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Repetitive process based design of PD-type iterative learning control laws for batch processes with time-delays ⋆

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Abstract: This paper uses linear matrix inequality techniques and the Kalman-Yakubovich-Popov lemma to design an iterative learning control law for a class of batch processes with state delays. The design procedure is based on the stability theory for repetitive processes, a class of 2D systems. A numerical example illustrates the new design and demonstrates that the design has advantages compared to the existing alternatives.

Keywords: Iterative learning control, repetitive processes, batch processes, linear matrix inequalities, systems with delays.

1. INTRODUCTION

Many industries, e.g., chemical, machinery manufacturing, electronic technology, pharmaceutical engineering, and agricultural products processing, use batch processing control systems to obtain high-quality products with high manufacturing efficiency. Therefore, the development of advanced batch process control technologies and performance assessment methods, see, e.g.,in (Lu et al., 2017; Wang et al., 2009), can result in improved productivity and quality in manufacturing-related industries.

Recent developments in batch process control technologies include using output information for each batch to improve the control for the subsequent one. Given that the processing time for each batch is finite and fixed, this is an application area for ILC. See, e.g., (Bristow et al., 2006; Wang et al., 2009) for early research results in this application area. As in other ILC applications, a variable is needed to specify the batch number and another to describe the dynamics within a batch. In this paper, the nonnegative integer k denotes the batch number, and t describes the finite duration of the dynamics within a batch. Moreover, the time taken to process a batch is denoted by $\alpha < \infty$.

In batch processing, there are two directions of information propagation, i.e., within each batch, where the duration is finite, and from batch to batch. The dynamics are defined over $(k, t) = [0\infty] \times [0, \alpha]$. Hence the dynamics of ILC applied to batch processing is a member of the class of 2D systems. In particular, the dynamics considered can be written as a repetitive process, i.e., a distinct class of 2D systems (Rogers et al., 2007). This setting allows simultaneous design for batch-to-batch (k) and batch dynamics (t).

The literature demonstrates that a P-type ILC law (where the control signal is proportional to the previous batch error only) can effectively control the average error in tracking the reference. However, in many industrial processes, this form of control may result in wasted production. Therefore, it is necessary to improve the control law to reduce the maximum error, where differential (D)-type control can improve the dynamic characteristics of the control system. e.g., reducing overshoot, shortening regulation time, and enhancing control accuracy.

This paper focuses on the design of ILC laws that combine P and D action, termed PD-type. The repetitive process setting is used for analysis control, and the novel contributions are as follows:

- the design of a PD-type ILC law in the frequency domain for a class of linear differential systems with time delays;
- batch-to-batch error convergence conditions as LMIs.

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Notation: $X \prec Y$ ($X \succ Y$) if the matrix $X-Y$ is negative definite (respectively, positive) matrix. 0 and I represent compatibly dimensioned null and identity matrices, respectively, A^* denotes the complex conjugate transpose of a matrix. Also, the symbol $(\star)$ denotes transposed elements in a symmetric matrix. The symbol $\mathrm{diag}\{X_1, X_2, \cdots, X_n\}$ denotes a block diagonal matrix with compatibly dimensioned diagonal blocks X_1, X_2, $\cdots$, X_n and $\mathrm{sym}(\Lambda)$ denotes the matrix $\Lambda + \Lambda^T$. Finally, for a square matrix, say H, $\rho(H)$ denotes the spectral radius, and $\otimes$ denotes the Kronecker product.

The following result is used in the analysis of this paper.

Lemma 1. (Gahinet and Apkarian, 1994) Given a symmetric matrix $\Upsilon \in \mathbb{R}^{q\times q}$ and two matrices Λ, Σ of column dimension q, there exists a matrix W such that the LMI

$$\Upsilon + \mathrm{sym}\left\{\Lambda^T W \Sigma\right\} \preceq 0,$$

holds if and only if the following two projection inequalities with respect to W are satisfied:

$$\Lambda^{\perp T}\Upsilon\Lambda^{\perp} \preceq 0, \ \ \Sigma^{\perp T}\Upsilon\Sigma^{\perp} \preceq 0, \tag{1}$$

where $\Lambda^{\perp}$ and $\Sigma^{\perp}$ are arbitrary matrices whose columns form a basis of the null spaces of Λ and Σ, respectively.

2. PROBLEM DESCRIPTION

This paper considers ILC applied to batch processing, i.e., a sequence of objects (indexed by the nonnegative integer k) where the processing of each of them occurs a finite duration $0 \le t \le \alpha$, where the dynamics are modeled as

$$\begin{aligned} \dot{x}_{k+1}(t) &= Ax_{k+1}(t) + A_d x_{k+1}(t-d) + Bu_{k+1}(t), \\ y_{k+1}(t) &= Cx_{k+1}(t), \end{aligned} \tag{2}$$

where $x_k(t) \in \mathbb{R}^n$, $y_k(t) \in \mathbb{R}^m$ and $u_k(p) \in \mathbb{R}^l$ are the state, output and input vectors, respectively. The time delay d is an unknown constant that satisfies $0 < d \le \overline{d}$, where $\overline{d}$ is a known upper bound. Since the initial state of each batch is reset, it is assumed that $x_k(t) = x_{0,k}$, $t \in [-d, 0]$.

Let $y_d(t)$, $0 \le t \le \alpha$, be a known reference trajectory. Then on batch k the error can be formed as

$$e_k(t) = y_d(t) - y_k(t) \tag{3}$$

and the control objective is to construct a control law that forces convergence in k of the error sequence $\{e_k(t)\}_k$, i.e.,

$$\lim_{k\to\infty} ||e_k(t)|| = 0, \ 0 \le t \le \alpha \tag{4}$$

where $||\cdot||$ is the norm on the underlying function space; in some practical applications, the convergence condition must be relaxed to within a suitably chosen (small) neighborhood of the origin. Moreover, it is also necessary to ensure acceptable dynamics in t.

The ILC law considered in this paper has the structure

$$u_{k+1}(t) = u_k(t) + \Delta u_{k+1}(t), \tag{5}$$

i.e., the control used for the previous batch plus a correction $\Delta u_{k+1}(t)$ that can make use of previous batch data, where the particular case considered is

$$\Delta u_{k+1}(t) = K_1 e_k(t) + K_2 \dot{e}_k(t) + K_3 e_k(t-d), \tag{6}$$

where K_1, K_2 and K_3 are gain matrices to be designed. Hence the control objective is to determine K_1, K_2 and K_3 of (6) such that the control input sequence generated by (5) over $0 \le t \le \alpha$ minimizes the tracking errors to achieve (4), and also ensure acceptable response in t, where this latter requirement is applications specific.

Remark 1. The control law of (6) reduces to the standard PD-type ILC law when $K_3 = 0$.

Remark 2. The structure of (6) is simpler when compared to the alternative given in (Tao et al., 2017) since it does not include state feedback. Specifically, complete state measurement may be difficult to implement; hence, the new control law structure is better suited to many physical applications.

3. REPETITIVE PROCESS BASED ILC DESIGN

Application of the control law (5)-(6) to (2) results in controlled batch process dynamics described by

$$\begin{aligned} \dot{x}_{k+1}(t) =& Ax_{k+1}(t) + A_d x_{k+1}(t-d) + BK_1 e_k(t) \\ &+ BK_2 \dot{e}_k(t) + BK_3 e_k(t-d) + Bu_k(t), \\ y_{k+1}(t) =& Cx_{k+1}(t). \end{aligned} \tag{7}$$

This model is a differential linear repetitive process with a delay in the state vector. The stability theory of these processes enables simultaneous control law design for error convergence and regulation of the temporal dynamics.

The Laplace transform is applied to (7) to provide a basis for frequency domain design. Note that this transform requires that the input and output signals are defined over an infinite time horizon and hence is not applicable in the current case due to the finite batch length. See, however, (Bristow et al., 2006; Rogers et al., 2007) for the details of how to avoid the effects due to the finite batch length. Applying this transform (where, e.g., the Laplace transform of $x_{k+1}(t)$ is written as $X_{k+1}(s)$) gives

$$\begin{aligned} sX_{k+1}(s) =& AX_{k+1}(s) + \mathrm{e}^{-sd} A_d X_{k+1}(s) \\ &+ BK_1 E_k(s) + sBK_2 E_k(s) \\ &+ \mathrm{e}^{-sd} BK_3 E_k(s) + BU_k(s), \\ Y_{k+1}(s) =& CX_{k+1}(s). \end{aligned} \tag{8}$$

Consequently

$$\begin{aligned} (sI - A - \mathrm{e}^{-sd} A_d) X_{k+1}(s) =& B\left(K_1 + sK_2 + \mathrm{e}^{-sd} K_3\right) E_k(s) \\ &+ BU_k(s), \\ Y_{k+1}(s) =& CX_{k+1}(s) \end{aligned} \tag{9}$$

and in case of $U_k(s) = 0$

$$\begin{aligned} Y_{k+1}(s) =& C(sI - A - \mathrm{e}^{-sd} A_d)^{-1} \\ &\times B(K_1 + sK_2 + \mathrm{e}^{-sd} K_3). \end{aligned}$$

Using the above results, the transfer function matrix coupling the previous trial error to the current trial output is

$$H(s) = G(s)[(K_1 + sK_2 + \mathrm{e}^{-sd} K_3) E_k(s)], \tag{10}$$

where

$$G(s) = C(sI - A - \mathrm{e}^{-sd} A_d)^{-1} B \tag{11}$$

is the transfer function matrix coupling the current trial input and output. Next, define

$$M(s,d) = sI - A - \mathrm{e}^{-sd} A_d$$

and assume that $M(s,d)$ is nonsingular, then

$$M(s,d)M(s,d)^{-1}=sM(s,d)^{-1} - \left(A+e^{-sd}A_d\right)M(s,d)^{-1} = I.$$

Rewriting this last equation as

$$sM(s,d)^{-1} = I+\left(A+e^{-sd}A_d\right)M(s,d)^{-1}$$

gives

$$H(s)=CM(s,d)^{-1}BK_1+CBK_2+CAM(s,d)^{-1}BK_2 + e^{-sd}(CA_dM(s,d)^{-1}BK_2+CM(s,d)^{-1}BK_3). \quad (12)$$

Next, focus on (9) again and assume that $E_k(s) = 0$. Then,

$$Y_{k+1}(s) = G(s)U_k(s),$$

where $G(s)$ is defined in (11). The next step is to consider the difference between successive batch errors since the propagation of the error from batch to batch must be analyzed, starting from

$$E_{k+1}(s) - E_k(s) = -G(s)(U_{k+1}(s) - U_k(s)). \quad (13)$$

Hence

$$E_{k+1}(s) = (I - H(s))E_k(s) \quad (14)$$

and therefore the tracking error converges as $k \to \infty$ if all eigenvalues of $I - H(s)$ have modulus less than unity, i.e.,

$$\rho\left(I - H\left(j\omega\right)\right) < 1, \forall\omega \in [0, \infty). \quad (15)$$

Also, the following condition in terms of matrix inequalities (LMIs) is the necessary and sufficient for (15) to hold

$$\left(I-H\left(j\omega\right)\right)^{*}P(j\omega)\left(I-H\left(j\omega\right)\right)-P(j\omega)\prec 0, \forall\omega \in [0, \infty), \quad (16)$$

where $P(j\omega) \succ 0$. Unfortunately, the dependence $P(j\omega)$ on ω is unknown, making the above inequality very hard to solve. Consequently, a constant P matrix over the entire frequency range is used, and the repetitive process-based analysis is applied.

3.1 *Repetitive process based analysis*

To formulate the ILC design problem in the repetitive process setting, consider the term $I - H(s)$, which can be rewritten as

$$I - H(s) = G_1(s) + G_2(s) + G_3(s), \quad (17)$$

where

$$\begin{aligned} G_1(s) &= I - CBK_2 - CM(s,d)^{-1}BK_1, \\ G_2(s) &= -C\left(A+e^{-sd}A_d\right)M(s,d)^{-1}BK_2, \\ G_3(s) &= -Ce^{-sd}M(s,d)^{-1}BK_3. \end{aligned} \quad (18)$$

Moreover (7) can be equivalently converted to the following three differential repetitive processes in a parallel connection,

$$\begin{aligned} \dot{x}_{k+1}(t) &= Ax_{k+1}(t) + A_d x_{k+1}(t-d) + B_{0v}e_k(t), \\ e_{k+1}(t) &= C_v x_{k+1}(t) + C_{dv}x_{k+1}(t-d) + D_{0v}e_k(t). \end{aligned} \quad (19)$$

where the subscript v denotes for the repetitive process number, i.e. $v = \{1, 2, 3\}$. Note that the matrices A and A_d are the same for all three processes. The remaining matrices for the first process model ($v = 1$) are

$$B_{01} = BK_1,\ C_1 = -C,\ C_{d1} = 0, D_{01} = (I–CBK_2), \quad (20)$$

for $v = 2$

$$B_{02} = BK_2,\ C_2 = -CA,\ C_{d2} = -CA_d, D_{02} = 0, \quad (21)$$

and for $v = 3$

$$B_{03} = BK_3,\ C_3 = 0,\ C_{d3} = -C, D_{03} = 0. \quad (22)$$

Given (15), the condition for tracking error convergence can be written as

$$\rho\left(G_1\left(j\omega\right)+G_2\left(j\omega\right)+G_3\left(j\omega\right)\right) < 1, \forall\omega \in [0, \infty). \quad (23)$$

However, even if (15) or (23) hold, poor transients in t occur even if convergence in k of the error sequence $\{e_k(t)\}_k$ in k occurs. (the dynamics in t occur over a finite duration where even an unstable linear system can only produce a bounded output in response to a bounded input.)

Replacing the condition in (15) by

$$\overline{\sigma}(I - H\left(j\omega\right)) < 1, \forall\omega \in [0, \infty), \quad (24)$$

where $\overline{\sigma}(\cdot)$ denotes the maximum singular value of its matrix argument, preventing the problem above from arising. Also

$$\|I - H\left(j\omega\right)\|_\infty \triangleq \max_{\omega\in[0,\infty)} \overline{\sigma}\left(I - H\left(j\omega\right)\right) < 1 \quad (25)$$

and this condition requires that the state matrix is stable. This is an $\mathcal{H}_\infty$ condition for batch-to-batch error convergence and implies that

$$\|e_k(t)\|_2 \leq \|\left(I - H\left(j\omega\right)\right)\|_\infty^k \|e_0(t)\|_2, \quad (26)$$

where $\|\cdot\|_2$ denotes the L_2 and hence monotonic batch-to-batch error convergence occurs.

The control synthesis problem is to design the closed-loop transfer function matrices $G_v(s)$, $v = 1, 2, 3$ such that

$$\begin{cases} \|G_1\left(j\omega\right)\|_\infty < 1 - \gamma_a - \gamma_b, \\ \|G_2\left(j\omega\right)\|_\infty < \gamma_a, \\ \|G_3\left(j\omega\right)\|_\infty < \gamma_b, \end{cases} \quad (27)$$

where γ_a and γ_b are given scalars satisfying $0 < \gamma_a < 1$ and $0 < \gamma_b < 1$. Then,

$$\begin{aligned} \|I-H\left(j\omega\right)\|_\infty &= \|G_1\left(j\omega\right)+G_2\left(j\omega\right)+G_3\left(j\omega\right)\|_\infty \\ &\leq \|G_1\left(j\omega\right)\|_\infty+\|G_2\left(j\omega\right)\|_\infty \\ &\quad +\|G_3\left(j\omega\right)\|_\infty<1. \end{aligned}$$

Hence, when the system of inequalities

$$\begin{cases} \overline{\sigma}\left(G_1\left(j\omega\right)\right) < 1 - \gamma_a - \gamma_b, \\ \overline{\sigma}\left(G_2\left(j\omega\right)\right) < \gamma_a, \\ \overline{\sigma}\left(G_3\left(j\omega\right)\right) < \gamma_b \end{cases} \quad (28)$$

hold monotonic batch-to-batch error convergence is ensured. The design problem is: given the batch process model and scalars $0 < \gamma_a < 1$ and $0 < \gamma_b < 1$, determine an ILC law of the form (5)-(6) such that the processes described by (19)-(22) are stable along the batch (trial).

By the triangle inequality, (27) can be rewritten as

$$\|G_v\left(j\omega\right)\|_\infty < \gamma_v, \quad (29)$$

for $v = 1, 2, 3$, where $\gamma_1 = 1 - \gamma_a - \gamma_b$, $\gamma_2 = \gamma_a$ and $\gamma_3 = \gamma_b$. The values of γ_a and γ_b must be selected based on knowledge of the particular example under consideration. The following theorem can now be established

Theorem 1. Suppose that an ILC law of the form (6) is applied to systems described by (2) with specified reference trajectory $y_d(t)$ over $0 \leq t \leq \alpha$. Then the resulting controlled dynamics can be written in the form (19). Moreover, this last representation is stable along the batch for all delays $d \in \left[0, \overline{d}\right]$ if and only if

i) $\rho(D_{0v}) < 1$,
ii) all eigenvalues of the matrix $(A + \mathrm{e}^{-j\omega d}A_d)$ have strictly negative real parts $\forall\omega \in [0,\infty)$ and $d \in \left[0,\bar{d}\right]$
iii) the inequality (23) holds $\forall\omega \in [0,\infty)$ and $d \in \left[0,\bar{d}\right]$.

The proof of this result follows from routine modifications to that for the corresponding result in (Rogers et al., 2007) for $d = 0$ (no time delay) (where a batch is termed a trial). Hence the details are omitted. Moreover, the first condition guarantees convergence in k, also known as asymptotic stability. This condition places no constraint on the state matrix and the dynamics in t. The obvious way to regulate the dynamics in t is to impose the second condition in the last result.

The second condition in Theorem 1 regulates the temporal dynamics for each of the batches but, in general, is not enough in all cases. An example to confirm this fact is given in (Rogers et al., 2007) (for the delay-free case but extends naturally to the current case). Instead, the third condition in this last result, stability along the batch, is required. This condition requires that the frequency content of y_d be attenuated at a geometric rate for all frequencies. Next, control law design is considered, resulting in LMI-based computations.

4. LMI-BASED DESIGN PROCEDURE

In this section, the main goal is to establish control law design algorithms. The route is based on a version of the KYP lemma and results in LMI-based computation of the control law matrices. The new results in this paper are based on a version of KYP lemma, which can directly ensure the control performance specifications over the entire frequency range and any delay value.

Lemma 2. Consider a differential linear repetitive process described by (19)-(22) with corresponding transfer-function matrix of (17). Suppose that scalars $\gamma_v \in (0,1)$ for $v = 1,2,3$ are given. Then (16) holds if there exist Hermitian matrices $\mathcal{P}_v(j\omega) \succ 0$, $v = 1,2,3$, such that

$$\begin{bmatrix} G_v(j\omega) \\ I \end{bmatrix}^* \begin{bmatrix} \mathcal{P}_v(j\omega) & 0 \\ 0 & -\gamma_v^2\mathcal{P}_v(j\omega) \end{bmatrix} \begin{bmatrix} G_v(j\omega) \\ I \end{bmatrix} \prec 0, \qquad (30)$$

$\forall\omega[0,\infty)$, where G_v, $v = 1,2,3$ are defined in (29).

The proof of this result follows by routine modification of that for the delay-free case, see (Rogers et al., 2007), and hence the details are omitted. Additionally, (30) holds provided (28) or (29) are feasible for $\mathcal{P}_v(j\omega) = I$, $v = 1,2,3$.

The following result is a version of Theorem 1 of (Zhang and Yang, 2012) to the single delay case over the entire frequency range ($\Psi = 0$). It gives an LMI-based sufficient condition, dependent on the upper bounds of time delay for an example to satisfy (29).

The following result is a version of Theorem 1 of (Zhang and Yang, 2012) to the single delay case over the entire frequency range ($\Psi = 0$). It gives an LMI-based sufficient condition, dependent on the upper bounds of time delay, for examples to satisfy (29).

Lemma 2. Let real symmetric matrices Π_v of compatible dimensions and any delay d satisfying $0 < d \leq \bar{d}$ be given. Also, consider the differential linear repetitive processes described by (19)-(22) with corresponding transfer-function matrix (17). Then the condition of (29) is satisfied if there exist $P_v \succ 0$, $Z_v \succ 0$ and symmetric matrices X_v, such that

$$\begin{aligned} &\begin{bmatrix} A & A_d & B_{0v} \\ I & 0 & 0 \end{bmatrix}^T \left(\Phi \otimes P_v + \Psi_0 \otimes \bar{d}Z_v\right) \begin{bmatrix} A & A_d & B_{0v} \\ I & 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} C_v & C_{dv} & D_{0v} \\ 0 & 0 & I \end{bmatrix}^T \Pi_v \begin{bmatrix} C_v & C_{dv} & D_{0v} \\ 0 & 0 & I \end{bmatrix} \\ &+ \begin{bmatrix} X_v - \bar{d}^{-1}Z_v & \bar{d}^{-1}Z_v & 0 \\ \bar{d}^{-1}Z_v & -X_v - \bar{d}^{-1}Z_v & 0 \\ 0 & 0 & 0 \end{bmatrix} \prec 0, \end{aligned} \qquad (31)$$

holds for $v = \{1,2,3\}$, where

$$\Phi = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \Psi_0 = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}.$$

Moreover, the following frequency domain inequality holds for the transfer function matrices defined in (18)

$$\begin{bmatrix} \sum_{v=1}^{3} G_v\left(j\omega\right) \\ I \end{bmatrix}^* \Pi_v \begin{bmatrix} \sum_{v=1}^{3} G_v\left(j\omega\right) \\ I \end{bmatrix} \prec 0, \forall\omega \in [0,\infty). \qquad (32)$$

Remark 3. This last result requires that the matrices P and Z are positive definite to guarantee that all eigenvalues of the matrix $(A + \mathrm{e}^{-j\omega d}A_d)$ have strictly negative real parts (Zhang and Yang, 2012). This requirement means that condition *ii)* of Lemma 1 is immediately satisfied.

The inequality conditions given in Lemma 2 are not convex and, therefore, cannot be solved using the numerical solvers directly (e.g., MATLAB packages as LMI CONTROL TOOLBOX or SEDUMI). The following transformations convert these conditions to a convex problem.

Firstly, by Lemma 2, the choice of $\Pi_v = \begin{bmatrix} I & 0 \\ 0 & -\gamma_v^2 I \end{bmatrix}$ in (31) is made. Then (32) implies that the conditions of (28) are satisfied for entire frequency range, i.e. $[0,\infty)$. Next by defining

$$\begin{aligned} \mathcal{M}_v = &\begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \left(\Phi \otimes P_v + \Psi_0 \otimes \bar{d}Z_v\right) \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T \\ &+ \begin{bmatrix} 0 & 0 \\ C_v^T & 0 \\ C_{dv}^T & 0 \\ D_{0v}^T & I \end{bmatrix} \Pi_v \begin{bmatrix} 0 & 0 \\ C_v^T & 0 \\ C_{dv}^T & 0 \\ D_{0v}^T & I \end{bmatrix}^T + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & X_v - \bar{d}^{-1}Z_v & \bar{d}^{-1}Z_v & 0 \\ 0 & \bar{d}^{-1}Z_v & -X_v - \bar{d}^{-1}Z_v & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \end{aligned} \qquad (33)$$

it follows that the inequalities of (31) can be rewritten as

$$\begin{bmatrix} A & A_d & B_{0v} \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}^T \mathcal{M}_v \begin{bmatrix} A & A_d & B_{0v} \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \prec 0. \qquad (34)$$

Next, introduce positive scalars β_v and select

$$\Lambda_v^{\perp} = \begin{bmatrix} A & A_d & B_{0v} \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \ \Sigma_v^{\perp} = \begin{bmatrix} I & 0 & 0 \\ -\beta_v I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix},$$

then by direct computation

$$\Lambda_v = \left[\, -I \;\; A \;\; A_d \;\; B_{0v} \,\right], \ \Sigma_v = \left[\, \beta_v I \;\; I \;\; 0 \;\; 0 \,\right]. \qquad (35)$$

Also,

$$\left(\Phi\otimes P_v+\Psi_0\otimes\bar dZ_v\right)=\left[\begin{array}{c|c}\bar dZ_v & P_v\\ \hline P_v & 0\end{array}\right] \quad (36)$$

and

$$\Sigma_v^{\perp T}\mathcal{M}\Sigma_v^{\perp}=\begin{bmatrix}\Sigma_{1v} & -\beta_vC_v^TC_{dv}-\beta_v\bar dZ_v & -\beta C_v^TD_{0v}\\ (\star) & C_{dv}^TC_{dv}-X_v-\bar d^{-1}Z_v & C_{dv}^TD_{0v}\\ (\star) & (\star) & D_{0v}^TD_{0v}-\gamma_vI\end{bmatrix}, \quad (37)$$

where $\Sigma_{1v}=-\beta_v^2(\bar dZ_v-X_v-C_v^TC_v)+\bar dZ_v$ for $v=1,2,3$. Consequently, when the following inequalities (see (34) and (37))

$$\Lambda_v^{\perp T}\mathcal{M}_v\Lambda_v^{\perp}\prec 0,\;\; \Sigma_v^{\perp T}\mathcal{M}_v\Sigma_v^{\perp}\prec 0$$

hold, Lemma 1 gives that there exists W_v such that

$$\mathcal{M}_v+\mathrm{sym}\left\{\Lambda_v^TW_v\Sigma_v\right\}\prec 0$$

is feasible for $v=1,2,3$. Since products exist among matrix variables, the last inequality is not in the LMI form. The following result solves this problem and reformulates the result of Lemma 2 into an LMI-based condition. This paper's main novel result is this theorem, enabling control law design.

Theorem 1. Suppose that a batch process described by (2) is required to repeatedly follow the given reference $y_d(t)$ over $0\le t\le\alpha$. Also, the let ILC law (6) be applied and assume that delay d satisfying $0<d\le\bar d$ is given. Additionally, let $\hat S_2$ be a given matrix and γ_v, β_v, be given scalars where $0<\gamma_v\le 1$, $\beta_v>0$ for $v=1,2,3$, resulting in dynamics described by (19)-(22) with corresponding transfer-function matrix (17). Moreover the condition of (29) is satisfied if there exist $\hat P_v\succ 0$, $\hat Z_v\succ 0$, $\hat S_1\succ 0$, $\hat S_3\succ 0$, symmetric matrices X_v, N_1, N_2 and N_3, together with positive scalars $\gamma_v>0$ such that

$$\begin{bmatrix}\bar d\hat Z_v-\mathrm{sym}\{\beta_v\hat S_v\} & \beta_vA\hat S_v-\hat S_v^T+\hat P_v & \beta_vA_d\hat S_v & \beta_vBN_v & 0\\ (\star) & \mathrm{sym}\{A\hat S_v\}-\bar d^{-1}\hat Z_v+\hat X_v & A_d\hat S_v+\bar d^{-1}\hat Z_v & BN_v & \hat S_v^TC_v^T\\ (\star) & (\star) & -\hat X_v-\bar d^{-1}\hat Z_v & 0 & S_v^TC_{dv}^T\\ (\star) & (\star) & (\star) & -\gamma_v^2I & D_{0v}^T\\ (\star) & (\star) & (\star) & 0 & -I\end{bmatrix}\prec 0. \quad (38)$$

hold for $v=\{1,2,3\}$. Moreover, if these LMIs are feasible, the corresponding matrices gain matrices K_1, K_2, and K_3 in updating law (6) are given by

$$K_1=N_1\hat S_1^{-1},\;K_2=N_2\hat S_2^{-1},K_3=N_3\hat S_3^{-1}. \quad (39)$$

Proof 1. Suppose that the LMI (38) is feasible. Then pre- and post-multiply (38) by diag $\left\{S_v^{-T},S_v^{-T},S_v^{-T},I,I\right\}$ and its transpose to obtain

$$\begin{bmatrix}\bar d\hat S^{-T}\hat Z_v\hat S^{-1}-\mathrm{sym}\{\beta_v\hat S^{-T}\} & \Upsilon_{12} & \beta_v\hat S^{-T}A_d & \beta_v\hat S^{-T}B_{0v}\hat S & 0\\ (\star) & \Upsilon_{13} & \hat S^{-T}A_d+\bar d^{-1}\hat S^{-T}\hat Z_v\hat S^{-1} & \hat S^{-T}B_{0v} & C_v^T\\ (\star) & (\star) & 0 & 0 & C_{dv}^T\\ (\star) & (\star) & (\star) & -\gamma_v^2I & D_{0v}^T\\ (\star) & (\star) & (\star) & 0 & -I\end{bmatrix}\prec 0. \quad (40)$$

where

$$\Upsilon_{12}=\beta_v\hat S^{-T}A-\hat S^{-1}+\hat S^{-T}\hat P\hat S^{-1}$$
$$\Upsilon_{13}=\mathrm{sym}\{\hat S^{-T}A\}-\bar d^{-1}\hat S^{-T}\hat Z_v\hat S^{-1}+\hat S^{-T}\hat X_v\hat S^{-1}$$

Next, introduce the following change of variables in (39)

$$S_v^{-1}=W_v,X_v=S_v^{-T}\hat X_vS_v^{-1},Z_v=S_v^{-T}\hat Z_vS_v^{-1},$$
$$P_v=S_v^{-T}\hat P_vS_v^{-1}.$$

Application of Schur's complement formula gives that (40) holds if and only if

$$\begin{bmatrix}-\mathrm{sym}\{\beta_vW_v\} & \beta_vW_v^TA-W_v & \beta_vW_v^TA_d & \beta_vW_v^TB_{0v}\\ (\star) & \mathrm{sym}\{A^TW_v\} & W_v^TA_d & W_v^TB_{0v}\\ (\star) & (\star) & 0 & 0\\ (\star) & (\star) & (\star) & 0\end{bmatrix}+\begin{bmatrix}\bar dZ_v & P_v & 0 & 0\\ (\star) & \Upsilon_{22} & C_v^TC_{dv}+\bar d^{-1}Z_v & C_{dv}^TD_{0v}\\ (\star) & (\star) & \Upsilon_{33} & C_{dv}^TD_{0v}\\ (\star) & (\star) & (\star) & D_{0v}^TD_{0v}-\gamma_vI\end{bmatrix}\prec 0$$

holds where

$$\Upsilon_{22}=C_v^TC_v-\bar d^{-1}Z_v+X_v,\Upsilon_{33}=-X_v-\bar d^{-1}Z_v+C_{dv}^TC_{dv}.$$

The above inequalities can be rewritten as

$$\mathcal{M}_v+\mathrm{sym}\left\{\Lambda_v^TW\Sigma_v\right\}\prec 0, \quad (41)$$

where $\mathcal{M}_v$, Λ_v and Σ_v are defined in (33) and (35) ,respectively. Then, given Lemma 1, it follows that (41) is feasible if and only if (31) hold and the proof is complete.

Remark 4. The matrix $\hat S_2$ in (38) cannot be declared as the matrix variable since the term D_{01} (for $v=1$) includes K_2 (or N_2) and this controller gain matrix is not multiplied by any matrix variable. Hence there is no possibility of introducing any (linearizing) change of variables.

5. CASE STUDY

To illustrate the application of the new design, the results of a numerical simulation on a two-stage chemical reactor with delayed recycle streams are given and discussed. Both of the reactors are isothermal continuous stirred tank reactors (CSTR). A reactor recycle does not increase the overall conversion and reduces the cost of a reaction and hence is commonly used in industrial applications. The input to be recycled has to be separated from the yields and then travel through pipes. The total recycle time, therefore, introduces delays in the state vector.

Following the detailed description in (Tao et al., 2017) the resulting batch process model is of the form (2) with

$$A=\begin{bmatrix}-2.5 & 0\\ 1 & -2.5\end{bmatrix},A_d=\begin{bmatrix}0 & 0.4\\ 0 & 0\end{bmatrix},B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

Also, the time-delay range is $0 < d \leq \bar{d} = 1$, with $d = 1$ in the simulation results. The state initial vector $x_k(0)$ and the input vector $u_k(0)$ are assumed to be zero $\forall k \geq 0$ and the reference trajectories are (see also Fig. 1):

$$y_{1d}(t) = \begin{cases} \frac{1}{20}t, & 0 \leq t < 40, \\ 2 + \frac{1}{120}(t-40), & 40 \leq t < 100, \\ 2.5 + \frac{1}{300}(t-100), & 100 \leq t < 250, \\ 3 & 250 \leq t \leq 300, \end{cases}$$

$$y_{2d}(t) = \begin{cases} \frac{1}{120}t, & 0 \leq t < 60 \quad, \\ 0.5 + \frac{1}{60}(t-60), & 60 \leq t < 120 \quad, \\ 1.5, & 120 \leq t < 150 \quad, \\ 1.5 + \frac{1}{100}(t-150), & 150 \leq t < 250 \quad, \\ 2.5, & 250 \leq t \leq 300 \quad. \end{cases}$$

To evaluate tracking performance from batch to batch, let e_{ik}, $i = 1, 2$ denote the tracking errors of output i and batch k. Then the convergence measure is the root mean square (RMS):

$$\text{RMS}(ik) = \frac{1}{300}\int_0^{300} (y_{id}(t) - y_{ik}(t))^2 \, \mathrm{d}t,$$

The smaller the value of this quantity, the better the tracking performance along the kth batch (i.e., in t). Application of Theorem 1 for $\beta_1 = 0.1$, $\beta_2 = 0.1$, $\beta_3 = 1$,

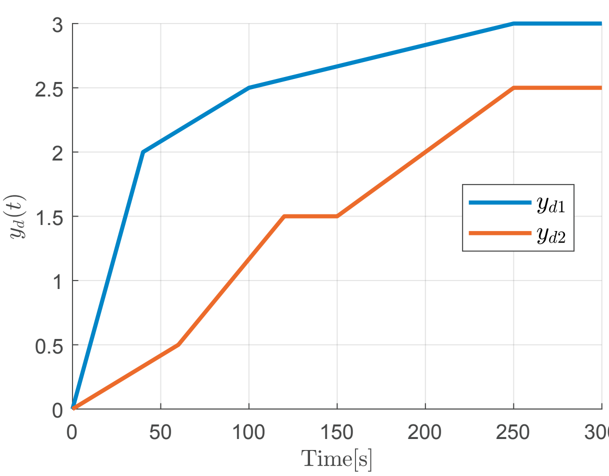

Fig. 1. The reference trajectories.

$\gamma_1 = 0.5$, $\gamma_2 = 0.3$, $\gamma_3 = 0.2$ and $\hat{S}_2 = \text{diag}\{0.5, 0.5\}$ gives

$$K_1 = \begin{bmatrix} 0.6680 & 0.1018 \\ 0.0745 & 0.6550 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.3226 & 0.0116 \\ 0.0149 & 0.3389 \end{bmatrix},$$

$$K_3 = 10^{-17} \cdot \begin{bmatrix} 0.0789 & -0.0472 \\ -0.0278 & 0.1332 \end{bmatrix}.$$

In this case, $K_3 \approx 0$ has little influence on the resulting performance (tracking error performance). The simulation results are shown in Fig. 2, where the tracking effectiveness of the ILC approach is assessed using the RMS output tracking errors for the first 40 batches and confirms the tracking errors' convergence. Furthermore, from Figure 2, it is seen that the developed design procedure yields rapid tracking error reduction from batch to batch, which is comparable with the design in (Tao et al., 2017). This new result does not use state feedback and finite frequency ranges.

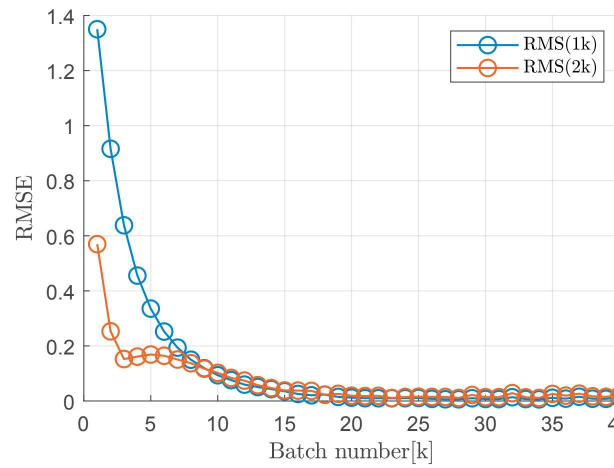

Fig. 2. The RMSE against batch number

6. CONCLUSIONS AND FURTHER RESEARCH

This paper has developed a new procedure for designing ILC laws for differential batch processes with time delays. The new design for PD-type control laws uses the repetitive process setting and the KYP lemma sufficient conditions for the convergence of the batch-to-batch error in the form of LMIs. Finally, a numerical example demonstrates the effectiveness of the new design. Future work will address the inclusion of multiple delays and uncertainty in the ILC design. Extending the theory to include disturbance attenuation is another possible topic for future work. Also, adding the output feedback controller is another area for potential future research.

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