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VARIABLE THERMAL CONDUCTIVITY AND MASS DIFFUSIVITY EFFECTS IN A FREE CONVECTIVE FLOW OF DOUBLY STRATIFIED NON-DARCIAN POROUS MEDIUM OVER A VERTICAL PLATE

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The research in this article is carried out to study incompressible and unsteady free convective flow on a semi-infinite isothermal vertical plate in a doubly stratified non-Darcian porous media with variable mass diffusivity and variable thermal conductivity. The governing non-linear partial differential equations of flow were calculated by applying an implicit finite difference scheme of the Crank-Nicolson type. Various parametric impacts on concentration profiles, temperature, velocity, as well as Sherwood number, Nusselt number, and skin friction, were examined and presented in graphs. It is examined that there exists a significant temperature decrease for high Darcy number in stratified fluids. Also, it is detected that the presence of stratification produces a considerable drop in skin friction while increasing the mass and heat transfer rate. Results from the present were compared to available solutions, and they matched up well.

Key words: mass diffusivity; non-Darcy; porous medium; stratification; thermal conductivity.

1. Introduction

Analysis of heat and mass transmission in porous media and thermal stratification is crucial in engineering and industrial settings, both theoretically and practically. The formation of layers in a fluid with thermal stratification results from temperature gradients. Refrigeration and air conditioning, geothermal reservoirs, petroleum industries, boundary layer controls, packed-bed catalytic reactors, building insulation, and heat exchange between the soil and atmosphere are all good examples of where thermal stratification porous mechanisms have been put to use. For past decades, combined buoyancy impacts in the free convective flow problem have received great interest. The problem has been analyzed by various authors using various techniques. This analysis on free convective mass and flow transfer effects in innumerable industrial and environmental applications such as in nuclear power plants, food processing, geophysical flows, chemical catalytic reactors, and polymer production, etc. Gebhart and Pera [1] used similarity solution to analyze the nonlinear problem of combined buoyancy effects induced by vertical natural convective flow.

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Numerical method of explicit finite difference scheme was used by Hellums and Churchill [2], Callahan and Marner [3], and Soundalgekar and Ganesan [4]. Later, in that line, several authors analyzed free convective fluid past vertical plate using various analytical and numerical techniques. In recent times, Abdul Kafoor Abdul Hakeem *et al.* [5] Investigated the radiation impacts on non-incompressible, free convective nanofluid flow by the method of Homotopy analysis employing a vertical plate.

Owing to the significant impact of convective free boundary layer flow in porous media, many authors widened the field and investigated the analysis of mass and heat transfer processes in porous medium. This is because of its diverse usage in industries and many ecological problems such as geothermal power plants, nuclear waste management, petroleum recovery, filtration processes, packed bed chemical reactors, etc. Non-Darcian inertia effects are explored in terms of their significance and the scope of their applicability in light of the various effects incorporated in [6-8]. Bejan and Nield [9], and Pop and Ingham [10] pioneered the studies in porous media intensively. Most of the initial studies on porous media were analyzed based on Darcy's law which exists for lower velocities and small porosity. Cheng and Minkowycz [11] pioneered free convective flow embedded in a porous media past the vertical plate which has been enlarged by many researchers like Raptis *et al.* [12], Lai and Kulacki [13], etc. later. As Darcy's law fell short in conditions where the flow has high velocity near the wall (inertial effects) and flow with boundary effects, researchers accounted for those effects by Forchheimer's extension and Brinkman's extension respectively in the later studies.

The effects of fluid flow such as micropolar fluid, nanofluid, second-grade fluid, power-law fluid, etc., in porous media on a vertical plate were also explored in innumerable researches. A.J. Chamkha *et al.* [14] inspected power-law non-Newtonian fluid flow in the non-Darcian porous media past a vertical plate. Farhad Ali *et al.* [15] observed the effect of II-grade fluid on an oscillating isothermal vertical plate in porous media. Similarly, it is equally fascinating to analyze the outcomes in a porous medium if the fluid is stratified. This is because stratification is another crucial aspect to be considered in mass and heat transfer analysis in porous media. Fluid stratification takes place due to the difference in fluid temperature, concentration., or density. The impact of thermal stratification in porous media and a detailed survey on the same can be found in Nield and Bejan [9]. Researches shows that the impacts of thermal stratification on the process of heat removal in porous media are substantial and a significant heat transfer has been observed, considering the thermally stratified effects in a porous medium. A comprehensive understanding of stratification is required in designing a nuclear reactor system, failing which leads to extreme crisis in reactors. In the case of pressurized water reactors, stratification of corium and metallic components may cause significant heat transfer effects. In the case of still water like ponds and lakes, examining the stratification of concentration and temperature differences of oxygen and hydrogen is of immense importance as this has an impact on the progress rate of every cultivated species. Also, the attainment of high energy effectiveness can be attained with improved stratification by solar engineers. In an earlier period, Chen and Eichhorn [16] explored a natural convection of thermally stratified medium over a heated vertical surface with the non-similarity method. In addition to this Srinivasan and Angirasa [17,18] made use of the finite difference technique to estimate this same type of problem.

Subsequently, due to immense applications of stratification in porous medium in various fields such as geophysical flows, problems of power production, etc., studying the convective boundary layer flow in stratified Darcian or non-Darcian porous media has received considerable interest. Hung and Chen [19] examined the convective free flow in thermally stratified porous media over an impermeable vertical plate by considering the non-Darcy effects. Magyari *et al.* [20] studied the 1-D glow on an infinite vertical plate embedded in thermal-porous stratified media for unsteady heat transfer convection. Where a considerable amount of work was also done on studying the impacts of thermal stratification mass and heat transfer in a porous medium, very few studies have been done on how mass and thermal stratification impact the mass and heat flow in porous mediums. Shalini Gupta and Rathish Kumar [21] studied the effect of mass and thermal stratification in a natural non-Darcy convection flow from a wavy vertical wall to porous media. Srinivasacharya and Surendar [22] examined the mixed convective along a vertical plate embedded in a doubly stratified porous media. Maria Naegu [23] worked on the analysis of free convective flow due to constant mass and heat fluxes over a wavy vertical wall in a doubly stratified non-Darcy porous medium.

However, the preceding studies were done with the assumption that physical properties have constant thermal conductivity and mass diffusion, but practical situations require properties with variable characteristics. Physical properties like thermal conductivity are prone to vary with temperature while mass diffusion coefficient varies with concentration. In the process of astonishing armor plate etc., it is essential to treat diffusivity as concentration-dependent because neglecting that dependency may not lead to the desired resultant product. Carl Wagner [24] investigated the problem of diffusion and studied that when the solid silver chloride is used as a solvent, due to the exchange of lead and silver ions this diffusion coefficient becomes proportional to the lead chloride's concentration. Willbanksrt [25] analyzed a 1D diffusion problem in a semi-infinite media with a linearly varying concentration in response to the diffusion coefficient.

In a similar behavior, the processes of refrigerating metallic plates in a cooling path and tinning of copper wires, etc. The temperature distributions are significantly dependent on the fluid properties. During such processes, as the material experiencing the treatment encounters a major change in its temperature, the change in its thermal conductivity may be significant. With this viewpoint, Ibrahim and Elbashbeshy [26] came up with a study on natural convection steady flow on a heated plate placed vertically by considering the thermal diffusivity and viscosity variations with changes in temperature values. Hassanien and Rashed [27] studied the mass diffusion, thermal conductivity, and variable viscosity, related to the non-Darcy-free convection on a cylinder placed horizontally in porous media. Natural convection through a non-isothermal vertical plate in a porous material saturated with a nanofluid is analyzed by Gorla and Chamkha [28]. Hamad *et al.* [29] examined mass and heat transfer effects within a porous plate by applying the thermal convection boundary condition and concentration-dependent diffusivity calculations while Hamad *et al.* [30] studied the mass and heat transfer effect, along boundary conditions of a porous plate with varying viscosities and variable thermal conductivities. Abdou [31] simulated the unsteady flow of the boundary layer when a stretch plate is in the porous media with thermal conductivity and viscosity dependent on temperature values. In a thermally stratified medium, the importance of oscillating mixed convective stratified fluid and heat transfer properties at various stages of a horizontal, non-conducting cylinder is addressed by Zia Ullah *et al.* [32]. Non-Newtonian fluid flow over an extensible Riga surface through a permeable medium is described by Yu-Ming Chu *et al.* [33]. Entropy generation, radiation, varying thermal conductivity, varying mass diffusivity, heat production, and convective circumstances are highlighted. The Cattaneo-Christov theory is used to analyze heat and mass flows.

In view of the significance of factors discussed above, besides examining the doubly stratified impacts in non-Darcy porous media, it is also interesting and realistic to consider the variable fluid properties like concentration and temperature depending on mass and thermal diffusivity respectively as they may play an important part in the mass and heat transfer process in doubly stratified porous media. Also, none of the former studies attempt to investigate the natural and unsteady convective flow on an isothermal vertical plate immersed in doubly stratified non-Darcian porous media having different thermal conductivity and variable mass diffusion. The analysis of the present work will aid in understanding the difficulties and complications in the problem of reactors and electrochemical processes. The effects of physical constraints on concentration, temperature, and velocity profiles with its skin friction values, Sherwood and Nusselt numbers were also analyzed and illustrated graphically.

2. Mathematical analysis

The problem of, laminar free convective, unsteady, viscous, and incompressible 2D flow on a semi-infinite vertical plate in a doubly stratified non-Darcian porous medium is taken for the present analysis. At the initial $t^{'}=0$ time, assume that the plate and the fluid were sustained at the same concentration and temperature. On $t^{'}>0$, time the C_w is the concentration and T_w is the temperature of the plate. Concentration and temperature in the ambient of the medium are considered to linearly increase with height while at $x=0$, $T_{\infty,0}$ and $C_{\infty,0}$ are considered respectively. Except for the body force terms, the thermal conductivity of the fluid, and diffusivity in concentration, all the properties of the fluid are unchanged. The thermal conductivities and concentration diffusivities of fluid were considered to linearly vary with fluid temperature and concentration respectively. A vertical *x*-axis is for values of the plate, and a *y*-axis is the normal to the plate, as depicted in Fig.1.

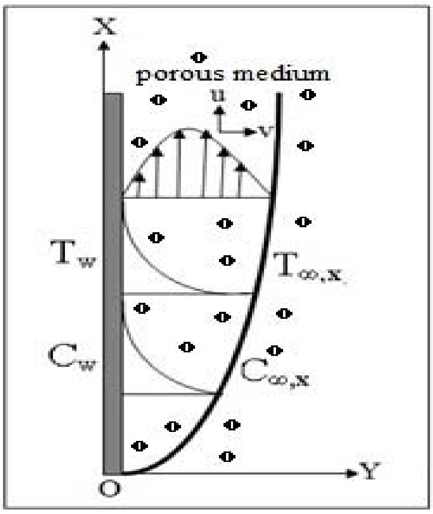

Fig.1. Physical model diagram.

With the assumption stated above, equations of the boundary layer that governs the flow, applying the Boussinesq's approximation (Herrmann Schlichting [34]) are given below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta_T(T'-T_{\infty,x})+g\beta_C(C'-C_{\infty,x})-\frac{\upsilon}{K_p}u-F_{ch}u^2, \tag{2.2}$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(k\frac{\partial T'}{\partial y}\right), \tag{2.3}$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=\frac{\partial}{\partial y}\left(D\frac{\partial C'}{\partial y}\right). \tag{2.4}$$

Initial and boundary conditions are as follows:

$$\begin{aligned}
&t'\le 0, \quad t'\le 0, \quad u=0, \quad T'=T_{\infty,x}, \quad C'=C_{\infty,x},\\
&t'>0, \quad t'\le 0, \quad v=0, \quad T'=T_w, \quad C'=C_w \quad \text{at} \quad y=0,\\
&t'\le 0, \quad u=0, \quad T'=T_{\infty,0}, \quad C'=C_{\infty,0} \quad \text{at} \quad x=0,\\
&u\to 0, \quad T'\to T_{\infty,x}, \quad C'\to C_{\infty,x} \quad \text{as} \quad y\to\infty.
\end{aligned} \tag{2.5}$$

The non-dimensional quantities are defined as follows:

$$X=\frac{x}{L};\quad Y=\frac{y}{L}Gr^{\frac{1}{4}};\quad U=\frac{uL}{\upsilon}Gr^{\frac{-1}{2}};\quad V=\frac{vL}{\upsilon}Gr^{\frac{-1}{4}};\quad T=\frac{\left(T'-T_{\infty,x}\right)}{\left(T_w-T_{\infty,0}\right)};\quad \Pr=\frac{\mu_0 c_p}{k_0};$$

$$C=\frac{\left(C'-C_{\infty,x}\right)}{\left(C_w-C_{\infty,0}\right)};\quad t=\frac{t'\upsilon}{L^2}Gr^{\frac{1}{2}};\quad Gr=\frac{g\beta L^3\left(T_w-T_{\infty,0}\right)}{\upsilon^2};\quad N=\frac{\left(T_w-T_{\infty,0}\right)}{\left(C_w-C_{\infty,0}\right)};\quad F=F_{ch}L;$$

$$S_T=\frac{\frac{dT_{\infty,x}}{dX}}{\left(T_w-T_{\infty,0}\right)};\quad S_M=\frac{\frac{dC_{\infty,x}}{dX}}{\left(C_w-C_{\infty,0}\right)};\quad \alpha=\frac{k_0}{\rho c_p};\quad Sc=\frac{\upsilon}{D_0};\quad Da=\frac{K_p Gr^{\frac{1}{2}}}{L^2}.$$

Here with the assumption that thermal conductivity varies linearly with temperature. Hence, these fluctuations in thermal conductivities in dimensionless temperature are written as (Elbashbeshy and Ibrahim [26])

$$k=k_0(1+\beta T), \tag{2.6}$$

where k_0 is the constant thermal conductivity of free stream fluid and β is the thermal conductivity parameter. Similarly, assume that the mass diffusivity varies linearly with concentration and hence the variation of concentration diffusivity in dimensionless concentration is written in the form (Hamad *et al.* [29]);

$$D=D_0(1+bC), \tag{2.7}$$

where D_0 is the constant concentration diffusivity of the free stream fluid and b is a parameter for mass or concentration diffusivity.
The dimensionless form of Eqs. (2.1) to (2.4) are attained as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.8}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+T+NC-\frac{1}{Da}U-FU^2, \tag{2.9}$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 T'}{\partial y^2}, \tag{2.10}$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2}-k_c'\left(C'-C_\infty\right). \tag{2.11}$$

Consequently, the boundary conditions given in Eq.(2.5) are reduced as follows,

$$\begin{aligned}&t\le 0,\quad U=0,\quad V=0,\quad T=0,\quad C=0,\\&t>0,\quad U=0,\quad V=0,\quad T=1-S_T X,\quad C=1-S_M X\quad \text{at}\quad Y=0,\end{aligned} \tag{2.12}$$

$$U = 0, \quad V = 0, \quad T = 0, \quad C = 0 \quad \text{at} \quad X = 0,$$
$$U \to 0, \quad T \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty. \tag{cont.2.12}$$

3. Numerical procedure

A Crank-Nicolson finite difference scheme with improved convergence speed and unconditional stability is applied for solving the non-linear, two-dimensional, coupled partial differential Eqs. (2.8)-(2.11) with boundary and initial conditions of Eq.(2.12).

An integral region is taken as a rectangle considering the values, $X_{\max} = 1$ and $Y_{\max} = 14$ sides, that correspond to it $Y = \infty$. After investigations, the $Y_{\max}$ was considered to be *14* and so boundary conditions of Eq.(2.12), two at the end are satisfied. In Fig.2, the *X*, *Y* and *t* directions, grid sizes are chosen as $\Delta t = 0.01$ $\Delta X = 0.05$, and $\Delta Y = 0.25$. Assuming the grid nodes *I* on *X*, *j* on *Y*, and *k* on *t* directions.

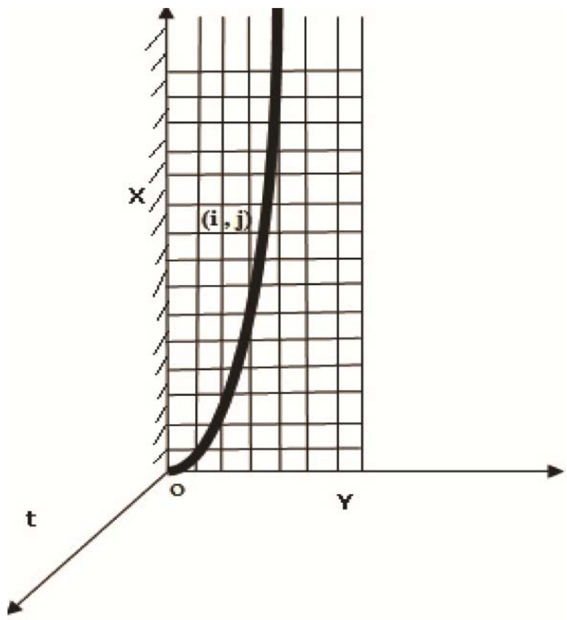

Fig.2. Discretization of domain.

All the *1*st and *2*nd order partial derivatives corresponding to spatial and time coordinates mentioned in Eqs. (2.8)-(2.11) are approximated as follows.

Using the finite difference formula, the *1*st-order derivatives w.r.t time are approximated by:

$$\frac{\left(\psi_{i,j}^{k+1} - \psi_{i,j}^{k}\right)}{\Delta t}. \tag{2.13}$$

With ψ signifying *U*, *V*, *T*, and *C* while *i* and *j* refer to the space coordinates, and *k* signifies time. The central finite formula is used to approximate the *1*st order derivatives with respect to *Y* as:

$$\frac{\left(\psi_{i,j-1}^{k+1} - \psi_{i,j+1}^{k+1} + \psi_{i,j-1}^{k} - \psi_{i,j+1}^{k}\right)}{4\Delta Y}. \tag{2.14}$$

The central finite formula is used to compute the 2^{nd}-order derivatives with respect to X and Y;

$$\frac{\left(\psi_{i,j-1}^{k+1}-2\psi_{i,j}^{k+1}+\psi_{i,j+1}^{k+1}+\psi_{i,j-1}^{k}-2\psi_{i,j}^{k}+\psi_{i,j+1}^{k}\right)}{2\Delta X^{2}}, \tag{2.15}$$

$$\frac{\left(\psi_{i,j-1}^{k+1}-2\psi_{i,j}^{k+1}+\psi_{i,j+1}^{k+1}+\psi_{i,j-1}^{k}-2\psi_{i,j}^{k}+\psi_{i,j+1}^{k}\right)}{2\Delta Y^{2}}, \tag{2.16}$$

respectively. The transformed difference Eqs. (2.8)-(2.11) obtained by finite difference method. There are tridiagonal systems on all the internal nodal points, for *i*-level particularly, the above difference equations were calculated with the Thomas algorithm same as that by Carnahan *et al.* [35]. The computations of C, T, U, and V were carried out on $(k+1)$ time intervals, this process repeats till a steady state is achieved at every nodal point. In this repeated process the difference of computation values (U, T, C) on two consecutive time levels lower than 10^{-5} for each grid is assumed to be steady state.

This scheme is considered to be stable unconditionally as shown by Soundalgekar and Ganesan [4] by using the Von-Neumann technique. As ΔX, ΔY and Δt decreases to zero; $O(\Delta t^{2}+\Delta X+\Delta Y^{2})$ the local truncation error tends to zero and ensures the compatibility of this system. The convergence of this scheme is ensured by proving the compatibility and stability of this scheme.

4. Skin friction, Sherwood and Nusselt number

Following is the dimensionless comparison of average and local values for Sherwood Number, Nusselt Number, and skin friction. Thus, the obtained non-dimensional values for the numbers:

$$\tau_X = Gr^{\frac{3}{4}}\left.\frac{\partial U}{\partial Y}\right|_{Y=0}, \tag{2.17}$$

$$Nu_X = -(1+\beta)Gr^{\frac{1}{4}}\frac{X\left.\frac{\partial T}{\partial Y}\right|_{Y=0}}{1-S_T X}, \tag{2.18}$$

sub-X $$Sh_X = -(1+b)Gr^{\frac{1}{4}}X\frac{\left.\frac{\partial C}{\partial Y}\right|_{Y=0}}{(1-S_M X)}. \tag{2.19}$$

Its non-dimensional form is estimated by:

$$\bar{\tau}_X = Gr^{\frac{3}{4}}\int_0^1\left.\frac{\partial U}{\partial Y}\right|_{Y=0}dX, \tag{2.20}$$

$$\overline{Nu}_X = -(1+\beta)Gr^{\frac{1}{4}}\int_0^1\frac{\left.\frac{\partial T}{\partial Y}\right|_{Y=0}}{(1-S_T X)}dX, \tag{2.21}$$

$$\overline{Sh}_X = -(1+b)Gr^{\frac{1}{4}} \int_0^1 \frac{\left.\dfrac{\partial C}{\partial Y}\right|_{Y=0}}{(1-S_M X)} dX . \tag{2.22}$$

In Eqs. (2.17)-(2.22) the values are calculated by applying Newton-Cotes closed integration formulas and derivatives using a *5*-pt. approximation formula

5. Results and discussion

To acquire an explicit vision of the problem, the numerical observations are presented with graphical illustrations. Also, the influence of the variable β thermal conductivity parameter is analyzed for air in the current investigation. Since β is considered between 0 and 6 for air (Elbashbeshy and Ibrahim [26]), the current analysis also examined for $\beta = 0,3,5$ and *6*. The variable concentration diffusivity parameter is inspected for $b = 0, 0.2, 0.4, 0.5$ and *0.6*. The influence of the inertial effect is examined for $F = 0.5, 1, 1.25$ and *2* and permeability for $Da = 0.5, 1$ and *2*. Mass and Thermal stratification for the range, $0 \le S_T \le 1$, respectively (Shalini Gupta and Rathish Kumar [21]), the study has been conducted analyzing the impacts of mass and thermal stratification. It was chosen for air with the Prandtl number *0.73*, while the Schmidt number is *0.2* (hydrogen). This Schmidt number value exhibits a physically buoyant gas diffusion in the boundary layer convective flow at $250°C$ and one atmospheric pressure. In order to observe physical parameters in action, the temperature, concentration, and velocity of fluid flows are represented graphically.

The particular solution of the current results is related to the available findings in the literature, to ascertain the precision of the result. Treating the fluid properties are constant where the doubly stratified porous medium is not present, on $N = 0.1$, $Sc = 0.94$, $\Pr = 0.7$, calculated velocity profiles were also compared to its available solution of Pera and Gebhart [1]. This comparison is illustrated in Fig.3. For $SM = 0$, $ST = 0.004$, $N = 2$, $Sc = 0.7$ and $\Pr = 0.7$, under the same conditions of constant fluid properties when the non-Darcy porous medium is not used, resulting concentration and temperature profiles are compared to those from Angirasa and Srinivasan [18], as shown in Fig.4. The current analysis's findings agree with the findings of earlier conducted methods as depicted in Figs 3 and 4.

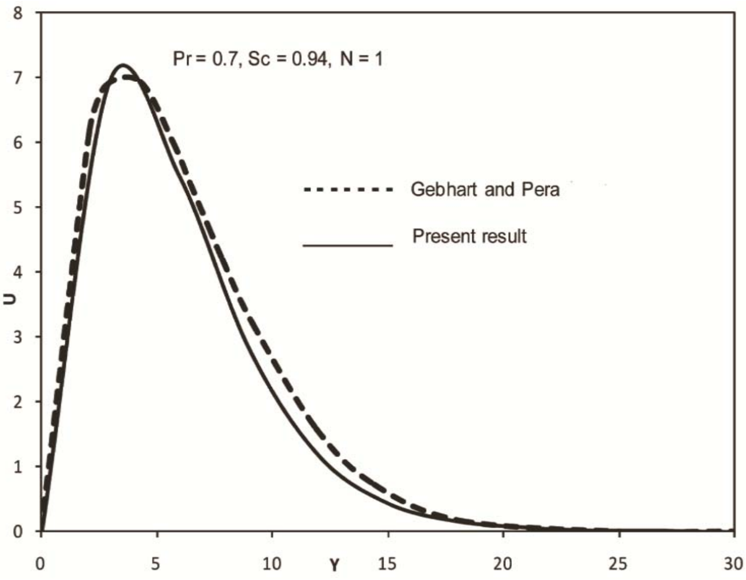

Fig.3. Velocity profiles comparison with Gebhart and Pera [1].

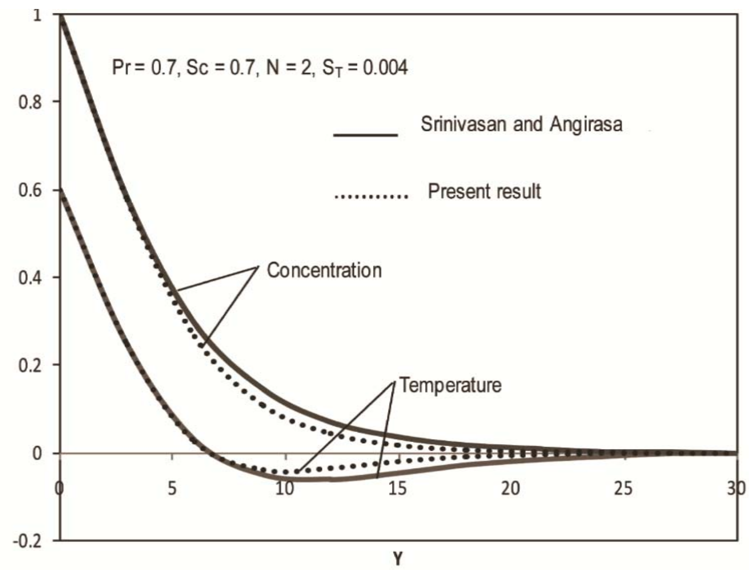

Fig.4. A comparison of Srinivasan and Angirasa's temperature and concentration profiles [18].

Figures 5 and 6 present the temperature and velocity profiles that were changed by S_T, the thermal stratification parameter. Temperature and velocity profiles descend as the temperature stratification parameter is increased. Moreover, subsequent to a certain increment in S_T, it is noticed that the dimensionless temperature attains negative values. The negative value attainment is because of an increase in thermal stratification inducing an increase in ambient temperatures with fixed height on a vertical location, the fluid present under is cooler as compared to the ambient temperature. Consequently, there is a downstream temperature defect and with S_T on the rise, this defect also increases. These characteristics have also been noted and expressed by Yang *et al.* [36] earlier. The decrease in temperature suppresses the buoyant force and leads to a deceleration in the velocity. After a certain state, the flow reverses due to a large temperature gradient after the fall in flow persists for some distance from the plate towards the free stream (Fig.7) portrays that increasing levels of S_T further raises the concentration. As a consequence of the decrement in temperature and velocity, its concentration level increases.

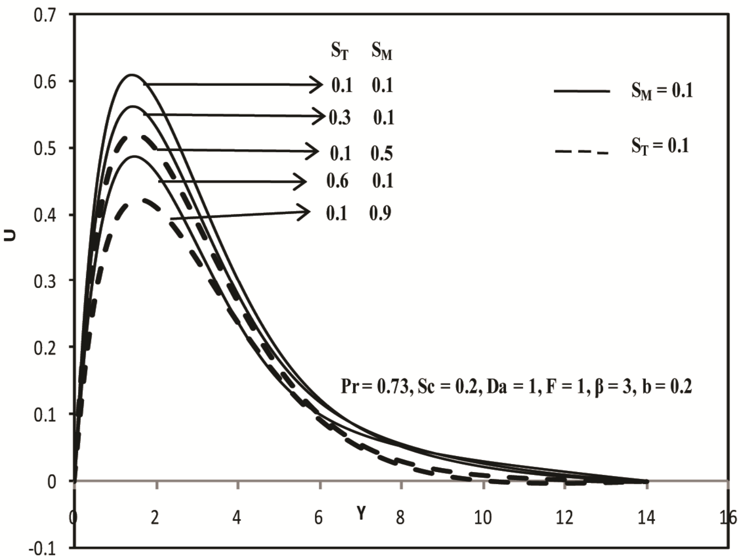

Fig.5. Influence of S_T and S_M on velocity profiles.

Figures 5 and 7 sketch an increment in S_M (mass stratification parameter) that falls down the profiles of concentration and velocity. The rise in mass stratification linearly raises the ambient conc. with height, similar to the impact of thermal stratification. When a reduction in concentration gradient is considered as an increasing buoyancy flow, the decreased buoyancy reduces the concentration gradient and also lowers its velocity. Furthermore, wall concentration decreases compared to ambient for a high mass stratification, and thus a negative value is arrived. From Fig.6, it is apparent that the rise in S_M escalates the temperature profiles. Thus, an increment in S_M, falls down the cooler region, with negative temperatures, until the position of the cold region vanishes.

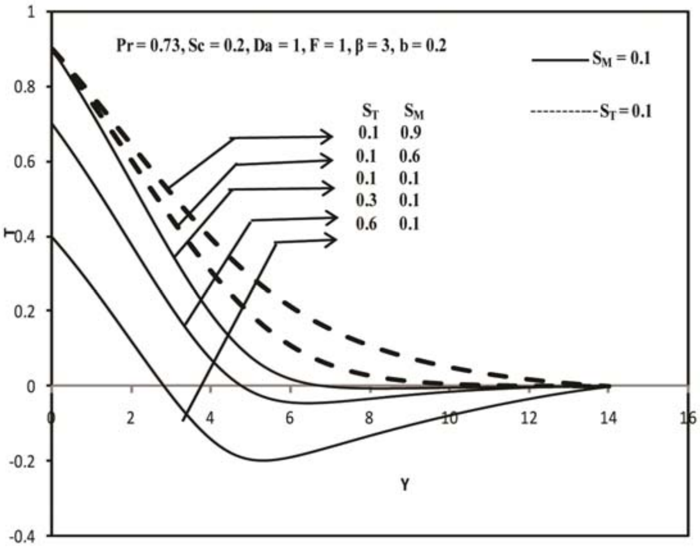

Fig.6. Influence of S_T and S_M on temperature.

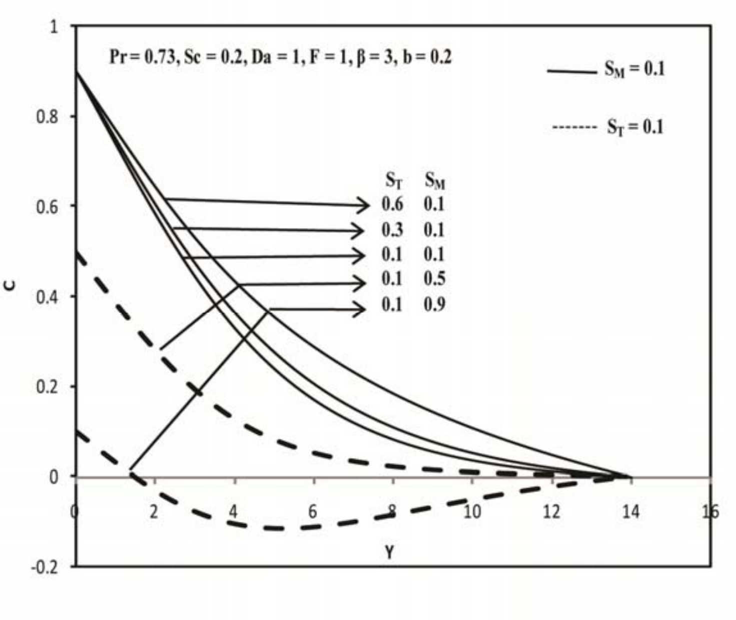

Fig.7. Influence of S_T and S_M on concentration.

Figure 8 depicts that the velocity of the flow accelerates for high Darcy number *Da*. Physically, for large Darcy numbers, the permeability increases, and hence the porosity on the medium is augmented. Thus, a rise in the fluid flow is observed. Due to this the fluid with a temperature less than the ambient comes up rapidly from below which increases the temperature gradient. Therefore, the temperature descends. Also, it is examined from Fig.9, that the temperature defect increases for higher Darcy numbers in stratified fluids. In a similar behavior, the species concentration declines for a high Darcy number as observed from Fig.10.

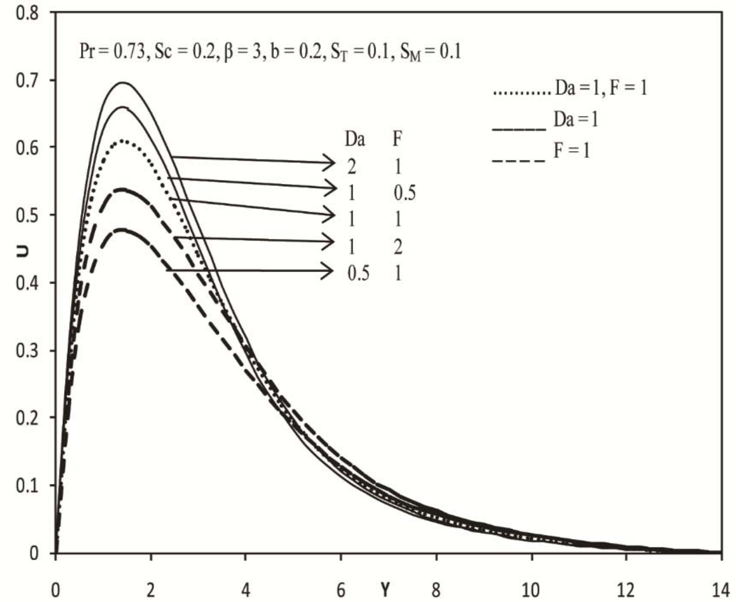

Fig.8. Darcy and inertial effects on velocity.

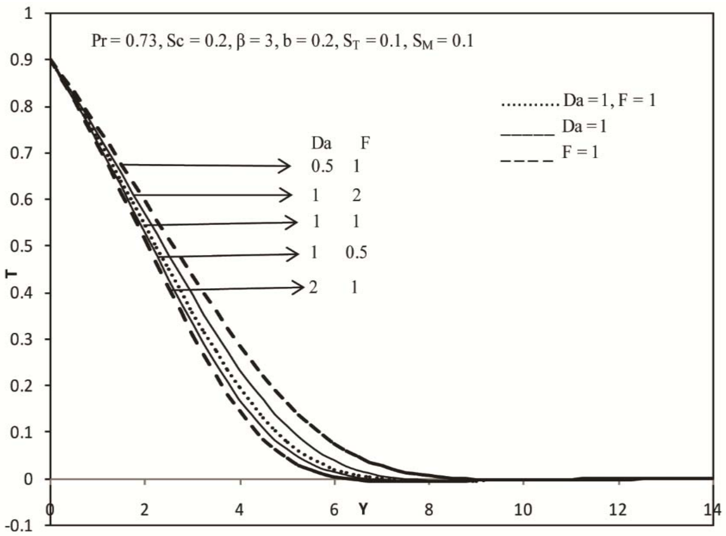

Fig.9. Darcy and inertial effects on temperature.

Figures 8 and 9 exhibit that an increment in Forchheimer number *F* decelerates the velocity and enhances the temperature. Physically, an increment in inertial effect in porous media represents that more resistance is generated to fluid motion which creates a drag and thereby impedes the flow velocity. This effect is observed when it is closer to the wall or when it is reversed away from the wall. As a consequence of the reduction in velocity the thermal boundary layer thickens. Figures 10 shows that higher Forchheimer drag decreases the concentration.

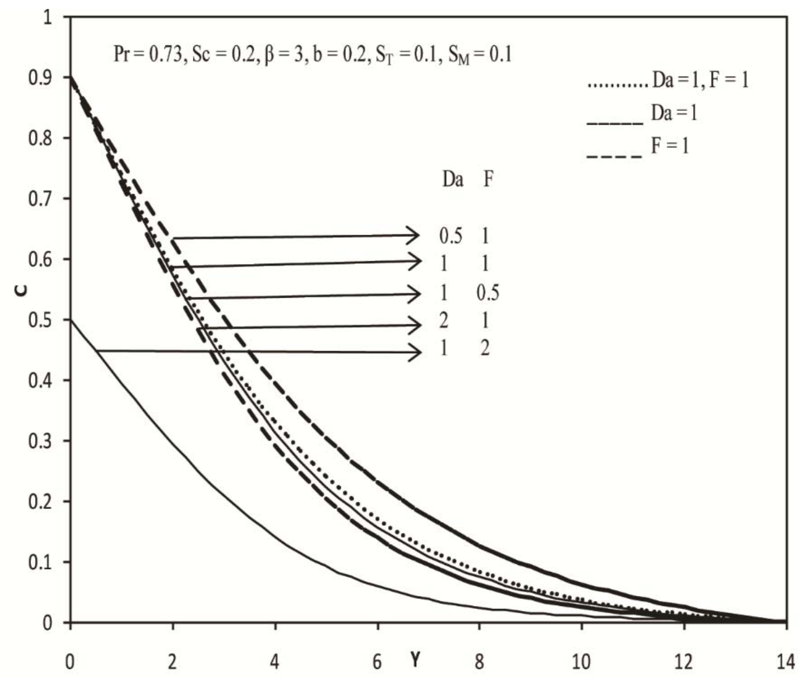

Fig.10. Darcy and inertial effects on concentration.

Figures 11 and 12 display temperature and velocity elevations along with β, the thermal conductivity parameter. It is evident that the fluid's thermal conductivity is enhanced with the rise in thermal conductivity parameter. This in turn increases the fluid temperature as well as the velocity. However, after a certain distance, the velocity of the fluid flow takes a reverse act. Also, Fig.13 exhibits retardation in species concentration as a parameter of thermal conductivity β rises. It is prominent from the figures that considerable error is encountered by ignoring the effect of variable thermal conductivity.

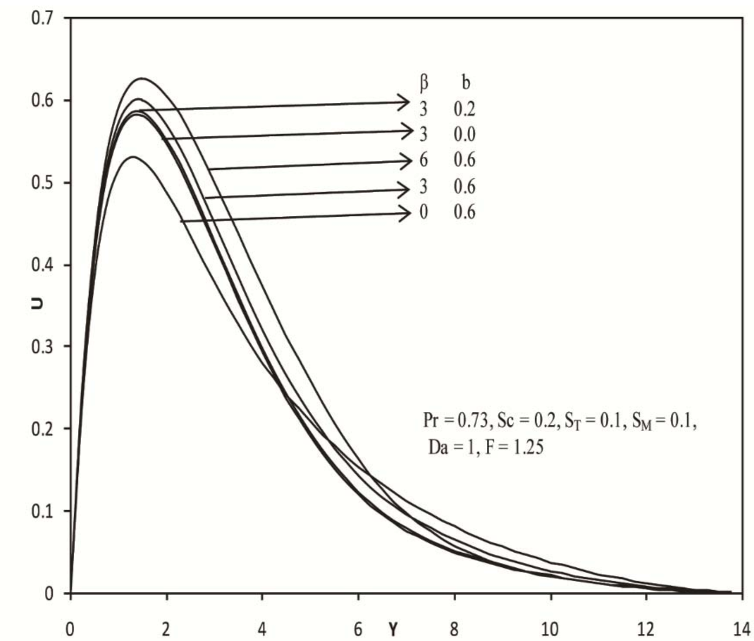

Fig.11. Impact of β and *b* on velocity profiles.

Figure 11 represents that an increment in variable mass diffusivity parameter *b* drops the velocity initially and then rises. It is noted from Figs 12 and 13 that, decrement in mass diffusivity parameter *b* results in decreasing temperature and enhances concentration profiles. Further, the velocity and concentration descend significantly when there is no mass diffusivity parameter as per the observation. Consequently, it is essential to consider the fluid properties as variable as it may lead to inaccurate prediction in the study of flow behavior.

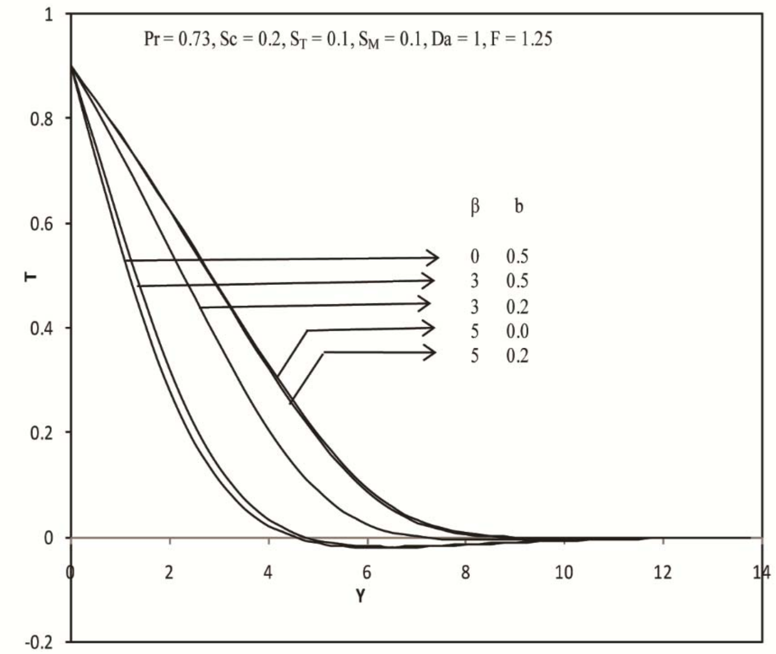

Fig.12. Impact of β and *b* on temperature.

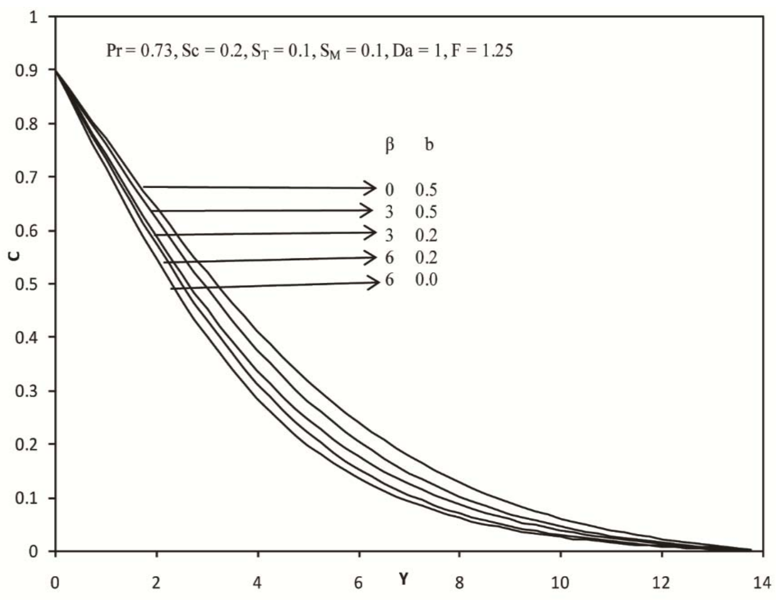

Fig.13. Impact of β and *b* on concentration.

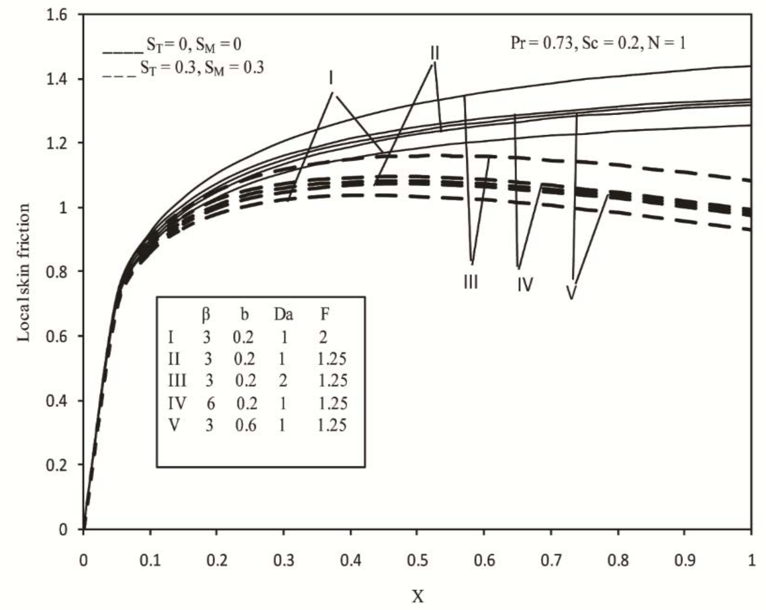

Fig.14. Local skin friction for variable *Da*, *F*, β, and *b*.

Figure 14 and 15 depict the impact on wall shear stress of the parameters in both stratified and unstratified flow. It is discovered that an increment in either β or *b* or *Da* increases the local and average skin friction while a decrement is noted for an increment in Forchheimer number *F*. For a high Darcy number, a rise in permeability enhances the velocity of the flow which gives rise to shear stress along the wall. On the contrary, an enhancement in the Forchheimer number develops the drag effect which in turn impedes the flow and hence inhibits the shear stress between the plate and fluid. Besides, it is detected that the presence of stratification produces a considerable drop in skin friction.

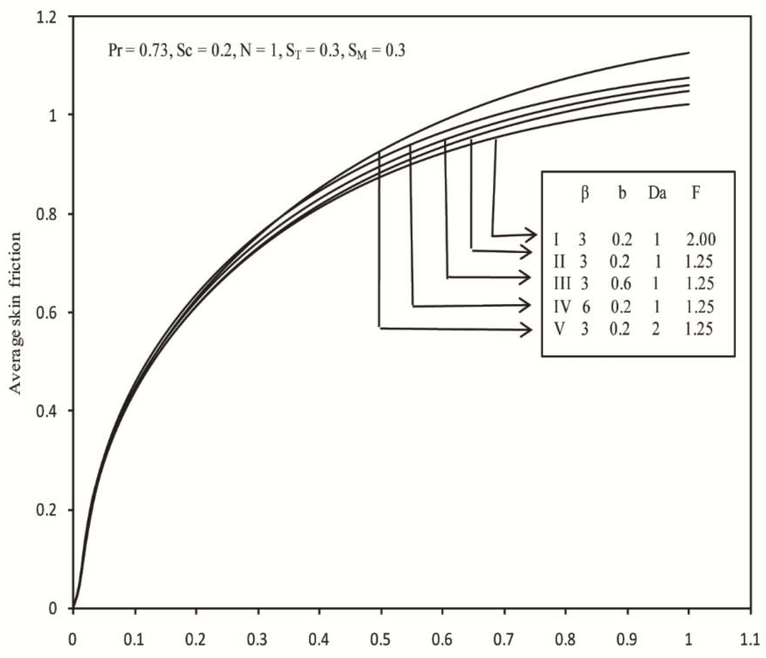

Fig.15. For different *Da*, *F*, β, and *b* the average skin friction.

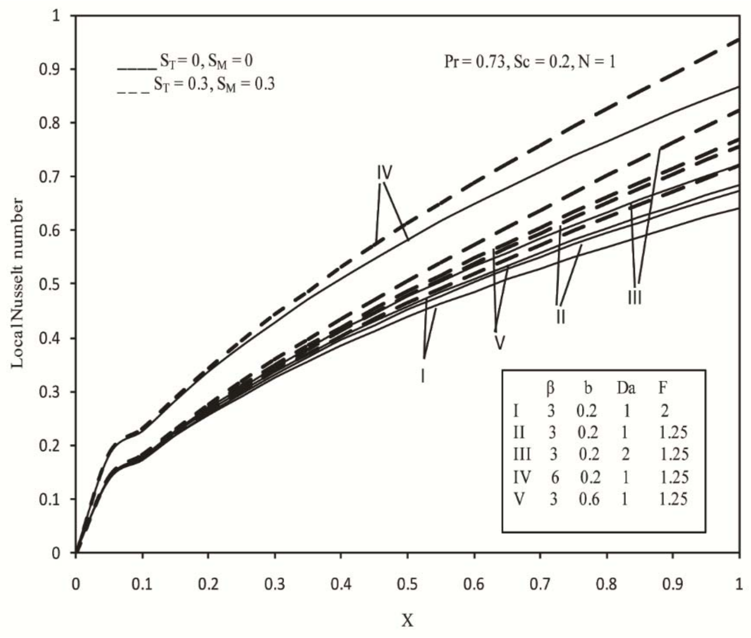

Fig.16. Local Nusselt number for different *Da*, *F*, β, and *b*.

Figure 16 illustrates that a lift up in either variable mass diffusivity parameter or Darcy number promotes the level of heat transfer. And is evident from Fig.9, that for a high Darcy number the permeability of the medium rises leading to a large temperature gradient and thereby increasing the Nusselt number. Also, when there is an enhancement in *F* lowers the Nusselt number. Physically, this is apparent as a rise in the Forchheimer drag increases the temperature and thereby reduces the temperature gradient. Hence, a reduction in the rate of heat transfers.

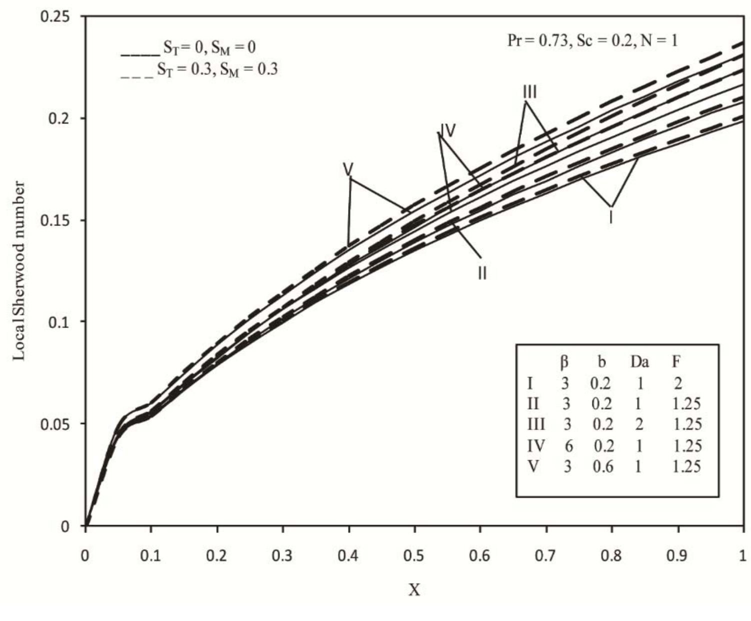

Fig.17. Local Sherwood number for different *Da*, *F*, β, and *b*.

Fig.17, shows that the local mass transfer rate increases for the rise in either variable mass diffusivity parameter thermal conductivity parameter, or Darcy number while decreases for the Forchheimer number. However, an increment in the mass diffusivity parameter or thermal conductivity parameter increases both average and local mass and heat transfer rate significantly as noted from Figs. 16-19. The onset of this significant increment is from the initial regime of the flow nearer to the wall. Further, it is noted that in the absence of stratification, a decrease in local Sherwood and Nusselt number is observed.

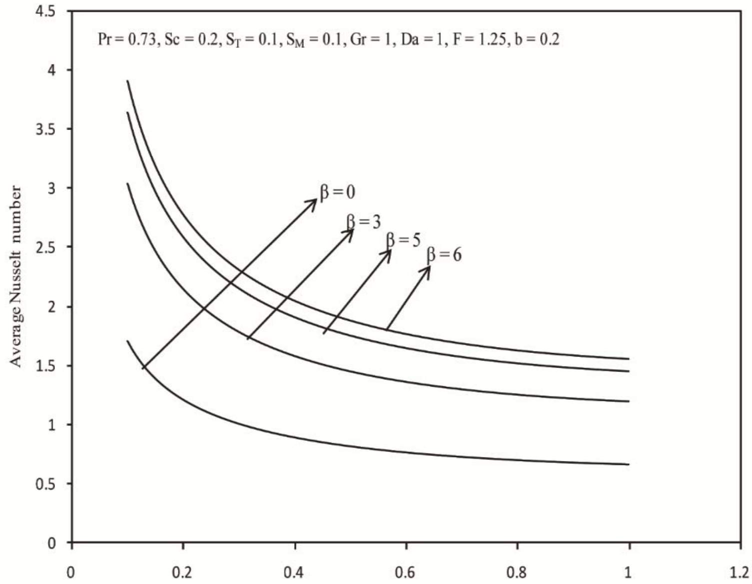

Fig.18. Effect of varying thermal conductivities on average Nusselt number.

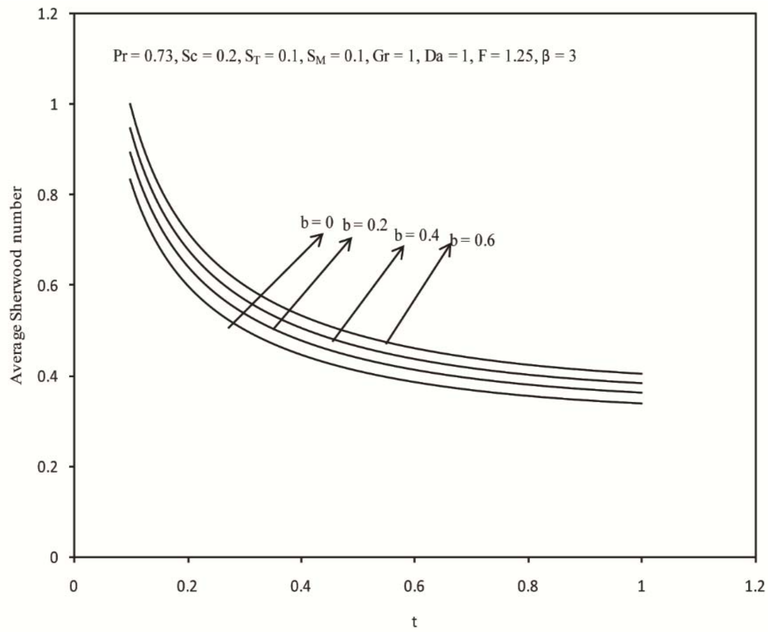

Fig.19. Effect of different mass diffusivities on average Sherwood number.

6. Conclusion

This analysis investigates an incompressible, transient, and free convective flow on a vertical plate in a non-Darcian doubly stratified media having varying mass diffusivity and changing thermal conductivity. The governing eqns. were calculated by using the Crank-Nicolson type finite difference scheme. Acquired results were compared with published results which were referred to in the literature. The impact of the physical parameter on average and local skin friction, Sherwood, and Nusselt numbers were discussed in the analysis. We validated the particular solutions presented in this paper by conducting a comparison with the methods presented in the paper published previously in the field. In comparison with the literature, our study results were in good agreement. We summarized our findings as follows:

1. Forchheimer number, or mass stratification parameter, or high thermal stratification parameter drops the velocity.
2. Thermal stratification lowers temperature and enhances concentration profiles, whereas mass stratification parameters and Forchheimer numbers have the opposite effect.
3. Due to higher values of S_T, the temperature defect arises and increases. The temperature defect increases for a higher Darcy number in stratified fluids.
4. For a high Darcy number, velocity accelerates while temperature and concentration decelerate.
5. The velocity escalates as the rise in thermal conductivity parameter, temperature, and retardation noted in species concentration whereas the reverse effect is observed for mass diffusivity parameter
6. Non-Darcian effect on wall shear stress is low in stratified fluid while a reverse is noted on the rate of heat and mass transmission.
7. An increment in the thermal conductivity or mass diffusivity parameter increases both the average and local heat transmission rate significantly.

Future research will address the local thermal equilibrium of the porous media with the nanofluid in more detail.

Nomenclature

b – parameter of mass diffusivity variable
C' – fluid concentration
C – dimensionless fluid concentration
D – coefficient of mass diffusion
D_0 – constant mass diffusivity of the free stream fluid
Da – Darcy number
F – dimensionless Forchheimer coefficient (or) number
F_{ch} – Forchheimer coefficient
Gr – thermal Grashof number
k – thermal conductivity
k_0 – constant thermal conductivity for free stream fluid
K_p – porous media permeability
N – parameter of buoyancy ratio
Nu_X – local Nusselt number
$\overline{Nu}_X$ – average Nusselt number
Pr – Prandtl number
Sc – Schmidt number
Sh_X – local Sherwood number
$\overline{Sh}_X$ – average Sherwood number

S_T – thermal stratification parameter

S_M – mass stratification parameter

t' – time

t – dimensionless time

T' – temperature of fluid

T – temperature of dimensionless fluid

U – dimensionless velocity component along the X-direction

u – velocity component along the plate

v – velocity component normal to the plate

V – dimensionless velocity component along Y-direction

x – spatial coordinate along the plate

X – dimensionless spatial coordinate along the plate

y – spatial coordinate normal to the plate

Y – dimensionless spatial coordinate normal to the plate

Greek symbols

α – thermal diffusivity

β – variable thermal conductivity parameter

β_C – volumetric coefficient of expansion with concentration

β_T – volumetric thermal expansion coefficient

ν – kinematic viscosity

ρ – density

$\overline{\tau_X}$ – average skin friction

τ_X – local skin friction

Subscripts

∞ – free stream conditions

w – wall conditions

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