) + \hat{D}_d\delta d_{k+1}(t), \end{aligned} \tag{27}$$

where

$$\hat{B}_1 = \begin{bmatrix} BK_1K \\ \Delta BK_1K \end{bmatrix}, \hat{B}_0 = \begin{bmatrix} BK_1K_4 \\ \Delta BK_1K_4 \end{bmatrix},$$
$$\hat{C} = -\widetilde{C}\hat{A}, \hat{D}_1 = -\widetilde{C}\hat{B}_1, \hat{D}_0 = I - \widetilde{C}\hat{B}_0, \hat{D}_d = -C\hat{B}_d.$$

and $\hat{A}$ is defined after (11). Introduce the augmented vector $\eta(t) = \begin{bmatrix}\xi_k^T(t) & e_k^T(t-1)\end{bmatrix}^T$. Then the dynamics of (27) can be written as

$$\begin{aligned} \eta_{k+1}(t+1) &= \begin{bmatrix} \hat{A} & \hat{B}_1 \\ 0 & 0 \end{bmatrix}\eta_{k+1}(t) + \begin{bmatrix} \hat{B}_0 \\ I \end{bmatrix} e_k(t) + \begin{bmatrix} \hat{B}_d \\ 0 \end{bmatrix}\delta d_{k+1}(t), \\ e_{k+1}(t) &= \begin{bmatrix} \hat{C} & \hat{D}_1 \end{bmatrix}\eta_{k+1}(t) + \hat{D}_0 e_k(t) + \hat{D}_d\delta d_{k+1}(t). \end{aligned} \tag{28}$$

The dynamics described by (28) have the structure of a discrete repetitive process. These processes are a distinct class of 2D systems characterized by a series of sweeps, termed trials in this paper and denoted by the nonnegative integer subscript k on variables, through a set of dynamics defined over a finite duration, termed the trial length, where for discrete dynamics the sample values are indexed by t, and if N denotes the number of samples, $0 \leq p \leq N-1$. Also N times the sampling period gives the trial length, denoted by α.

Once a trial is complete, the process resets to the starting location and the next trial begins. On any trial, the output from the previous trial acts as a forcing function on the next and thereby contributes to its dynamics, see [8] for a detailed treatment for linear examples, including references to their industrial origins. The unique control problem for these processes is that the sequence of pass profiles can include oscillations that increase in amplitude from trial-to-trial (k) and cannot be regulated by standard control action. Repetitive processes are a class of 2D systems that operate over the restricted quadrant $(k, t) \in [0, \infty] \times [0, \alpha]$. In the repetitive process interpretation of the ILC dynamics, $\xi_{k+1}(t)$ is the state vector on trial $k+1$ and $e_k(t)$ is the error on trial k.

The stability theory for repetitive processes applied to ILC requires that a bounded initial trial error produces a sequence of bounded trial outputs. Moreover, this property can be applied over the finite and fixed trial length, or uniformly, i.e., for all possible trial lengths, where this last property can be analyzed mathematically by considering $\alpha \to \infty$. The first of these properties is termed asymptotic stability and the second stability along the trial. Moreover, the asymptotic stability property guarantees trial-to-trial error convergence but is independent of the state dynamics. In particular, due to the finite trial length, trial-to-trial error convergence can occur for unstable systems.

Stability along the trial is the name given to the second property. Moreover, a necessary condition for this property is that the state matrix is stable, i.e., the dynamics along a given trial are stable, but this condition is only necessary for stability along the trial. This property also requires that the frequency content of any trial decays geometrically from trial-to-trial. Even though these properties can be checked graphically, the last, in particular, does not provide a basis for control law design, except in a few special cases.

One alternative to the frequency domain approach is to formulate conditions for stability along the trial in terms of linear matrix inequalities (LMIs). Define the system matrix Ξ for (28) as

$$\Xi = \begin{bmatrix} \Xi_1 & \Xi_2 \\ \Xi_3 & \Xi_4 \end{bmatrix} \tag{29}$$

where

$$\Xi_1 = \begin{bmatrix} \hat{A} & \hat{B}_1 \\ 0 & 0 \end{bmatrix}, \ \Xi_2 = \begin{bmatrix} \hat{B}_0 \\ I \end{bmatrix}$$

$$\Xi_3 = \begin{bmatrix} \hat{C} & \hat{D}_1 \end{bmatrix}, \ \Xi_4 = \hat{D}_0$$

Then it can be shown [8] that a sufficient condition for stability along the trial of (28) is that there exists a matrix $S > 0$ such that

$$\Xi^T S \Xi - S \prec 0 \tag{30}$$

This last expression forms the 2D Lyapunov equation condition for stability along the trial and is both necessary and sufficient in some cases, the most relevant being single-input single-output systems. As shown in the rest of this paper, it immediately leads to conditions for control law design in terms of LMIs.

4. ILC tuning

Firstly, the nominal case, $\Delta = 0, d_{k+1}(t) = 0$, is considered, and the following theorem gives an LMI condition for stability along the trial.

Theorem 2. *Consider an indirect-type ILC scheme described as a discrete, repetitive process described by (28) in the nominal case with no disturbance and matrices L, K_1 and K_2 available from Theorem 1. Then stability along the trial hold if there exist compatibly dimensioned matrices $W_1 > 0, W_2 > 0, W_3 > 0$, R_1 and R_2 such that*

$$\Psi_1 = \begin{bmatrix} \Theta_1 & \Theta_2 \\ (*) & \Theta_1 \end{bmatrix} \prec 0, \tag{31}$$

where

$$\Theta_1 = \begin{bmatrix} -W_1 & 0 & 0 \\ (*) & -W_2 & 0 \\ (*) & (*) & -W_3 \end{bmatrix},$$

$$\Theta_2 = \begin{bmatrix} \widetilde{A}W_1 & B_1K_1R_1 & B_1K_1R_2 \\ 0 & 0 & W_3 \\ -\widetilde{C}\widetilde{A}W_1 & -\widetilde{C}B_1K_1R_1 & W_3 - \widetilde{C}B_1K_1R_2 \end{bmatrix}.$$

If this last LMI is feasible, then the matrices in the control law (23) are given by

$$K = R_1W_2^{-1}, K_4 = R_2W_3^{-1}, K_3 = K + K_4. \tag{32}$$

Proof. The nominal system model is obtained by setting $\Delta A = 0, \Delta B = 0, d_{k+1} = 0$ and hence

$$\begin{aligned} \hat{A} &= \widetilde{A}, \\ \hat{B}_1 &= B_1K_1K, \\ \hat{B}_0 &= B_1K_1K_4, \end{aligned} \tag{33}$$

Applying the Schur's complement formula, (30) can be written as

$$\begin{bmatrix} -S^{-1} & \Xi \\ (*) & -S \end{bmatrix} \prec 0, \tag{34}$$

where

$$\Xi = \begin{bmatrix} \hat{A} & \hat{B}_1 & \hat{B}_0 \\ 0 & 0 & I \\ \hat{C} & \hat{D}_1 & \hat{D}_0 \end{bmatrix}, S = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}.$$

Pre- and post- multiplying this last expression by $\text{diag}\begin{bmatrix} I & I & I & S_1^{-1} & S_2^{-1} & S_3^{-1} \end{bmatrix}$, and setting $W_1 = S_1^{-1}, W_2 = S_2^{-1}, W_3 = S_3^{-1}, KW_2 = R_1, K_4W_3 = R_2$, gives the LMI (31), and the proof is complete.

The following result enables ILC design in the presence of uncertainty characterised by (4) and repetitive disturbances, i.e., those for which (24) (applied to the disturbance) hold.

Theorem 3. *Consider an indirect-type ILC scheme described by a discrete, repetitive process of the form (28) in the presence of time-varying uncertainties characterised by (4) and a repetitive disturbance that satisfies (24). Suppose also that matrices L, K_1 and K_2 are available from Theorem 1. Then this process is stable along the trial if there exists a compatibly dimensioned diagonal matrix $W > 0$, together with compatibly dimensioned matrices $Q_1, Q_2, Q_3 > 0$, $Q_4 > 0$, and a scalar $\varepsilon_1 > 0$ such that the following LMI is feasible*

$$\Psi_2 = \begin{bmatrix} O_1 & O_2 & O_4 \\ (*) & O_3 & O_5 \\ (*) & (*) & O_6 \end{bmatrix} \prec 0, \tag{35}$$

where

$$O_1 = \begin{bmatrix} -W & 0 \\ (*) & -Q_4 \end{bmatrix},$$

$$O_2 = \begin{bmatrix} \widetilde{A}W & B_1K_1Q_1 & B_1K_1Q_2 \\ -\widetilde{C}\widetilde{A}W & -\widetilde{C}B_1K_1Q_1 & Q_4 - \widetilde{C}B_1K_1Q_2 \end{bmatrix},$$

$$O_3 = \begin{bmatrix} -W & 0 & 0 \\ (*) & -Q_3 & 0 \\ (*) & (*) & -Q_4 + Q_3 \end{bmatrix}, O_4 = \begin{bmatrix} 0 & \varepsilon_1 D_0 \\ 0 & -\varepsilon_1 \widetilde{C} D_0 \end{bmatrix},$$

$$O_5 = \begin{bmatrix} (FW)^T & 0 \\ (F_bK_1Q_1)^T & 0 \\ (F_bK_1Q_2)^T & 0 \end{bmatrix}, O_6 = \begin{bmatrix} -\varepsilon_1 I & 0 \\ (*) & -\varepsilon_1 I \end{bmatrix}.$$

If this last LMI is feasible, then the matrices in the control law (23) are given by

$$K = Q_1Q_4^{-1}, K_4 = Q_2Q_4^{-1}, K_3 = K + K_4. \tag{36}$$

Proof. Consider the Lyapunov function

$$\begin{aligned} V(k,t) &= V_1(t,k) + V_2(k,t), \\ V_1(t,k) &= \delta\xi_{k+1}^T(t) S \delta\xi_{k+1}(t), \\ V_2(k,t) &= e_k^T(t-1)P_1 e_k(t-1) + e_k^T(t)(P_2 - P_1)e_k(t), \end{aligned} \quad (37)$$

where $S = \mathrm{diag}\begin{bmatrix} S_1 & S_2 \end{bmatrix} > 0, P_2 > 0, > P_1 > 0$ and $P_2 - P_1 > 0$. Then

$$\begin{aligned} \Delta V_1(t,k) &= V_1(t+1,k) - V_1(t,k) \\ &= \delta\xi_{k+1}^T(t+1) S \delta\xi_{k+1}(t+1) - \delta\xi_{k+1}^T(t) S \delta\xi_{k+1}(t), \\ \Delta V_2(k,t) &= V_2(k+1,t) - V_2(k,t) \\ &= e_{k+1}^T(t)P_2 e_{k+1}(t) - e_k^T(t-1)P_1 e_k(t-1) \\ &\quad - e_k^T(t)(P_2 - P_1)e_k(t), \\ \Delta V(k,t) &= \Delta V_1(t,k) + \Delta V_2(k,t) \\ &= H_{k+1}^T(t) \varPi H_{k+1}(t), \end{aligned} \quad (38)$$

where

$$H_{k+1}(t) = \begin{bmatrix} \delta\xi_{k+1}(t) \\ e_k(t-1) \\ e_k(t) \end{bmatrix},$$

$$\varPi = \begin{bmatrix} \hat{A} & \hat{B}_1 & \hat{B}_0 \\ \hat{C} & \hat{D}_1 & \hat{D}_0 \end{bmatrix}^T \begin{bmatrix} S & 0 \\ (*) & P_2 \end{bmatrix} \begin{bmatrix} \hat{A} & \hat{B}_1 & \hat{B}_0 \\ \hat{C} & \hat{D}_1 & \hat{D}_0 \end{bmatrix} - \begin{bmatrix} S & 0 & 0 \\ (*) & P_1 & 0 \\ (*) & (*) & P_2 - P_1 \end{bmatrix}.$$

Then stability along the trial holds [8] if $\Delta V(k,t) < 0$ for all k and t, which is equivalent to the requirement that $\varPi < 0$ for any $H_{k+1}(t) \neq 0$.

By the Schur's complement formula, $\varPi < 0$ is equivalent to

$$\Gamma = \begin{bmatrix} -S^{-1} & 0 & \hat{A} & \hat{B}_1 & \hat{B}_0 \\ (*) & -P_2^{-1} & \hat{C} & \hat{D}_1 & \hat{D}_0 \\ (*) & (*) & -S & 0 & 0 \\ (*) & (*) & (*) & -P_1 & 0 \\ (*) & (*) & (*) & (*) & -P_2 + P_1 \end{bmatrix} < 0, \quad (39)$$

Moreover, from the state space model and the uncertainty description (4),

$$\hat{B}_1 = (B_1 + \Delta B_1)K_1 K, \hat{B}_0 = (B_1 + \Delta B_1)K_1 K_4, \quad (40)$$

where

$$\Delta B_1 = \begin{bmatrix} 0 \\ \Delta B \end{bmatrix} = D_0 \Delta F_b,$$

Pre- and post- multiplying (39) by $\mathrm{diag}\begin{bmatrix} I & I & S^{-1} & P_2^{-1} & P_2^{-1} \end{bmatrix}$, and letting $W = S^{-1}, Q_1 = KP_2^{-1}, Q_2 = K_4 P_2^{-1}, Q_3 = P_2^{-1}P_1 P_2^{-1}, Q_4 = P_2^{-1}$, gives that

$$\Gamma = \begin{bmatrix} O_1 & O_2 + \Delta O_2 \\ (*) & O_3 \end{bmatrix} < 0, \quad (41)$$

where

$$\Delta O_2 = \begin{bmatrix} \Delta\widetilde{A}W & \Delta B_1 K_1 Q_1 & \Delta B_1 K_1 Q_2 \\ -\widetilde{C}\Delta\widetilde{A}W & -\widetilde{C}\Delta B_1 K_1 Q_1 & -\widetilde{C}\Delta B_1 K_1 Q_2 \end{bmatrix}.$$

This last inequality is still nonlinear due to the system uncertainties, but (41) can be rewritten as

$$\Omega_1 + X_1 \Delta Y_1 + Y_1^T \Delta^T X_1^T < 0, \quad (42)$$

where

$$\begin{aligned} \Omega_1 &= \begin{bmatrix} O_1 & O_2 \\ (*) & O_3 \end{bmatrix}, \\ X_1 &= \begin{bmatrix} D_0^T & (-\widetilde{C}D_0)^T & 0 & 0 & 0 \end{bmatrix}^T, \\ Y_1 &= \begin{bmatrix} 0 & 0 & FW & F_b K_1 Q_1 & F_b K_1 Q_2 \end{bmatrix}. \end{aligned}$$

Applying Lemma 1, gives that (42) is equivalent to

$$\Omega_1 + \varepsilon_1 X_1 X_1^T + \varepsilon_1^{-1} Y_1^T Y_1 < 0, \quad (43)$$

Moreover, application of the Schur's complement formula, and then pre- and post- multiplying the result by $\mathrm{diag}\begin{bmatrix} I & \varepsilon_1^{1/2} I & \varepsilon_1^{1/2} I \end{bmatrix}$, (43) can be written as

$$\begin{bmatrix} \Omega_1 & Y_1^T & \varepsilon_1 X_1 \\ (*) & -\varepsilon_1 I & 0 \\ (*) & (*) & -\varepsilon_1 I \end{bmatrix} < 0, \quad (44)$$

and the proof is complete.

Theorem 4. *Consider an indirect-type ILC scheme described by a discrete, repetitive process of the form (28) in the presence of time-varying uncertainties characterized by (4) and a non-repetitive disturbance that does not satisfy (24). Suppose also that matrices L, K_1 and K_2 are available from Theorem 1. Then this process is stable along the trial with H_∞ disturbance attenuation performance level γ if there exists a diagonal matrix $W > 0$, matrices Q_1, Q_2, $Q_3 > 0$, $Q_4 > 0$, and scalars $\varepsilon_2 > 0$, and $\gamma > 0$ such that*

$$\Psi_3 = \begin{bmatrix} O_1 & O_2 & O_7 & O_{10} \\ (*) & O_3 & O_8 & O_{11} \\ (*) & (*) & O_9 & O_{12} \\ (*) & (*) & (*) & O_{13} \end{bmatrix} < 0, \quad (45)$$

where

$$O_7 = \begin{bmatrix} \hat{B}_d & 0 \\ -\widetilde{C}\hat{B}_d & 0 \end{bmatrix}, O_8 = \begin{bmatrix} 0 & (-\widetilde{C}\widetilde{A}W)^T \\ 0 & (-\widetilde{C}B_1 K_1 Q_1)^T \\ 0 & (Q_4 - \widetilde{C}B_1 K_1 Q_2)^T \end{bmatrix},$$

$$O_9 = \begin{bmatrix} -\rho & (-\widetilde{C}\hat{B}_d)^T \\ (*) & -I \end{bmatrix}, O_{10} = \begin{bmatrix} 0 & \varepsilon_2 D_0 \\ 0 & -\varepsilon_2 \widetilde{C}D_0 \end{bmatrix},$$

$$O_{11} = \begin{bmatrix} (FW)^T & 0 \\ (F_b K_1 Q_1)^T & 0 \\ (F_b K_1 Q_2)^T & 0 \end{bmatrix}, O_{12} = \begin{bmatrix} 0 & 0 \\ 0 & -\varepsilon_2 \widetilde{C}D_0 \end{bmatrix},$$

$$O_{13} = \begin{bmatrix} -\varepsilon_2 I & 0 \\ (*) & -\varepsilon_2 I \end{bmatrix}, \rho = \gamma^2 I.$$

If the above LMI is feasible, then the matrices in the control law (23) are given by

$$K = Q_1 Q_4^{-1}, K_4 = Q_2 Q_4^{-1}, K_3 = K + K_4. \tag{46}$$

Proof. In the presence of a non repetitive disturbance, the repetitive process (28) is robustly stable along the trial with H_∞ disturbance attenuation performance $\gamma > 0$ if

$$\|e_{k+1}(t)\|_2 < \gamma \|\delta d_{k+1}(t)\|_2 , \tag{47}$$

where $0 \le t \le \alpha$,

$$\|e_{k+1}(t)\|_2 = \sqrt{\sum_{k=0}^{\infty}\sum_{t=0}^{\alpha} e_{k+1}^T(t) e_{k+1}(t)},$$

$$\|\delta d_{k+1}(t)\|_2 = \sqrt{\sum_{k=0}^{\infty}\sum_{t=0}^{\alpha} \delta d_{k+1}^T(t)\delta d_{k+1}(t)}.$$

Define the H_∞ control objective function

$$J = \|e_{k+1}(t)\|_2^2 - \gamma^2 \|\delta d_{k+1}(t)\|_2^2 , \tag{48}$$

Then, since zero initial state is assumed, $x_k(0) = 0, e_k(0) = 0, \xi_{k+1}(0) = 0$, (48) can be rewritten as

$$\begin{aligned} J &< \|e_{k+1}(t)\|_2^2 - \gamma^2 \|\delta d_{k+1}(t)\|_2^2 + V(\infty, \alpha+1) - V(0,0) \\ &= \sum_{k=0}^{\infty}\sum_{t=0}^{\alpha} [e_{k+1}^T(t)e_{k+1}(t) - \gamma^2 \delta d_{k+1}^T(t)\delta d_{k+1}(t) + \Delta V(k,t)] \\ &= \sum_{k=0}^{\infty}\sum_{t=0}^{\alpha} Z_{k+1}^T \Phi Z_{k+1}, \end{aligned} \tag{49}$$

where

$$Z_{k+1}(t) = \begin{bmatrix} \delta\xi_{k+1}(t) \\ e_k(t-1) \\ e_k(t) \\ \delta d_{k+1}(t) \end{bmatrix},$$

$$\Phi = \begin{bmatrix} \hat{A} & \hat{B}_1 & \hat{B}_0 & \hat{B}_d \\ \hat{C} & \hat{D}_1 & \hat{D}_0 & \hat{D}_d \end{bmatrix}^T \begin{bmatrix} S & 0 \\ (*) & I + P_2 \end{bmatrix} \begin{bmatrix} \hat{A} & \hat{B}_1 & \hat{B}_0 & \hat{B}_d \\ \hat{C} & \hat{D}_1 & \hat{D}_0 & \hat{D}_d \end{bmatrix} - \begin{bmatrix} S & 0 & 0 & 0 \\ (*) & P_1 & 0 & 0 \\ (*) & (*) & P_2 - P_1 & 0 \\ (*) & (*) & (*) & \gamma^2 I \end{bmatrix}.$$

By Theorem 2, robust stability of the controlled dynamics holds if objective function $J \prec 0$, which is equivalent to $\Phi \prec 0$. Also, write the matrix Φ in the form

$$\Phi = \begin{bmatrix} M_1^T \\ M_2^T \end{bmatrix} (\mathrm{O} + I_1^T I_1) \begin{bmatrix} M_1 & M_2 \end{bmatrix} - \begin{bmatrix} M_3 & 0 \\ (*) & \gamma^2 I \end{bmatrix}, \tag{50}$$

where

$$M_1 = \begin{bmatrix} \hat{A} & \hat{B}_1 & \hat{B}_0 \\ \hat{C} & \hat{D}_1 & \hat{D}_0 \end{bmatrix}, M_2 = \begin{bmatrix} \hat{B}_d \\ \hat{D}_d \end{bmatrix}, I_1 = \begin{bmatrix} 0 & I \end{bmatrix},$$

$$M_3 = \begin{bmatrix} S & 0 & 0 \\ (*) & P_1 & 0 \\ (*) & (*) & P_2 - P_1 \end{bmatrix}, \mathrm{O} = \begin{bmatrix} S & 0 \\ (*) & P_2 \end{bmatrix}.$$

For the convenience of application of the Schur's complement, write the matrix Φ as

$$\Phi = \begin{bmatrix} M_1^T \\ M_2^T \end{bmatrix} \mathrm{O} \begin{bmatrix} M_1 & M_2 \end{bmatrix} + \left(\begin{bmatrix} M_1^T \\ M_2^T \end{bmatrix} I_1^T I_1 \begin{bmatrix} M_1 & M_2 \end{bmatrix} - \begin{bmatrix} M_3 & 0 \\ (*) & \gamma^2 I \end{bmatrix} \right),$$

By applying Schur's complement formula, the robust stability condition $\Phi \prec 0$ holds if

$$\begin{bmatrix} -\mathrm{O}^{-1} & M_1 & M_2 \\ (*) & -M_3 + M_1^T I_1^T I_1 M_1 & M_1^T I_1^T I_1 M_2 \\ (*) & (*) & -\gamma^2 I + M_2^T I_1^T I_1 M_2 \end{bmatrix} \prec 0$$

Then above LMI can be written as

$$\begin{bmatrix} -\mathrm{O}^{-1} & M_1 & M_2 \\ (*) & -M_3 & 0 \\ (*) & (*) & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} 0 \\ M_1^T I_1^T \\ M_2^T I_1^T \end{bmatrix} I \begin{bmatrix} 0 & I_1 M_1 & I_1 M_2 \end{bmatrix} \prec 0$$

and applying Schur's complement formula again, which gives

$$\phi = \begin{bmatrix} -\mathrm{O}^{-1} & M_1 & M_2 & 0 \\ (*) & -M_3 & 0 & (I_1 M_1)^T \\ (*) & (*) & -\gamma^2 I & (I_1 M_2)^T \\ (*) & (*) & (*) & -I \end{bmatrix} \prec 0, \tag{51}$$

Also on pre- and post- multiplying the last expression by $\mathrm{diag}\begin{bmatrix} I & I & S^{-1} & P_2^{-1} & P_2^{-1} & I & I \end{bmatrix}$ and letting $W = S^{-1}, Q_1 = K P_2^{-1}, Q_2 = K_4 P_2^{-1}, Q_3 = P_2^{-1} P_1^{-1} P_2^{-1}, Q_4 = P_2^{-1}, \gamma^2 I = \rho$, gives

$$\begin{bmatrix} O_1 & O_2 + \Delta O_2 & O_7 \\ (*) & O_3 & O_8 + \Delta O_8 \\ (*) & (*) & O_9 \end{bmatrix} \prec 0, \tag{52}$$

where

$$\Delta O_8 = \begin{bmatrix} 0 & (-\widetilde{C}\Delta\widetilde{A}W)^T \\ 0 & (-\widetilde{C}\Delta B_1 K_1 Q_1)^T \\ 0 & (-\widetilde{C}\Delta B_1 K_1 Q_2)^T \end{bmatrix}.$$

This last matrix is not an LMI but, proceeding as in the proof of the previous theorem, (52) can be written as

$$\begin{bmatrix} O_1 & O_2 + \Delta O_2 & O_7 \\ (*) & O_3 & (*) \\ (*) & (O_8 + \Delta O_8)^T & O_9 \end{bmatrix} \prec 0, \tag{53}$$

or

$$\Omega_2 + X_2 \Delta_2 Y_2 + Y_2^T \Delta^T X_2^T \prec 0, \tag{54}$$

where

$$\Omega_2 = \begin{bmatrix} O_1 & O_2 & O_7 \\ (*) & O_3 & (*) \\ (*) & (O_8)^T & O_9 \end{bmatrix},$$
$$X_2 = \begin{bmatrix} D_0^T & (-\widetilde{C}D_0)^T & 0 & 0 & 0 & 0 & (-\widetilde{C}D_0)^T \end{bmatrix}^T,$$
$$Y_2 = \begin{bmatrix} 0 & 0 & FW & F_bK_1Q_1 & F_bK_1Q_2 & 0 & 0 \end{bmatrix}.$$

Moreover, on applying Lemma 1, (54) is equivalent to

$$\Omega_2 + \varepsilon_2 X_2 X_2^T + \varepsilon_2^{-1} Y_2^T Y_2 < 0, \tag{55}$$

and pre- and post- multiplying by diag$\begin{bmatrix} I & \varepsilon_2^{1/2} I & \varepsilon_2^{1/2} I \end{bmatrix}$, gives that (55) can be written as

$$\begin{bmatrix} \Omega_2 & Y_2^T & \varepsilon_2 X_2 \\ (*) & -\varepsilon_2 I & 0 \\ (*) & (*) & -\varepsilon_2 I \end{bmatrix} < 0. \tag{56}$$

and the proof is complete.

5. Case Study

This section gives the results of a simulation-based study on applying the new designs in this paper using a model of a batch processing system. Included is a comparison with a direct ILC law design. In ILC, the trial-to-trial error convergence is of critical importance, and in keeping with many other papers in the literature, this property is measured using the root mean square (RMS) tracking error on each trial, defined for trial k as

$$RMS = \sqrt{\frac{1}{\alpha}\sum_{t=1}^{\alpha} e_k^2(t)}. \tag{57}$$

The performance along a trial is assessed as per standard systems.

The example is for one form of injection molding. As a typical batch process, injection molding plays an important role in transforming polymer granules into various shapes and types of products like cups or compact discs even precision lens. It includes three stages: filling, packing/holding, and cooling. During filling, injection screw pushes the polymer melt into the mold cavity and after that, the process switches into packing-holding stage. In the second stage, additional polymer is added under a certain pressure to compensate the shrinkage caused by material cooling and solidification until the entrance of mold cavity freezing and isolating. Then the third stage came for continuously cooling.

For the packing stage, a critical process variable to be controlled is the nozzle pressure, which significantly influences product quality. As studied in [22], based on the open-loop tests and analysis, the model of the nozzle packing pressure response to the hydraulic control valve opening can be identified as

$$P_\Delta(z^{-1}) = \frac{1.2390(0.10)z^{-1} - 0.9282(0.14)z^{-2}}{1 - 1.607(0.08)z^{-1} + 0.6086(0.08)z^{-2}}$$

where the numbers in the brackets show the typical norm bounds of the parameter perturbations. Then the model in converted into state-space model and considering the situation in input delay and disturbance as follow:

$$\begin{aligned} x_{k+1}(t+1) &= \left(\begin{bmatrix} 1.607 & 1 \\ -0.6086 & 0 \end{bmatrix} + \Delta A(t)\right) x_{k+1}(t) \\ &+ \left(\begin{bmatrix} 1.2390 \\ -0.9282 \end{bmatrix} + \Delta B(t)\right) u_{k+1}(t-\tau) \\ &+ \begin{bmatrix} 1 \\ 0 \end{bmatrix} d_{k+1}(t), \\ y_{k+1}(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_{k+1}(t), \end{aligned} \tag{58}$$

where $\Delta A(t)$ and $\Delta B(t)$ representing the uncertainty are given later in the section.

The input delay is $\tau = 4$, trail length is $\alpha = 200$, and the reference trajectory is taken as

$$r_d(t) = \begin{cases} 200, & 0 < t \le 100; \\ 200 + 5(t-100), & 100 < t \le 120; \\ 300, & 120 < t \le 200, \end{cases} \tag{59}$$

which is representative of the reference trajectories used in the injection molding process. Moreover, to protect the inject machine from conflict of the suddenly signal, the initial part of the trajectory is smoothed by a user-specified prefilter for practical implementation [13]. In this study, the filter is taken as $G_f(z) = (z^{-1} + z^{-2})/(3 - z^{-1})$. Note that the inclusion of this filter is application specific and is not required in other applications.

The new design in this paper is compared against a direct type ILC law, which has the structure

$$\begin{aligned} u_{k+1}(t) &= u_k(t) + K_1^d \delta\hat{x}_k(t+1) + K_2^d e_k(t+\tau) \\ &+ K_3^d[e_k(t+\tau+1) - e_k(t+\tau)]. \end{aligned} \tag{60}$$

This law also uses (7), and for clarity, the observer gain is denoted by L^d in all cases considered below.

Application of this last ILC law results in the controlled dynamics

$$\begin{aligned} \zeta_{k+1}(t+1) &= \hat{A}^d \zeta_{k+1}(t) + \hat{B}_1^d e_k(t-1) \\ &+ \hat{B}_0^d e_k(t) + \hat{B}_d \delta d_{k+1}(t) \\ e_{k+1}(t) &= \hat{C}^d \zeta_{k+1}(t) + \hat{D}_1^d e_k(t-1) \\ &+ \hat{D}_0^d e_k(t) + \hat{D}_d \delta d_{k+1}(t), \end{aligned} \tag{61}$$

where

$$\begin{aligned} \zeta_{k+1}(t) &= \begin{bmatrix} \delta\hat{x}_{k+1}(t-\tau) \\ \delta\hat{e}_{k+1}(t-\tau) \end{bmatrix}, \\ \hat{A}^d &= \begin{bmatrix} A + BK_1^d & L^dC \\ \Delta A + \Delta BK_1^d & A + \Delta A - L^dC \end{bmatrix}, \\ &= A_1 + B_1\widetilde{K}^d + \widetilde{L}^d + \Delta\widetilde{A}^d, \\ \widetilde{K}^d &= \begin{bmatrix} K_1^d & 0 \end{bmatrix}, \widetilde{L}^d = \begin{bmatrix} 0 & L^dC \\ 0 & -L^dC \end{bmatrix}, \\ \Delta\widetilde{A}^d &= D_0\Delta\left(F_1 + F_b\widetilde{K}^d\right), \hat{B}_1^d = \begin{bmatrix} BK^d \\ \Delta BK^d \end{bmatrix}, \\ \hat{B}_0^d &= \begin{bmatrix} BK_3^d \\ \Delta BK_3^d \end{bmatrix}, K^d = K_2^d - K_3^d, \\ \hat{C}^d &= -\widetilde{C}\hat{A}^d, \hat{D}_1^d = -\widetilde{C}\hat{B}_1^d, \hat{D}_0^d = I - \widetilde{C}\hat{B}_0^d. \end{aligned} \tag{62}$$

and the remaining matrices are defined in (13). Introduce the augmented vector $\vartheta_{k+1}(t) = \begin{bmatrix} \zeta_{k+1}(t)^T & e_k^T(t-1) \end{bmatrix}^T$. Then the last state space model can be written as

$$\begin{cases} \vartheta_{k+1}(t+1) = \mathbb{A}^d\vartheta_{k+1}(t) + \mathbb{B}^d e_k(t) + \mathbb{B}_d\delta d_{k+1}(t) \\ e_{k+1}(t) = \mathbb{C}^d\vartheta_{k+1}(t) + \mathbb{D}^d e_k(t) + \mathbb{D}_d\delta d_{k+1}(t), \end{cases} \tag{63}$$

where

$$\begin{aligned} &\mathbb{A}^d = \begin{bmatrix} \hat{A}^d & \hat{B}_1^d \\ 0 & 0 \end{bmatrix}, \mathbb{B}^d = \begin{bmatrix} \hat{B}_0^d \\ I \end{bmatrix}, \mathbb{B}_d = \begin{bmatrix} \hat{B}_d \\ 0 \end{bmatrix}, \\ &\mathbb{C}^d = \begin{bmatrix} \hat{C}^d & \hat{D}_1^d \end{bmatrix}, \mathbb{D}^d = \hat{D}_0^d, \mathbb{D}_d = \hat{D}_d. \end{aligned} \tag{64}$$

This last model is a discrete repetitive process and the system matrix is

$$\Xi^d = \begin{bmatrix} \hat{A}^d & \hat{B}_1^d & \hat{B}_0^d \\ 0 & 0 & I \\ \hat{C}^d & \hat{D}_1^d & \hat{D}_0^d \end{bmatrix}. \tag{65}$$

Hence the repetitive process-based results developed above can be used to design this law. Three designs are completed, denoted as Cases 1,2 and 3, respectively, and compared in this section, for the particular example given above. In each case, the direct-type ILC law is first designed, followed by the indirect-type, and then simulation results are given and discussed.

Case 1: — nominal dynamics, i.e., $\Delta A(t) = 0$, $\Delta B(t) = 0$ and also the disturbance $d_{k+1}(t)$ is zero. The controlled dynamics for the direct-type law, in this case, is governed by the LMI condition of Theorem 2, and hence this property holds if there exist compatibly dimensioned matrices $W_1 \succ 0, W_2 \succ 0, W_3 \succ 0$, and hence the block diagonal matrix $W = \text{diag}\begin{bmatrix} W_1 & W_2 & W_3 \end{bmatrix}$, and matrices G_1, G_2, Z, R^d such that the following LMI is feasible

$$\begin{bmatrix} \Theta_1 & \Theta_2^d \\ (*) & \Theta_1 \end{bmatrix} \prec 0, \tag{66}$$

where Θ_1 is defined in Theorem 2 and

$$\Theta_2^d = \begin{bmatrix} A_1W_1 + B_1Z + R^d & B_1G_1 & B_1G_2 \\ 0 & 0 & W_3 \\ -\widetilde{C}\left(A_1W_1 + B_1Z + R^d\right) & -\widetilde{C}B_1G_1 & W_3 - \widetilde{C}B_1G_2 \end{bmatrix},$$

$G_1 = K^dW_2, G_2 = K_3^dW_3, Z = \widetilde{K}^dW_1, R^d = \widetilde{L}^dW_1$.
Hence

$$\begin{aligned} &\begin{bmatrix} L^d \\ -L^d \end{bmatrix}\begin{bmatrix} 0 & C \end{bmatrix} = R^dW_1^{-1}, \begin{bmatrix} K_1^d & 0 \end{bmatrix} = ZW_1^{-1}, \\ &K^d = G_1W_2^{-1}, \ K_3^d = G_2W_3^{-1}, K_2^d = K^d + K_3^d. \end{aligned} \tag{67}$$

The matrices in the control law (60) in this case are

$$\begin{aligned} K_1^d &= \begin{bmatrix} -1.2286 & -0.7253 \end{bmatrix}, \ K_2^d = 0.5735, \\ K_3^d &= 0.4089, \ L^d = \begin{bmatrix} 1.4021 \\ -0.4633 \end{bmatrix}. \end{aligned}$$

For the indirect-type ILC law, applying (22) (the form of the result of Theorem 1 for nominal dynamics) and then Theorem 2 gives

$$\begin{aligned} K_1 &= 1.0637, K_2 = \begin{bmatrix} -0.0048 & -0.5203 \end{bmatrix}, \\ K_3 &= 0.7335, \ K_4 = 0.5930, \ L = \begin{bmatrix} 1.3762 \\ -0.9085 \end{bmatrix}. \end{aligned}$$

The tracking results for this example are given in Fig. 2, and the RMS plots against trial number are shown in Fig. 3. It is seen that the indirect-type ILC law achieves very close to perfect tracking of the reference trajectory after 3 trials. In contrast, the direct-type alternative needs 6 trials to achieve the same outcome. These results demonstrate that the new design can improve performance, and the following two case studies investigate performance in the presence of model uncertainty and repetitive (Case 2) and nonrepetitive disturbances (Case 3).

In this paper, the uncertainty is represented by (4), and in both cases below, the uncertainty is defined by

$$\begin{aligned} &D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, F_a = \begin{bmatrix} 0.08 & 0 \\ 0.08 & 0 \end{bmatrix}, F_b = \begin{bmatrix} 0.1 \\ 0.14 \end{bmatrix}, \\ &\Delta(t) = \begin{bmatrix} \kappa(t) & 0 \\ 0 & \kappa(t) \end{bmatrix}, \end{aligned}$$

where $\kappa(t)$ is a random variable taking values in $[-0.5, 0.5]$.

Case 2: — design and performance in the presence of time-varying process uncertainties and repetitive disturbances. In this case, the controlled dynamics for the direct-type law are governed by the LMI condition of Theorem 3. Suppose that there exist compatibly dimensioned matrices $W \succ 0$, and G_1, G_2, Z, R^d, and a positive scalar $\varepsilon_1^d \succ 0$ such that the following LMI is feasible

$$\begin{bmatrix} \Theta_1 & \Theta_2^d & \Theta_4^d \\ (*) & \Theta_1 & \Theta_5^d \\ (*) & (*) & \Theta_6^d \end{bmatrix} \prec 0, \tag{68}$$

where

$$\Theta_4^d = \begin{bmatrix} 0 & \varepsilon_1^d D_0 \\ 0 & 0 \\ 0 & -\varepsilon_1^d \widetilde{C} D_0 \end{bmatrix}, \ \Theta_5^d = \begin{bmatrix} (F_1W + F_bZ)^T & 0 \\ (F_bG_1)^T & 0 \\ (F_bG_2)^T & 0 \end{bmatrix},$$

$$\Theta_6^d = \begin{bmatrix} -\varepsilon_1^d I & 0 \\ (*) & -\varepsilon_1^d I \end{bmatrix}.$$

Then control law matrices in (60) for this case are given by

$$\begin{aligned} &\begin{bmatrix} L^d \\ -L^d \end{bmatrix}\begin{bmatrix} 0 & C \end{bmatrix} = R^dW_1^{-1}, \begin{bmatrix} K_1^d & 0 \end{bmatrix} = ZW_1^{-1}, \\ &K^d = G_1W_2^{-1}, \ K_3^d = G_2W_3^{-1}, K_2^d = K^d + K_3^d, \end{aligned} \tag{69}$$

and for the example considered

$$\begin{aligned} K_1^d &= \begin{bmatrix} -0.5892 & 0.0750 \end{bmatrix}, \ K_2^d = 0.4148, \\ K_3^d &= 0.3589, \ L^d = \begin{bmatrix} 0.0580 \\ 0.0506 \end{bmatrix}. \end{aligned}$$

Applying Theorem 1 (the LMI (14)) with the minimum parameter γ_c and Theorem 3 gives the following control law matrices for the new indirect-type design

$$\begin{aligned} K_1 &= 0.8710, \ K_2 = \begin{bmatrix} 0.2059 & -0.6992 \end{bmatrix}, \\ K_3 &= 0.2799, \ K_4 = 0.2665, \ L = \begin{bmatrix} 1.5065 \\ -0.5394 \end{bmatrix}, \\ \gamma_c &= 5.2622 \times 10^{-5}. \end{aligned}$$

The simulation results are shown in Fig. 4 - 7, where the repetitive disturbance is $d_{k+1}(t) = 10sin(t)$. Evidently, in the

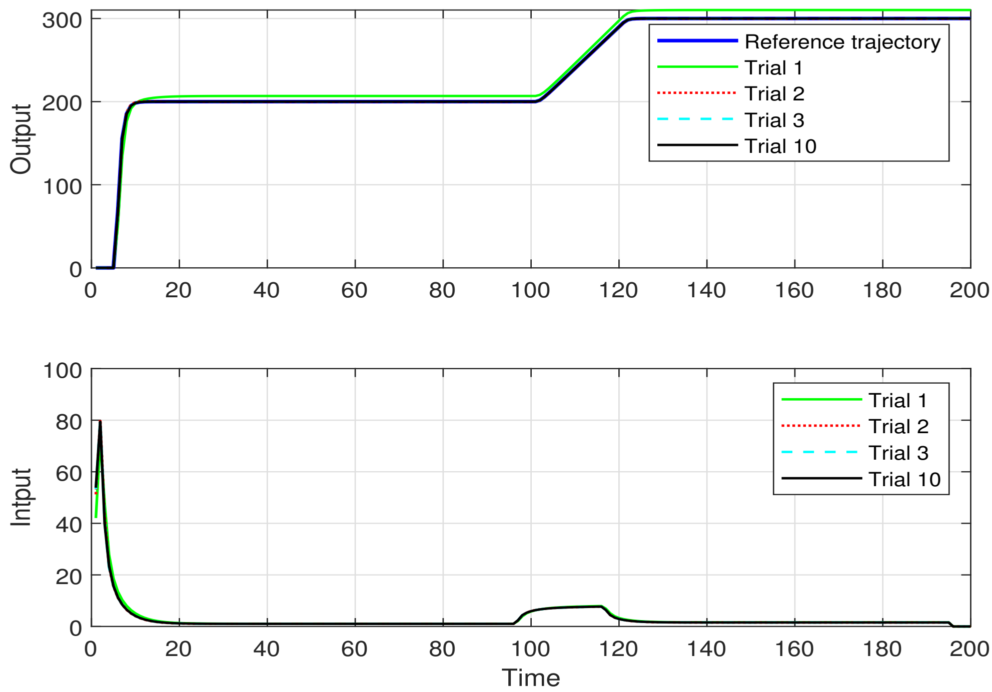

Figure 2: Output and input signals for the indirect-type nominal system for Case 1.

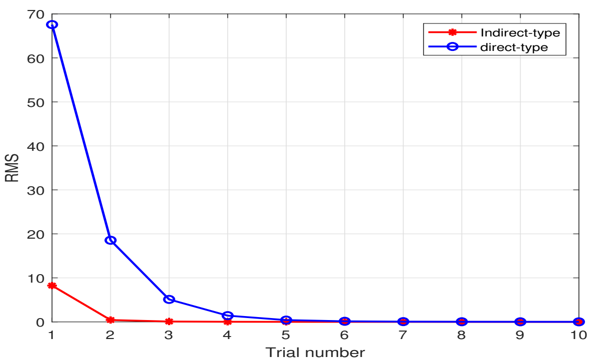

Figure 3: RMS of indirect and direct type designs against the trial number for Case 1.

presence of the time-varying uncertainties and repetitive disturbance, the output of the system tracks the reference trajectory asymptotically, i.e., the tracking error converges to a minimum value. Also, compared with direct-type ILC, the new design results in a faster tracking error convergence.

Case 3:

A procedure for design in the case of nonrepetitive distrubance is given in the following algorithm

Suppose that there exists matrices $W_1 > 0, W_2 > 0, W_3 > 0$, and matrices G_1, G_2, Z, R^d and positive scalar $\varepsilon_2^d > 0$ with the disturbance attenuation performance level γ^d, such that the

Algorithm 1 Indirect-type ILC with nonrepetitive disturbance systems

Input: : The nominal state-space model matrices and reference trajectory ($r_d(t)$), the trial length (α), the maximum number of trials considered (denoted by M), the nonrepetitive disturbance $d_{k+1}(t)$ and the time-varying uncertainty description (the matrices D, F_a, F_b, and Δ. Apply Theorems 1 and 4 to obtain the predictor and controllers parameters L, K_1, K_2, K_3, K_4 and set $y_{r,0} = 0$.

Output: $y_k(t), u_k(t)$ and RMS

for do $k = 0 : N$

for do $t = 0 : \alpha$

Calculate the set-point $y_{r,k}(t)$ by ILC law (23).

Apply $u_k(t)$ to the batch process (3) and predictor (7) and record the output trajectory $y_k(t)$ together with the set-point error $e_{r,k}(t)$ and tracking error $e_k(t)$.

Calculate the RMS for the current trial k using (57).

end for

end for

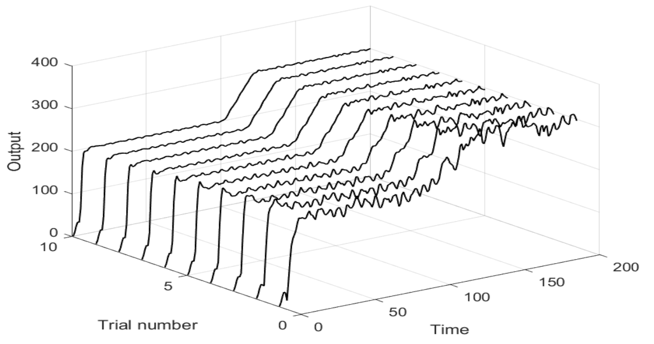

Figure 4: Outputs under the indirect-type ILC design for Case 2.

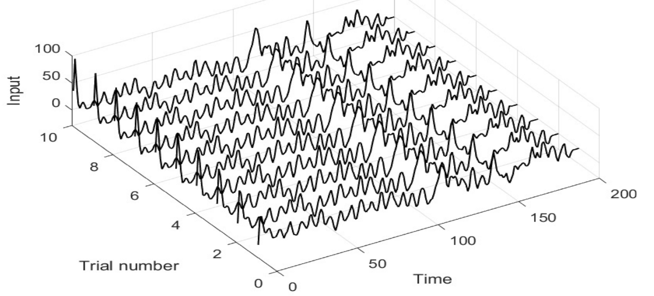

Figure 5: Inputs under the indirect-type ILC design for Case 2.

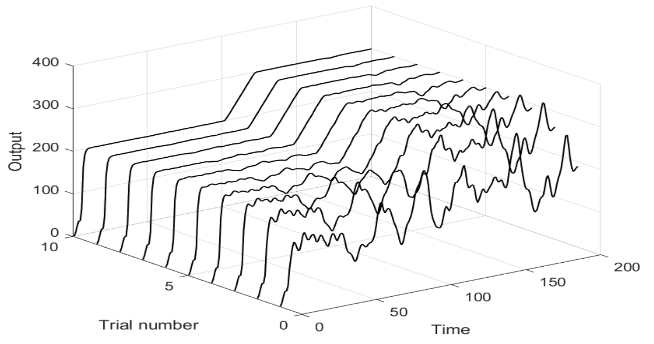

Figure 6: Outputs under the direct-type ILC design for Case 2.

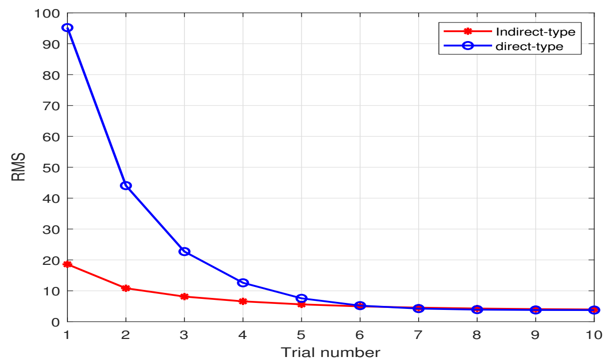

Figure 7: RMS of indirect and direct type control for Case 2.

following LMI is feasible

$$\begin{bmatrix} \Theta_1 & \Theta_2^d & \Theta_7^d & \Theta_{10}^d \\ (*) & \Theta_1 & \Theta_8^d & \Theta_{11}^d \\ (*) & (*) & \Theta_9^d & \Theta_{12}^d \\ (*) & (*) & (*) & \Theta_{13}^d \end{bmatrix} \prec 0, \tag{70}$$

where

$$O_7^d = \begin{bmatrix} \hat{B}_d & 0 \\ 0 & 0 \\ -\widetilde{C}\hat{B}_d & 0 \end{bmatrix}, O_8^d = \begin{bmatrix} 0 & \left(-\widetilde{C}A_1W_1 - \widetilde{C}B_1Z\right)^T \\ 0 & \left(-\widetilde{C}B_1G_1\right)^T \\ 0 & \left(W_3 - \widetilde{C}B_1G_2\right)^T \end{bmatrix},$$

$$O_9^d = \begin{bmatrix} -\rho^d & \left(-\widetilde{C}\hat{B}_d\right)^T \\ (*) & -I \end{bmatrix}, O_{10}^d = \begin{bmatrix} 0 & \varepsilon_2^d D_0 \\ 0 & 0 \\ 0 & -\varepsilon_2^d D_0 \end{bmatrix},$$

$$O_{11}^d = \begin{bmatrix} (A_1W_1 + B_1Z)^T & 0 \\ (F_bG_1)^T & 0 \\ (F_bG_2)^T & 0 \end{bmatrix}, O_{12}^d = \begin{bmatrix} 0 & 0 \\ 0 & -\varepsilon_2\widetilde{C}D_0 \end{bmatrix},$$

$$O_{13}^d = \begin{bmatrix} -\varepsilon_2 I & 0 \\ (*) & -\varepsilon_2 I \end{bmatrix}, \rho^d = (\gamma^d)^2 I.$$

Then the matrices in the control law (60) are

$$\begin{bmatrix} L^d \\ -L^d \end{bmatrix}\begin{bmatrix} 0 & C \end{bmatrix} = R^d W_1^{-1}, \begin{bmatrix} K_1^d & 0 \end{bmatrix} = ZW_1^{-1}, K^d = G_1W_2^{-1},$$
$$K_3^d = G_2W_3^{-1}, K_2^d = K^d + K_3^d. \tag{71}$$

For the example considered

$$\begin{aligned} K_1^d &= \begin{bmatrix} -1.3507 & -0.8744 \end{bmatrix}, K_2^d = 0.8200 \\ K_3^d &= 0.8019, \ L^d = \begin{bmatrix} 0.7617 \\ -0.6018 \end{bmatrix}, \\ \gamma^d &= 110.7080. \end{aligned}$$

Applying Theorem 1 (the LMI (14)) with the minimum parameter γ_c and Theorem 4 in this case gives

$$\begin{aligned} K_1 &= 0.8710, K_2 = \begin{bmatrix} 0.2059 & -0.6992 \end{bmatrix}, \\ K_3 &= 0.2548, \ K_4 = 0.2545, \ L = \begin{bmatrix} 1.5065 \\ -0.5394 \end{bmatrix}, \\ \gamma_c &= 5.2622 \times 10^{-5}, \gamma = 32.1822. \end{aligned}$$

Consider also the case when $d(k+1,t) = 10sin(t) + w(t)$, where $w(t)$ varies arbitrarily over $[-5, 5]$ (the disturbance is the sum of a repetitive term and a non-repetitive term)

The results for this case are given in Figs. 8– 14

Finally, to assess the influence of the time delay, this case was recomputed for a delay $\tau = 10$. The output sequence is shown in Fig 11 and the RMS plots for $\tau = 4$ and $\tau = 10$ are shown in Figs 13 and Fig. 14. These latter results indicate that the design can be applied for a range of time delays.

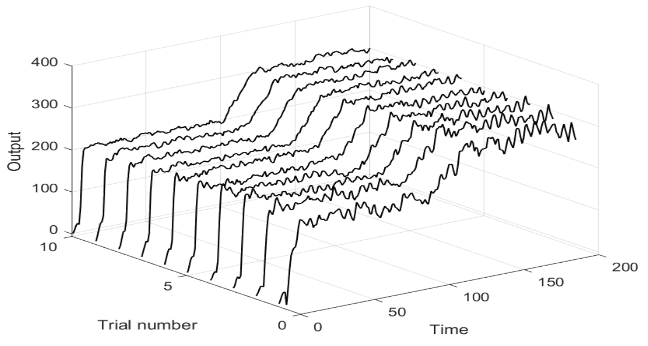

Figure 8: Outputs under the indirect-type ILC design for Case 3 with $\tau = 4$.

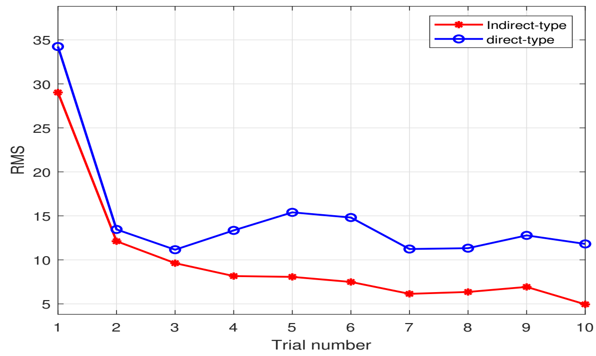

Figure 10: RMS of indirect and direct designs for Case 3 with $\tau = 4$.

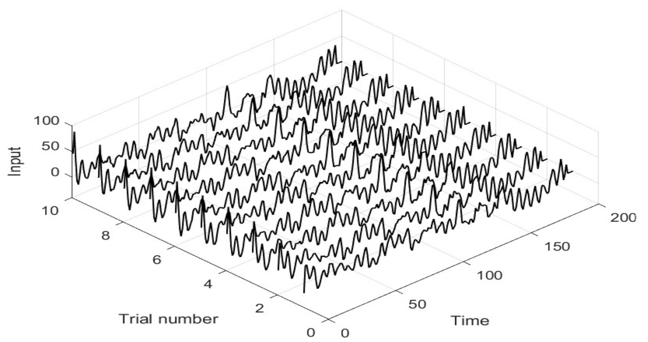

Figure 9: Inputs under the indirect-type ILC design for Case 3 with $\tau = 4$.

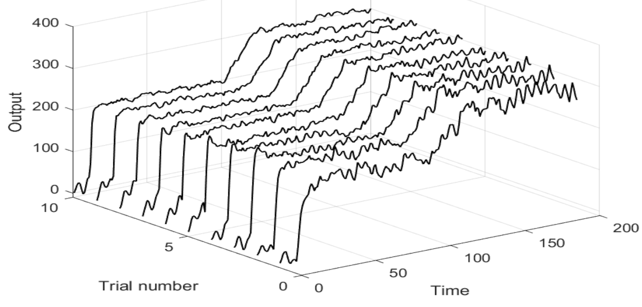

Figure 11: Outputs under the indirect-type ILC design for Case 3 with $\tau = 10$.

6. Conclusions and Future Research

This paper has addressed the problem of the design of an indirect ILC scheme for batch processes with time-varying uncertainties, input delay, and disturbances. The analysis covers three cases, where the latter two focus on repetitive and non-repetitive disturbances, respectively. In the last case, H_∞ disturbance attenuation is used as an ILC design alone can only compensate for the presence of a repetitive component in the disturbance. The designs are LMI based, and their performance has been highlighted by three case studies on a physically-based model.

The current design uses an observer to reconstruct state variables. One area for possible future research is to develop algorithms that do not require access to the state vector entries. Also, further research is possible to provide a range of results for different descriptions of the uncertainty.

CRediT authorship contribution statement

Hongfeng Tao: Conceptualization, Methodology, Supervision, Recourses, Writing - review & editing. **Junhao Zheng:** Conceptualization, Methodology, Writing - original - draft, Data curation, Software, Investigation, Visualization. **Junyu Wei:** Conceptualization, Methodology, Supervision, Recourses, Writing - review & editing. **Wojciech Paszke:** Supervision, Recourses, Writing - review & editing. **Eric Rogers:** Supervision, Recourses, Writing - review & editing. **Vladimir Stojanovic:** Supervision, Recourses, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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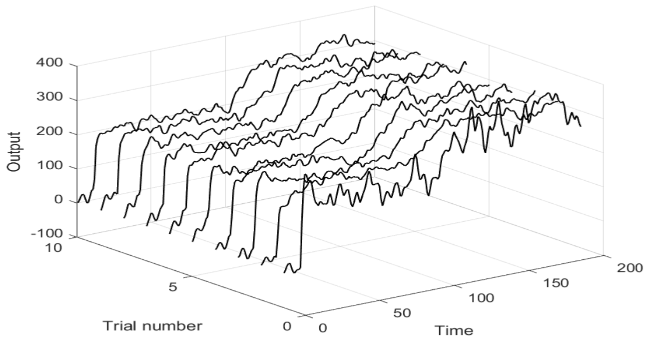

Figure 12: Outputs under the direct-type ILC design for Case 3 with $\tau = 10$.

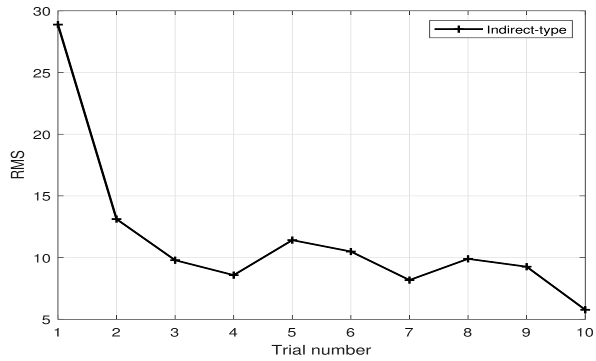

Figure 14: RMS of indirect design for Case 3 with $\tau = 10$.

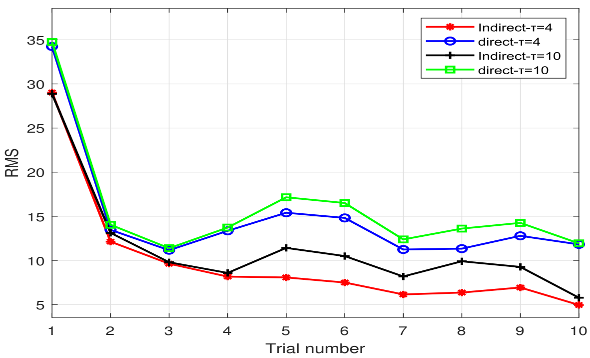

Figure 13: RMS of direct and indirect designs for Case 3 with $\tau = 4$ and $\tau = 10$.

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