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ANALYSIS OF LIQUID SLOSHING FREQUENCIES IN A PARTIALLY FILLED 3D RECTANGULAR TANK

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This research investigates liquid sloshing in a 3D rigid rectangular tank. The impact of rigid baffle on sloshing frequencies has been studied. The mathematical model has been developed using potential theroy. The boundary value problem has an analytical solution in terms of velocity potential with undetermined frequency. We get a system of homogeneous algebraic equations using boundary and free surface conditions. The frequencies are calculated using the non-trivial solution condition. Frequencies of baffled tank are computed for various filling levels. The effects of filling level on frequencies are identified. ANSYS software is used to report the liquid domain and rigid baffle mode forms.

Key words: frequency; rectangular tank; rigid baffle; sloshing mitigation.

1. Introduction

Liquid sloshing is a phenomena that happens when a liquid-free surface moves within a liquid tank that is partially filled with any kind of liquid. When a liquid inside partially filled tank behaves in motion and collaborates with some tank wall, pressure due to this collaboration may affect the solidity of the tank and the supporting structure. It has significant consequences on the stability of the vehicle carrying liquid tanks. It causes undesired hydrodynamic forces, which can be hazardous to ship, truck, rocket, and satellite structural stability and mobility. The problem of sloshing leaves a burden on various industries like the aerospace industry, offshore industry, civil, mechanical, nuclear engineers, physicists, and mathematicians. Sloshing can cause catastrophic harm to water and oil storage tanks. To control the effect of sloshing on storage tanks, the researchers use different types of baffles and movable devices in the tanks. The use of baffles on the free surface has the advantage of shifting the fundamental frequency to a higher value, which minimizes the amount of sloshing mass involved in the structure's dynamic response. The effects of liquid vibrations on circular cylindrical storage tanks have been the subject of extensive investigation, but only a small number of researchers have examined the harm caused by sloshing to cylindrical tanks. Gavrilyuk *et al.* concentrated on finding the basic answers to the liquid vibrations problem in a vertical cylindrical tank with a damping device. They created a numerical method that accurately depicts the analytical characteristics of the velocity potential near the baffle edge [1]. Abadi and Ansari found the solution to nonlinear sloshing problems using the modal method [2]. They came to the conclusion that raising the upper phase density lowers oscillation amplitude. The tank-baffle system, which is built of lightweight materials, was subjected to coupled analysis using the finite element method by Biswal and Bhattacharyya [3]. They noticed that the maximum reduction in liquid and structural response is achieved by moving the baffle closer to the free surface. N. Choudhary *et al.* reported the liquid vibrations modes and frequencies in the annular area of hollow cylinderical tank using ANSYS software [4]. They found a monotone relationship between frequency and the baffle's inner radius. Elena *et al.* used boundary element method to numerically simulate the free surface amplitudes in cylindrical liquid filled tank [5]. Chen *et al.* adopted MPS approach to investigate the frequencies of fluid filled tanks with baffles [6]. The results demonstrated that adding vertical baffles to the tanks significantly improved their capacity to

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decrease liquid sloshing. Saghi *et al.* created a hydroelastic model to forecast the sloshing loads caused by sway on flexible trapezoidal and rectangular tanks [7].

Most of the research is done on rectangular and cylindrical tanks using numerical approach. Brar and Singh focused on the fluid-structure interactions in an elliptical tank [8]. They used modeling and experiments to investigate the fluid movement in the tank. They conducted their research using various baffle configurations and concluded that using both a horizontal and vertical baffle together is sufficient to reduce the sloshing that is created in the tank. Eswaran and Saha reviewed a few experimental studies carried out in previous years [9]. They highlighted the various parameters like the effect of baffles, tuned liquid dampers, magnetic field, electric field, etc, which is responsible for liquid sloshing in tanks. Gedikli and Erguven described how a rigid baffle affected the liquid's seismic behavior in a rigid cylindrical tank [10]. They repoted the seismic response and modal analysis using superpostion principle and boundary element method, respectively. Gnitko *et al.* studied the small amplitude liquid oscillations in a stiff, half-filled tank with baffles [11]. Using numerical experiment, they found a suitable place for installing baffles in the tank. Saghi *et al.* proposed a dual-baffled rectangular tank with various designs to lessen the sloshing impact [12]. The results demonstrated that the dual-baffled arrangement is superior to the single-baffled in decreasing the sloshing at low angular frequencies. Researchers Choudhary *et al.* looked into how flexible membranes affected liquid vibrations in rigid circular cylindrical shells [13]. They came to the conclusion that a key factor in reducing sloshing in cylindrical tanks is the height of the barrier placement. Kumar and Choudhary used numerical techniques to reduce sloshing in a circular cylindrical tank by putting a rigid baffle in the middle of the liquid's free surface [14]. It is noted that a baffle on the tank's free surface lowers sloshing. Wang *et al.* conducted a numerical investigation of liquid vibrations in half-filled circular cylindrical tank with various rigid barriers that are subjected to pitching excitation [15].

Many academicians have been investigating the sloshing effects using baffles, however the usage of several baffles has not gained much popularity. Hosseini and Farshadmanesh examined the impact of several vertical barriers on sloshing in rectangular tanks using the finite element method [16]. They observed that two or three vertical baffles could minimize the sloshing impact to a great extent. Jamalabadi investigated the effects of covering a rigid, cylindrical fuel tank with a thin plate [17]. He studied the fundamental angular velocity of linked fluid-structure motion as a parameter. His findings showed that mass ratio is the most critical parameter for detecting the natural frequency. Maleki *et al.* investigated how baffles could lessen the quake responses of a cylindrical, seismically isolated fluid storage reservoir [18]. Matsui *et al.* studied the dynamic behaviour of a floating cover in a cylindrical liquid storage container under wind loads [19]. Zhou *et al.* created a numerical model to investigate the charaterstics of liquid oscillation in a cylindrical-shaped tank with several solid barriers [20].

There aren't many analytical studies in literature that look at how baffles reduce liquid vibrations in rectangular tanks. As a result, the current approach is developed to handle the fluid behavior in a vessel with rigid baffles.

2. Mathematical model

A *3D* rectangular tank (Fig.1) of length a, width b, and height h is considered. The tank in the consideration is partially filled with a liquid upto height h. The liquid is assumed to be incompressible and inviscid.

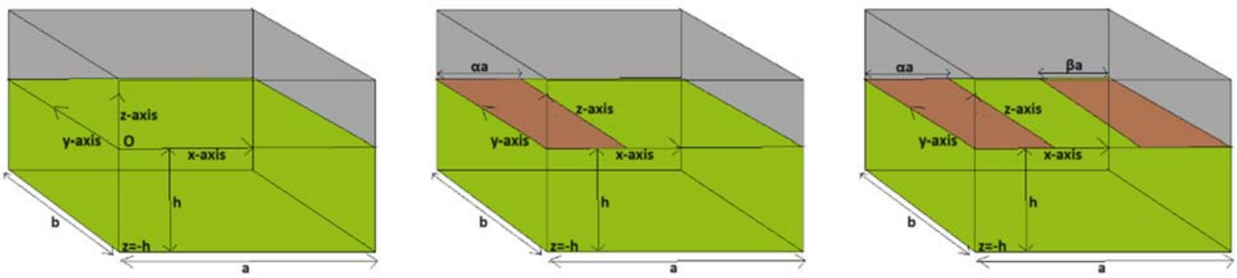

Fig.1. Tank geometries.

The mathematical modeling of the problem is done using the cartesian coordinate system (x, y, z), where z is taken in a vertically upword direction. $z = 0$ indicates the mean location of the liquid surface inside the tank. The rectangular tank's bottom is represented by $z = -h$. The following three cases are discussed here.
A three-dimensional rectangular tank
1. has no cover on the free surface.
2. has a single baffle partially covering the free surface from the tank's left side.
3. has two baffles from the tank's left and right sides, partially covering the free surface.

Under these assumptions, fluid flow can be described from the following boundary value problem

$$\nabla^2 \Phi(x, y, z, t) = 0, \tag{2.1}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

To explore the behaviour of free surface oscillations, this mathematical equation must be solved with the requisite side wall boundary conditions

$$\frac{\partial \Phi}{\partial x} = 0, \quad \text{at} \quad x = 0 \quad \text{and} \quad x = a, \tag{2.2}$$

$$\frac{\partial \Phi}{\partial y} = 0, \quad \text{at} \quad y = 0 \quad \text{and} \quad y = b, \tag{2.3}$$

$$\frac{\partial \Phi}{\partial z} = 0, \quad \text{at} \quad z = -h. \tag{2.4}$$

Solution for BVP (2.1)-(2.4) is given by

$$\Phi(x, y, z, t) = \\ = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{A_{mn} \cosh\left(\dfrac{\pi \sqrt{n^2 a^2 + m^2 b^2}\,(z+h)}{ab} \right)}{\cosh\left(\dfrac{\pi \sqrt{\left(n^2 a^2 + m^2 b^2\right)} h}{ab} \right)} \cos\left(\frac{m \pi x}{a} \right) \cos\left(\frac{n \pi y}{b} \right) \times \exp^{\iota \omega t}, \tag{2.5}$$

where A_{mn} is the constants of integration, and ω is the natural frequency, are unknowns. The boundary conditions at $z = 0$ are utilized to determine unknowns.

2.1. Tank with no cover

To treat the considered problem for the case when the free surface has no cover, we have a free surface condition. In this case, we get

$$\frac{\partial^2\Phi}{\partial t^2}+g\frac{\partial\Phi}{\partial z}=0, \quad \text{at} \quad z=0 \quad \text{in} \quad 0<x<a \quad \text{and} \quad 0<y<b. \tag{2.6}$$

Substituting the velocity potential given by Eq.(2.5) in Eq.(2.6) the following homogeneous system of equations is obtained

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{A_{mn}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)}{\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)}\times\left[-\omega^2\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)+\right.$$
$$\left.+g\frac{\pi\sqrt{n^2a^2+m^2b^2}}{ab}\sinh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)\right]=0. \tag{2.7}$$

We choose $M_1 - 1$ and $N_1 - 1$ points in $0<x<a$ and $0<y<b$ satisfying Eq.(2.7), to get

$$\sum_{m=0}^{M_1-1}\sum_{n=0}^{N_1-1}\frac{A_{mn}\cos\left(\frac{m\pi m_1}{M_1}\right)\cos\left(\frac{n\pi n_1}{N_1}\right)}{\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)}\left[-\omega^2\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)+\right.$$
$$\left.+g\frac{\pi\sqrt{n^2a^2+m^2b^2}}{ab}\sinh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)\right]=0, \tag{2.8}$$

for $m_1 = 1,.....,M_1 - 1$ and $n_1 = 1,......,N_1 - 1$. Eq.(2.8) represents $M_1 \times N_1$ homogenous algebraic equations in the integration constants A_{mn} for $m = 0,1,2,.....,M_1 - 1$ and $n = 0,1,2,.....,N_1 - 1$. It represents a homogenous system of equations of the form $AX = 0.$ The solution to this system is found using matrix theory.

2.2. Tank with single cover

Here, the free surface is covered by a baffle of size αa, with $(\alpha < 1)$ which gives rise to two connditions at $z = 0$:

$$\frac{\partial^2\Phi}{\partial t^2}+g\frac{\partial\Phi}{\partial z}=0, \quad \text{on} \quad z=0 \quad \text{in} \quad \alpha a<x<a \quad \text{and} \quad 0<y<b. \tag{2.9}$$

$$\frac{\partial\Phi}{\partial z}=0, \quad \text{on} \quad z=0 \quad \text{in} \quad 0<x\le\alpha a \quad \text{and} \quad 0<y<b. \tag{2.10}$$

Substituting the velocity potential given by Eq.(2.5) in Eqs (2.9)-(2.10) the following homogeneous system of equations is obtained

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{A_{mn}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)}{\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)}\left[-\omega^2\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)+\right.$$
$$\left.+g\frac{\pi\sqrt{n^2a^2+m^2b^2}}{ab}\sinh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)\right]=0. \tag{2.11}$$

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{A_{mn}}{\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)}\frac{\pi\sqrt{n^2a^2+m^2b^2}}{ab}\times$$
$$\times\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sinh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)=0. \tag{2.12}$$

We choose M_1 and $N_1 - 1$ points in $0 < x \le \alpha a$ and $0 < y < b$, satisfying Eq.(2.12), to get

$$\sum_{m=0}^{M_1+M_2-1}\sum_{n=0}^{N_1-1}\frac{A_{mn}}{\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)}\frac{\pi\sqrt{n^2a^2+m^2b^2}}{ab}\times$$
$$\times\cos\left(\frac{m\pi\alpha m_1}{M_1}\right)\cos\left(\frac{n\pi n_1}{N_1}\right)\sinh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)=0, \tag{2.13}$$

for $m_1 = 1,2,......,M_1, n_1 = 1,2,......,N_1 - 1$.

Similarly, we choose $M_2 - 1$ and $N_1 - 1$ points *in* $\alpha a < x < a$ *and* $0 < y < b$, satisfying Eq.(2.11), to get

$$\sum_{m=0}^{M_1+M_2-1}\sum_{n=0}^{N_1-1}\frac{A_{mn}}{\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)}\cos\left(m\pi\left(\alpha+\frac{(1-\alpha)m_2}{M_2}\right)\right)\times$$
$$\times\cos\left(\frac{n\pi n_1}{N_1}\right)\left[-\omega^2\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)+\right. \tag{2.14}$$

$$+g\frac{\pi\sqrt{n^2a^2+m^2b^2}}{ab}\sinh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)\Bigg]=0, \qquad \text{(cont.2.14)}$$

for $m_2 = 1,2,......,M_2 - 1,\ n_1 = 0,1,2,......,N_1 - 1$.

Equations (2.13)-(2.14) represent $(M_1 + M_2) \times N_1$ homogenous algebraic equations in the integration constants A_{mn} for $m = 0,1,2,.....,M_1 + M_2 - 1$ and $n = 0,1,2,.....,N_1 - 1$. It represents a homogenous system of equations of the form $AX = 0$.

2.3. Tank with two covers

Here, the free surface is covered by two baffles of size αa and size βa where $(\alpha < 1, \beta < 1)$, respectively, from both sides of the wall. This arragement gives the following conditions at $z = 0$:

$$\frac{\partial^2\Phi}{\partial t^2}+g\frac{\partial\Phi}{\partial z}=0, \quad \text{on} \quad z=0 \quad \text{in} \quad \alpha a<x<\beta a \quad \text{and} \quad 0<y<b. \tag{2.15}$$

$$\frac{\partial\Phi}{\partial z}=0, \quad \text{on} \quad z=0 \quad \text{in} \quad 0<x\le\alpha a,\ \ \beta a\le x\le a \quad \text{and} \quad 0<y<b. \tag{2.16}$$

Substituting the velocity potential given by Eq.(2.5) in Eqs (2.15)-(2.16) the following homogeneous system of equations is obtained

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{A_{mn}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)}{\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)}\Bigg[-\omega^2\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)+$$
$$+g\frac{\pi\sqrt{n^2a^2+m^2b^2}}{ab}\sinh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)\Bigg]=0. \tag{2.17}$$

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{A_{mn}}{\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)}\frac{\pi\sqrt{n^2a^2+m^2b^2}}{ab}\times$$
$$\times\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sinh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)=0. \tag{2.18}$$

We choose M_1 and $N_1 - 1$ points *in* $0 < x \le \alpha a$ and $0 < y < b$ satisfying Eq.(2.18) to get

$$\sum_{m=0}^{M_1+M_2+M_3-1}\sum_{n=0}^{N_1-1}\frac{A_{mn}}{\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)}\times\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}}{ab}\times$$
$$\times\cos\left(\frac{m\pi\alpha m_1}{M_1}\right)\times\cos\left(\frac{n\pi n_1}{N_1}\right)\times\sinh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)=0, \tag{2.19}$$

for $m_1 = 1,2,......,M_1, n_1 = 1,2,......,N_1 - 1$.
We choose M_2 and $N_1 - 1$ points *in* $\beta a \le x < a$ *and* $0 < y < b$ satisfying Eq. (2.18) to get

$$\sum_{m=0}^{M_1+M_2+M_3-1}\sum_{n=0}^{N_1-1}\frac{A_{mn}}{\cosh k\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)}\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}}{ab}\times$$
$$\times\cos\left(m\pi\left(\beta+\frac{(1-\beta)m_2}{M_2}\right)\right)\times\cos\left(\frac{n\pi n_1}{N_1}\right)\times\sinh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)=0, \tag{2.20}$$

for $m_2 = 0,1,2,......,M_2, n_1 = 0,1,2,......,N_1$.
Similarly, we choose $M_3 - 1$ and $N_1 - 1$ points in $\alpha a < x < \beta a$ and $0 < y < b$, satisfying Eq.(2.17), to get

$$\sum_{m=0}^{M_1+M_2+M_3+1}\sum_{n=0}^{N_1+1}\frac{A_{mn}}{\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)}\cos\left(m\pi\left(\alpha+\frac{(\beta-\alpha)m_3}{M_3}\right)\right)\cos\left(\frac{n\pi n_1}{N_1}\right)\times$$
$$\left[-\omega^2\cosh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)+g\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}}{ab}\times\sinh\left(\frac{\pi\sqrt{\left(n^2a^2+m^2b^2\right)}h}{ab}\right)\right]=0, \tag{2.21}$$

for $m_3 = 1,2,......,M_3 - 1, n_1 = 1,2,......,N_1 - 1$.

Equations (2.19)-(2.21) represent $\left(M_1 + M_2 + M_3\right) \times N_1$ homogenous algebraic equations in the integration constants A_{mn} for $m = 0,1,2,.....,M_1 + M_2 + M_3 - 1$ and $n = 0,1,2,.....,N_1 - 1$. It represents a homogenous system of equations of the form $AX = 0$.

3. Numerical results

The sloshing frequencies and free surface response are received using ANSYS software and proposed analytical method. For numerical simulations, for all considered cases, paramerters describing the liquid filled tank structure are taken as $b = 2\ m$, $a = 1\ m$, and $h = 0.5\ m$, respectively. Baffles width are considered, namely $\alpha = 0.3\ m$ and $\beta = 0.3\ m$.

3.1. Tank with no cover

The modes of the first six axisymmetric vibrations of the un-baffled rectangular tank are reported in Fig. 2. In Figure 2, the pressure contours of the $3D$ rectangular tank without baffles at different frequencies are reported. These contours show sloshing of the un-baffled $3D$ rectangular tank. The frequencies of the un-baffled $3D$ rectangular tank are presented in Table 1. The lowest frequency, which corresponds to the first mode, is provided. The first frequency corresponding to each mode is shown in Table 1.

Table 1. Frequencies of un-baffled tank.

Mode	Frequency (Hz)
1	*0.50594*
2	*0.84616*
3	*0.84616*
4	*0.90668*
5	*1.0384*
6	*1.0724*

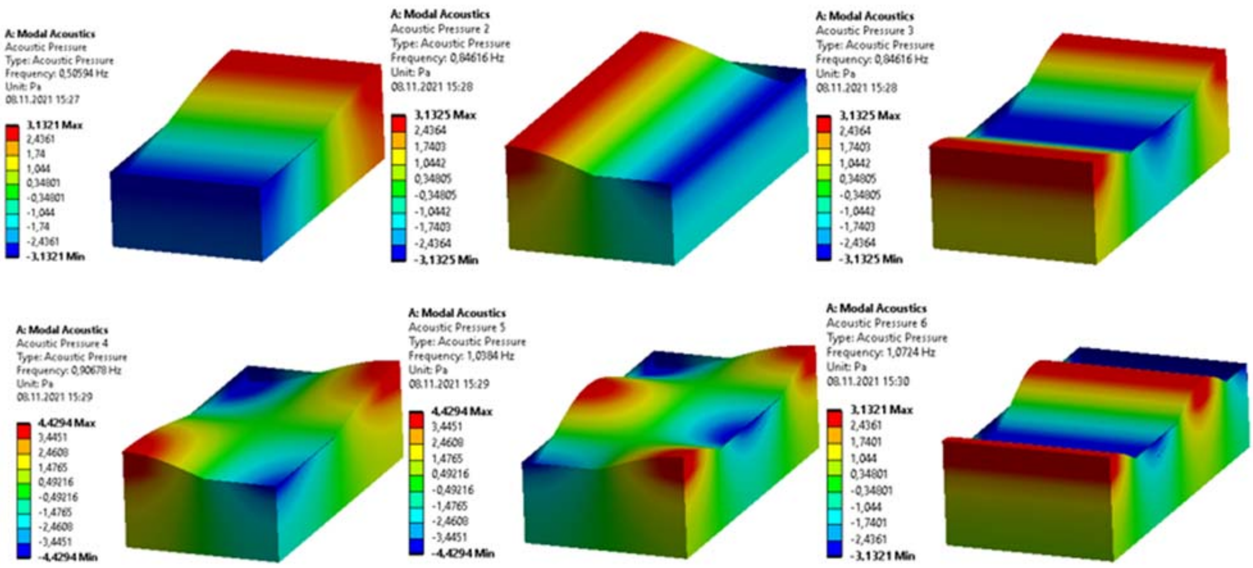

Fig.2. Liquid oscillations in un-baffled rectangular tank.

Comparison of results obtained by proposed analytical method, finite element method with Abramson [21] and NZSEE [22] are displayed in Table 2.

In Table 3, frequencies of axisymmetric modes are presented for various values of h. It is observed that the value of frequency increases significantly for the higher value of h. The frequency value is increased until it reaches the filling level $h = 0.8\,m$. The difference in frequency values is insignificant after $h = 0.8\ m$. The filling level $h = 1\,m$ has the highest frequency.

Table 2. Comparison of results.

$b \times a \times h$ (m)	Mode No.	Frequency(Hz)			
		Abramson [22]	NZSEE(1986) [23]	Analytical Method	Finite Element Method
4×3×1	1	0.36	0.36	0.35	0.35
	2	0.76	0.76	0.75	0.75
4×3×2	1	0.42	0.42	0.42	0.42
	2	0.77	0.77	0.76	0.76
4×3×3	1	0.44	0.44	0.43	0.43
	2	0.77	0.77	0.76	0.76

The relationship of frequencies vs. filling level h is shown in Fig.3 for various values of *h*. For small filling levels of *h*, a rapid rise in sloshing frequency is observed. Sloshing frequency increases as the filling level increases to $h = 0.6\,m$, but the difference in increments reduces beyond $h = 0.6\,m$. It is also noted that sloshing frequency is highest at filling level $h = 1\ m$. Higher frequency is reported, corresponding to a higher value of h.

Table 3. Frequencies (*Hz*) for an un-baffled liquid tank for different values of *h*.

n	h(m)									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.2466	0.3446	0.4140	0.4662	0.5059	0.5361	0.5589	0.5760	0.5888	0.5983
2	0.4873	0.6593	0.7581	0.8146	0.8462	0.8634	0.8727	0.8777	0.8804	0.8819
3	0.4873	0.6593	0.7581	0.8146	0.8462	0.8634	0.8727	0.8777	0.8804	0.8819
4	0.5427	0.7272	0.8268	0.8795	0.9067	0.9205	0.9274	0.9308	0.9325	0.9334
5	0.6786	0.8858	0.9800	1.0211	1.0384	1.0456	1.0486	1.0499	1.0504	1.0506
6	0.7171	0.9286	1.0199	1.0575	1.0724	1.0783	1.0806	1.0815	1.0819	1.082

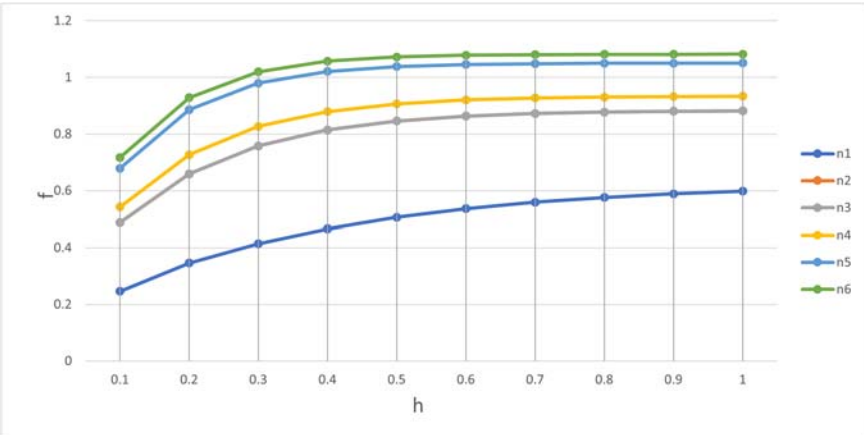

Fig.3. Frequencies versus filling level.

3.2. Tank with single cover

In Figure 4, the first six modes of liquid vibrations for a baffled rectangular tank with baffle width considered as $\alpha = 0.3\,m$ are reported. When the baffle of width $\alpha = 0.3\,m$ is placed on the free surface, it reduces the sloshing significantly. Sloshing on a un-baffled surface is more compared to baffled surface. This is clearly shown in Fig. 4 . The first frequency corresponding to each mode is shown in Table 4 . The baffled 3D rectangular tank frequencies are presented in Table 3 .

Table 4. Frequencies of baffled tank with baffle width $\alpha = 0.3\ m$.

Mode	Frequency (Hz)
1	0.57434
2	0.90286
3	1.075
4	1.1111
5	1.1322
6	1.2416

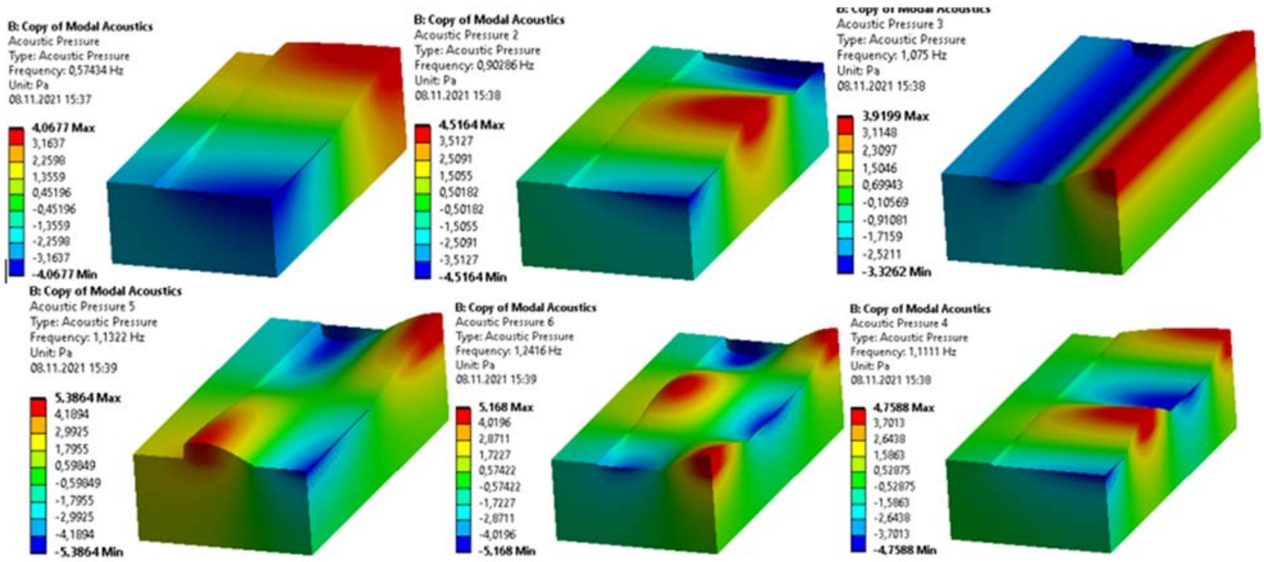

Fig.4. Modes of liquid oscillations in baffled rectangular tank.

In Table 5 , frequencies corresponding to axisymmetric modes are presented for varrying values of liquid height h . It is observed that the value of frequency increases significantly for the higher value of h . In this case, also, frequency value is increased up to filling level $h = 0.8\ m$. The highest frequency value for filling level $h = 1\ m$ is $\omega = 1.2494\ Hz$, which is greater than the frequency value $\omega = 1.082\ Hz$.

Table 5: Frequencies (Hz) for baffled liquid tank

n	h(m)									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.2853	0.3973	0.4751	0.5321	0.5743	0.6056	0.6288	0.6459	0.6584	0.6676
2	0.5347	0.7180	0.8188	0.8736	0.9029	0.9182	0.9264	0.9306	0.9329	0.9340
3	0.6858	0.8984	1.0008	1.0505	1.0751	1.0874	1.0938	1.0972	1.099	1.0999
4	0.7518	0.9717	1.0632	1.0979	1.1111	1.1161	1.118	1.1188	1.1191	1.1192
5	0.7589	0.9746	1.069	1.1123	1.1322	1.1419	1.1469	1.1496	1.1511	1.1521
6	0.8942	1.1202	1.2013	1.2305	1.2416	1.2461	1.248	1.2488	1.2492	1.2494

The relationship of frequencies vs. filling level h is shown in Fig. 5. Figure 5 shows a drastic change in sloshing frequencies when the free surface has a cover of width $\alpha = 0.3\ m$ from the tank's left side. It is evident that frequency increases as the filling level increases up to $h = 0.8\ m$, while the difference in increment reduces beyond $h = 0.8\ m$.

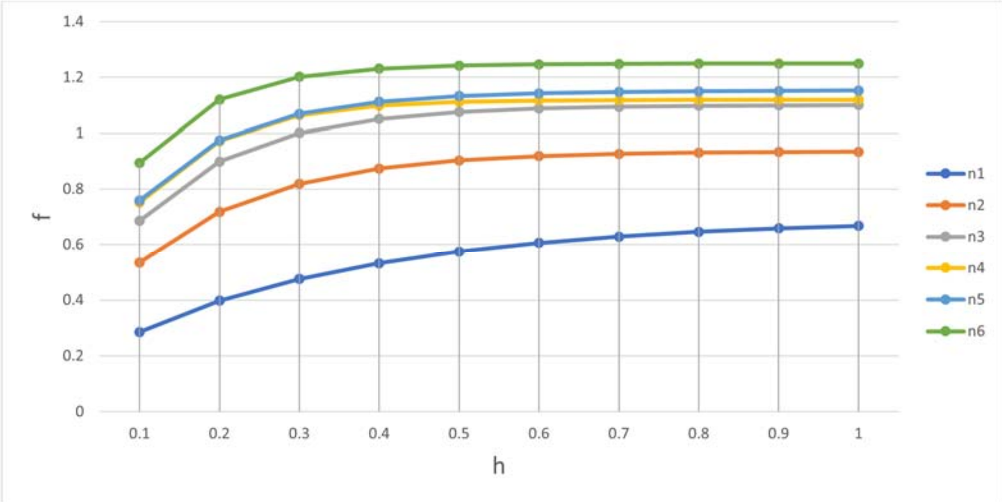

Fig.5. Frequencies versus filling level.

3.3. Tank with two covers

The first six modes for a baffled rectangular tank with two covers of widths $\alpha = 0.3\ m$ and $\beta = 0.3\ m$ on the liquid-free surface are shown in Fig. 6. In this figure, it is observed that sloshing occurs only in the middle portion of the tank. In the rest region, it is fully damped by rigid baffles. The first frequency corresponding to each mode is given in Table 6. The frequencies of baffled 3D rectangular tank are presented in Table 5 with parameters $b = 2\ m, a = 1\ m, \alpha = 0.3\ m, \beta = 0.3\ m$ and $h = 1\ m$.

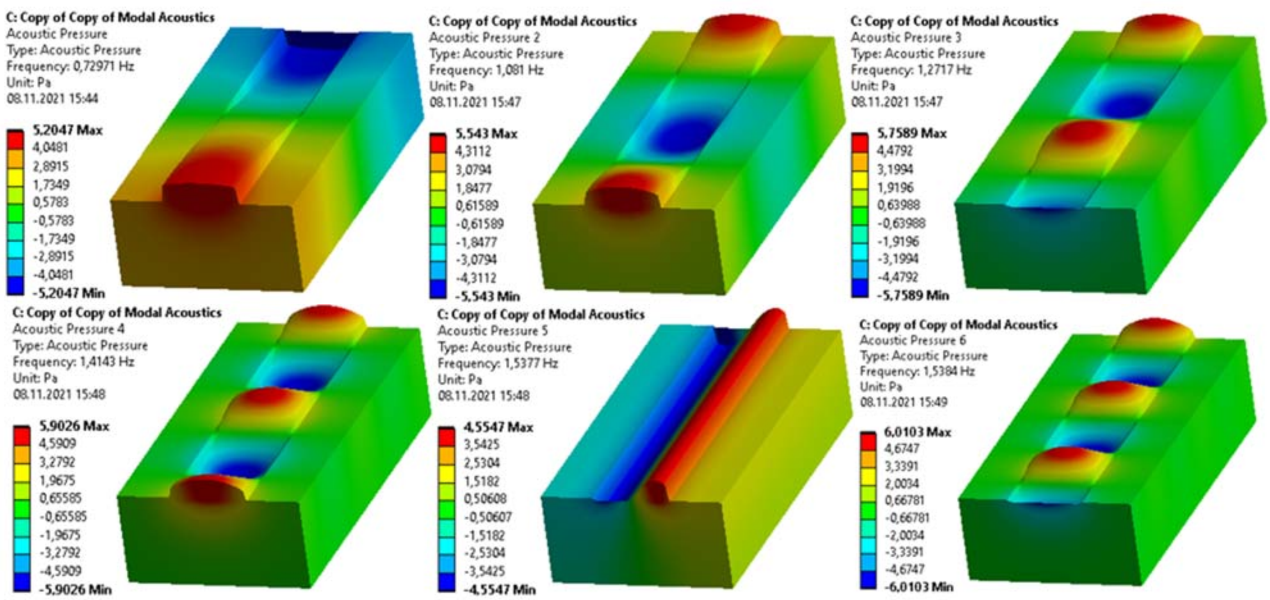

Fig.6. Modes of liquid oscillations in baffled rectangular tank.

Table 6. Frequencies of baffled tank with baffle width $\alpha = 0.3\ m$ and $\beta = 0.3\ m$.

Mode	Frequency (Hz)
1	0.72971
2	1.081
3	1.2717
4	1.4143
5	1.5377
6	1.5384

Table 7 shows the frequency of axisymmetric modes for various h values. The value of frequency increases dramatically as the value of h increases. The frequency value is also increased until it reaches the filling level $h = 0.8\ m$. The increase in frequency values after $h = 0.8\ m$ is minimal. The greatest frequency value for $h = 1\ m,$ in this case, is $\omega = 1.5573$, which is higher than the frequencies reported in previous cases.

Table 7. Frequencies (Hz) for dual baffled liquid tank.

n	h(m)									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.3750	0.51793	0.6133	0.6808	0.7297	0.7654	0.7916	0.8108	0.8249	0.8353
2	0.6814	0.89489	1.0004	1.0536	1.0810	1.0953	1.1028	1.1068	1.109	1.1101
3	0.9227	1.1513	1.2323	1.2612	1.2717	1.2758	1.2773	1.2779	1.2781	1.2782
4	1.1248	1.3412	1.3965	1.4106	1.4143	1.4154	1.4157	1.4158	1.4158	1.4158
5	1.1415	1.3905	1.4805	1.5191	1.5377	1.5387	1.5388	1.5388	1.5388	1.5388
6	1.2099	1.4511	1.53	1.5371	1.5384	1.5476	1.5525	1.5551	1.5565	1.5573

Figure 7 depicts the relationship between frequencies and filling level h for varying values of h. When the free surface of the container is occupied by baffles of width $\alpha = 0.3\ m$ and $\beta = 0.3\ m$ on both sides, sloshing frequencies increase rapidly compared to the preceding cases. The lowest frequency is gradually increased. It has also been noted that as the filling levels increase, so does the frequency.

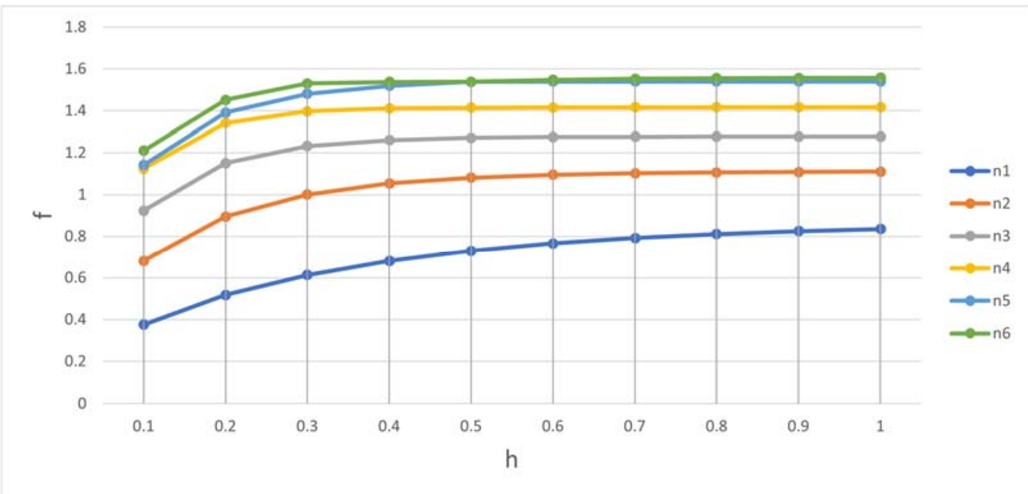

Fig.7. Frequencies versus filling level.

4. Conclusions

The purpose of this study is to see how rigid baffles affect sloshing. A semi-analytical approach is used for this investigation, and numerical experiments are done for validation of the results. The sloshing frequencies in the setups are studied using potential wave theory. The results from analytical and numerical experiments are compared. The effect of baffles and filling level of liquid in the tank is studied. Following are the major conclusions:

1. With a rising filling level and free surface coverage of liquid in the container, the frequency value rises. A sharp increase in frequency is observed up to $h = 1m$ of filling level.
2. The frequency is shifted to a greater value when baffles are placed on the free surface. When two baffles are installed on the free surface from both side walls, a greater frequency is seen.
3. Insertion of baffles has a more significant impact on the lowest frequency.
4. Rigid baffles can be used to prevent liquid sloshing in containers. The approach provided here should work for any structure in similar surface plane.

Future research endeavors to develop mathematical models to study liquid sloshing involving nonlinearity.

One can investigate the effects of multiple baffles (including vertical and horizontal) on liquid sloshing in tanks. One can also examine the sloshing problems in elastic tanks.

Nomenclature

a – tank length

A_{mn} – the constants of integration

b – tank width

h – tank height

(x,y,z) – the cartesian coordinate system

ω – the natural frequency,

$\Phi(x,y,z,t)$ – the velocity potential

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