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SENSITIVITY ANALYSIS AND UNCERTAINTY QUANTIFICATION OF STIFFNESS MODULUS USING INDIRECT TENSILE TEST

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The stiffness modulus is a critical element in pavement engineering, as its accurate measurement directly reflects the load-bearing capacity of the pavement section. Many researchers utilize indirect tensile tests in laboratory studies to determine this modulus. The pavement's performance, as well as other properties of the pavement layers, can be directly influenced by this modulus. It also provides insights into the resistance of the asphalt mix to permanent deformation due to repeated loads, a common issue in flexible pavements. The stiffness modulus holds a significant role in pavement and pavement materials-related works and designs. This research aims to address uncertainties in stiffness modulus measurements of asphalt mixtures, which can lead to unreliable results and poor decisions in pavement construction. The paper presents theoretical and experimental findings on the uncertainty and sensitivity analysis in identifying influential parameters in stiffness modulus measurement. Key factors such as experimental setup, interface roughness, sample shape, and plate-sample contact were evaluated for their impact on measurement uncertainty. The study found that test repeatability is the primary contributor to measurement error, accounting for approximately *79.72*%, followed by thickness measurement, contributing around *19.75*%.

Key words: stiffness modulus, indirect tensile testing, uncertainty quantification, sensitivity analysis, asphalt mixtures.

1. Introduction

Stiffness modulus is the key property of a pavement material that provides resistance and energy distribution through deformation under several loading patterns. Various studies have reported using the stiffness modulus over the years, either to estimate pavement direction under a wide range of loading on rural and urban highways, or to understand the impact of climate on material properties under various laboratory conditions. Note that material parameters and external factors greatly influence the stiffness modulus. Such parameters significantly influence the predicted results. Therefore, quantification of sensitivity and epistemic uncertainty in pavement design is required.

It is important to know the stiffness modulus of an elastic material like bituminous binder, fine-graded asphalt concrete, chip-sealed wearing course, and reinforced geocell base in order to understand how it acts. Most pavement mechanics theories use this modulus as a key component to represent the mechanical properties of materials. The definition of stiffness modulus, which is the ratio of normal stress to axial deformation, is similar to the definition used to measure Young's modulus, which is also called elastic modulus. Numerous research fields, including deformation modulus, dynamic modulus, and resilient modulus, can predict the stiffness modulus of geomaterials, including asphaltic materials.

Investigating uncertainties reveals the root causes of overall uncertainty, thereby facilitating the refinement and advancement of experimental methodologies. Contemporary scientific endeavors inherently accompany the precise quantification of measurement results with a certain level of doubt regarding their accuracy.

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Various influences and disturbances, whether random or systematic, contribute to measurement variability, underscoring the formulation of concepts like measurement uncertainty.

Uncertainty quantification, or UQ, is a crucial component in tackling these issues. We used the Guide to the Expression of Uncertainty in Measurement (GUM), [1] to provide a systematic framework for evaluating and communicating measurement uncertainty. By using this method, researchers and practitioners can measure how confident they are in the outcomes of their measurements, which ultimately leads to better decision-making in engineering processes. While the GUM Guide serves as a primary reference document for measurement uncertainty of stiffness modulus measurements, other publications [2], can also provide valuable insights. When assessing uncertainty in the stiffness modulus experimentally, researchers can employ the B-type approach, the statistical A-type approach predicated on arithmetic averages for a series of measurements, or both methodologies to obtain the combined uncertainty [3].

Recent advancements in the field have highlighted the need for better methods that incorporate UQ into routine testing protocols. Researchers have been examining a range of statistical and computational techniques in an attempt to gain a better understanding of the elements that lead to the unpredictability of material properties. However, the literature still lacks comprehensive application of these methods, particularly in obtaining the stiffness modulus from indirect tensile tests.

The accuracy of stiffness modulus measurement using indirect tensile testing [4] is significantly influenced by the test system and procedure, encompassing factors such as dimension measurement devices, alignment system, gauge determination system, force measurement accuracy, extensometer accuracy, test machine stiffness and testing software, Barksdale *et al.* [5]. have extensively discussed the several sources of measurement errors, emphasizing the necessity of understanding the quantitative implications of each component to enhance test reliability [6].

The quantification of specimen elongation frequently utilizes strain gauges due to their cost-effectiveness, simplicity of operation, and adequate precision. Nevertheless, the principal challenge arises during their installation, particularly when the specimen possesses a diminutive cross-sectional area, as noted by Kostic *et al.* [7].

The recommendations outlined in the publication known as "The Guide to the Expression of Uncertainty in Measurement" [1] delineates uncertainty as the doubt inherent in measurement results, providing a structured framework for uncertainty evaluation.

In accordance with widely recognized protocols, the streamlined methodology for uncertainty assessment may be delineated as per the ensuing timetable [8]:

- Determine the precise measurements, both direct and indirect.
- Identify the different sources of uncertainty and assign a value to them based on the appropriate probability distribution.
- Evaluate the typical uncertainty for each element.
- Evaluate the impact of each element on the combined type uncertainty.
- The system calculates the expanded uncertainty, adding it to the verification result.

It is important to underscore that the evaluation of uncertainty can lead to a better understanding of the method employed. Particularly, the uncertainty budget is a valuable tool in enhancing the precision of the testing method. The accurate assessment identifies the primary source of uncertainty in the method used, as well as negligible factors [9].

Gabauer, [10] advocates to group major factors affecting tensile testing results of asphalt mixtures' stiffness to optimize uncertainty calculations, categorizing uncertainties into distinct types based on their quantifiability. Notably, Type A uncertainties, derived from empirical data, offer greater accuracy compared to Type B uncertainties [11], which rely on a priori information, such as a calibration certificate, manufacturer's specifications, or an expert's assessment.

Following uncertainty quantification, sensitivity analysis becomes imperative to discern the relative importance of different uncertainty sources, enabling researchers to allocate resources effectively for uncertainty reduction [12]. In [13], an expanded polynomial chaos framework was utilized to assess how variations in input distribution parameters affect the output distribution functions. This involved computing the sensitivities of the response function concerning the distribution parameters of the random variables, the sensitivity is determined by calculating the partial derivatives of the model concerning its input factors [12]. We perform sensitivity analysis to determine the most impactful uncertain variables in the stiffness modulus. This paper talks about the results of an experiment that used indirect tension testing on cylindrical specimens to find out important things about figuring out stiffness modulus. This type of testing is usually called the Indirect Tensile Stiffness Modulus test. While adhering to the EN 12697-26 standard's guidelines, the study extends beyond mere quantification of measurement uncertainty. The study looks at the difficulties of getting accurate stiffness modulus measurements on laboratory indirect tensile testing machines. The goal is to find factors that affect these measurements and suggest ways to keep the uncertainty within acceptable limits.

2. Materials and methods

2.1. Materials

The analyzed sample was high modulus asphalt mixture (reference BBME *0/10*) [14]. We performed stiffness modulus measurements on eight Marshall specimens made of the same mixture. We used the indirect tensile test to determine the stiffness modulus of each sample eight times. We collected modified stiffness modulus data as part of the statistical analysis, yielding eight values of the obtained stiffness modulus.

2.2. Experimental tests

2.2.1. Indirect tensile stiffness modulus test of asphalt mixture

For conducting the tests, a hydraulic press with a capacity of *50 kN* is utilized. Its primary characteristics include:

- Utilization of specialized software for determining various testing variables.
- Generation of desired loading signals, as well as controlling the press and the thermoregulated enclosure, inclusion of all essential components for experiment programming.

The press operation is orchestrated through dedicated software, enabling the definition of different parameters and creation of desired loading signals. This software encompasses control functionalities for the press, thermal enclosure, and all essential elements for experiment programming.

Measurements are carried out within a designated room in the test laboratory, preserved under room temperature environmental conditions.

The stiffness modulus-testing device is enclosed in a temperature-regulated chamber. The device has undergone external calibration and validation. This enclosure ensures a constant test temperature ranging from *0* to *40°C* in close proximity to the test samples, with a margin of error of ± *0.5°C*. Control charts facilitate the metrological monitoring of the machine.

Two linear sensors, identified as LVDTs (Linear Variable Differential Transformers), form the deformation measuring system, able to precisely measure the transient diametral deformation of the specimen while applying a charge pulse, with a margin of error of ±*1 µm*.

- Employing a force sensor allows for the measurement of the applied load with a precision of *2*%. Displacement sensors and force sensors undergo separate calibration procedures conducted by an external entity.
- Thermocouples specially tailored and calibrated by an external entity, are utilized to measure the temperature of the specimen, storage conditions, and testing environment with a precision of ± *0.1°C*.
- An externally calibrated caliper allows for the measurement of specimen thicknesses and diameters with an accuracy of ±*0.1 mm*.

The uncertainties associated with these sensors (dimensional (caliper), temperature (thermocouple), displacement (LVDT), force) are propagated according to established principles. The evaluation of standard uncertainty encompasses factors such as resolution, calibration, drift, sensor sensitivity, load rise time, and temperature, drawing upon scientific reasoning and incorporating all relevant information available.

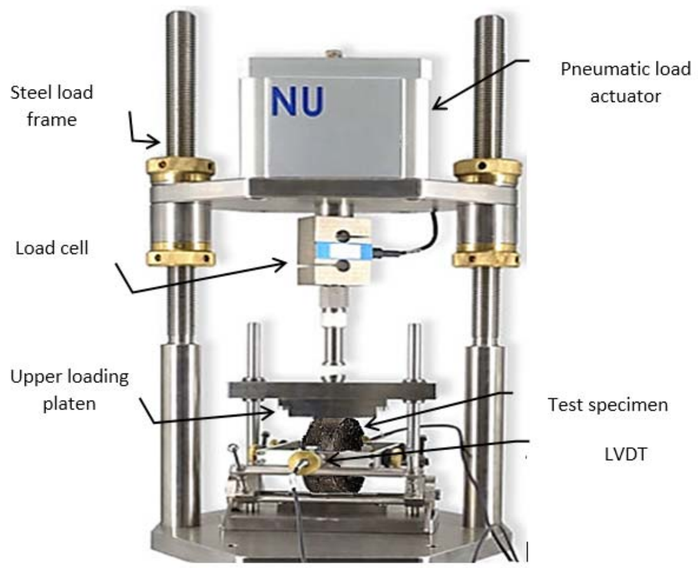

Fig.1. Apparatus employed for the indirect tensile stiffness modulus test (EN 12697-26), [6].

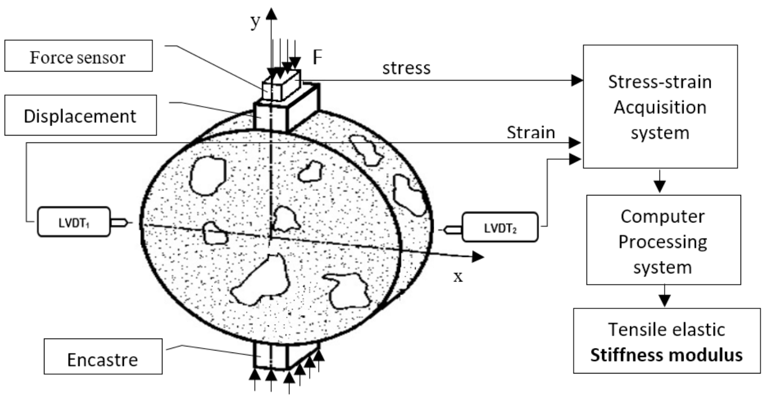

Fig.2. The method of measuring the rigidity modulus.

2.2.2. Stiffness modulus

Tests were conducted using the indirect tensile method on compacted samples with a diameter of *100 mm*, a thickness of *53 mm*, and a density of *2451 kg/m*^{3}, adhering to the standard EN 12697-26: (2004) standard [6]. These specimens were subjected to a *6*-hour conditioning period at the test temperature before testing (Fig.3).

The EN 12697-30 [15] standard guides the preparation of cylindrical specimens in the laboratory to characterize a high modulus bituminous concrete (BBME) with *0/10 mm* granularity. First, we heat the aggregates and the bituminous binder separately to the appropriate temperature, following the EN 12697-35 [16] standard. Once the materials reach the correct temperature, they undergo mechanical mixing to achieve a homogeneous mixture, adhering to the EN 12697-35 [16] standard's recommendations. We then introduce the hot mixture into standardized cylindrical molds, which have a diameter of *100 mm* and a thickness of *53 mm*, as specified in the standard. A laboratory press compacts the specimens by applying a number of gyrations defined in the EN 12697-31 [17] standard. This step ensures a level of compaction representative of field implementation. Once the compaction is complete, we cool the specimens and deform them, always following the guidelines of the EN 12697-30 standard.

Fig.3. Test specimen.

To determine the rigidity modulus of elasticity, a pneumatic press was utilized in a temperature-controlled chamber as shown in Figs (1 and 2). The testing methodology involved subjecting specimens to cyclic compressive forces applied diametrically, with horizontal diameter variation measurements recorded (Fig.3).

Table 1. Testing conditions for the indirect tensile tests.

Horizontal deformation µ*m*	*5* (with an error margin ± *2*)
Pulse repetition period *s*	*3* (with an error margin ± *0.1*)
Load rise-time *ms*	*124* (with an error margin ± *4*)
Frequency *Hz*	*10 Hz*
Number of pulses	*10*
Essay temperature °*C*	*15°C*
Poisson's ratio	*0.35*
Load area factor *k*	*0.6*

Table 2. The results of the indirect tensile tests.

Identification of the specimen	AB-C1/1	AB-C1/2	AB-C1/3	AB-C1/4	AB-C1/5	AB-C1/6	AB-C1/7	AB-C1/8
Diameter *mm*	*100*	*100*	*100*	*100*	*100*	*100*	*100*	*100*
Thickness *h mm*	*53*	*53*	*53*	*53*	*53*	*53*	*53*	*53*
Density kg/m^3	*2451*	*2451*	*2451*	*2451*	*2451*	*2451*	*2451*	*2451*
Load *N*	*2.26*	*2.37*	*2.39*	*2.3*	*2.14*	*2.18*	*2.28*	*2.25*
Load area factor *k*	*0.65*	*0.65*	*0.65*	*0.65*	*0.65*	*0.65*	*0.65*	*0.65*
Horizontal displacement *z* µ*m*	*4.2*	*4.5*	*4.8*	*4.1*	*4.5*	*4.1*	*4.3*	*4.1*
Rise-time *ms*	*123.5*	*124.2*	*122.6*	*126.1*	*123.9*	*124.5*	*122.1*	*124.5*
Measured Stiffness Modulus S_m *MPa*	*6689*	*6841*	*6975*	*6304*	*6345*	*6310*	*6304*	*6309*
Adjusted Stiffness Modulus S'_m *MPa*	*6897*	*7046*	*7182*	*6488*	*6475*	*6460*	*6466*	*6456*
The average value of the $\overline{S}_m$ *MPa*	*6509.625*							
The average value of the $\overline{S}'_m$ *MPa*	*6683.75*							

The load pulse duration was set at *124* milliseconds (Rise-time), with the load application lasting *3.0* seconds from the onset of loading until the subsequent cycle commenced. Adjustments were made to the load magnitude to induce a momentary expansion of *5* microns in the horizontal diameter of the specimen. The test was conducted at room temperature (*15°C*) in stress control mode, with a half-sinusoidal repeated load pattern applied at a frequency of *5 Hz* (refer to Tab.1).

During the test, ten initial conditioning charge pulses were applied for press calibration. This was followed by a series of five charge pulses along with measurements of horizontal diameter variations and applied load. The rigidity modulus was calculated from these five charging pulses using formula (2.1) [18], with results summarized in Tab.2. The desired horizontal deformation was set at *5* μ*m*, and the peak load was adjusted as per the deformation measurement. Information from five charge pulses was recorded, and Figs 1 and 2 illustrate the test steps. Specimens were conditioned at the test temperature in the enclosure for *6* hours prior to testing.

In accordance with Hooke's law, relevant to the domain of elastic deformations, the stiffness modulus can be calculated by employing Eq.(2.1), [8]:

$$S_m = \frac{F(\vartheta + 0.27)}{z\,h} \text{ [6]}, \tag{2.1}$$

where:

- S_m : stiffness modulus determined through measurements *MPa*;
- h : average thickness of the specimen *mm*;
- F : applied vertical load *N*;
- ϑ: Poisson's coefficient;
- z : observed lateral displacement μ*m*.

The adjustment of the measured stiffness modulus should conform to the formula specified in European Standards EN 12697-26, [6].

$$S'_m = S_m\left[(1 - 0.322\ (LogS_m - 1.82)(0.60\ -\ k)\right], \tag{2.2}$$

where:

- S'_m : adjusted stiffness modulus *MPa*;
- *k*: load area factor;
- we take $\Delta k = 0.60 - k = 0.1$.

The charge area factor *k* is defined as the ratio of the area under the have sine charge curve (from pulse start to peak charge) to the area of the bounding rectangle (which is calculated as the product rise time and maximum load) [19]. This load pulse area represents the energy transmitted to a sample and should ideally maintain uniformity across each charge application. The load area factor adjusts the stiffness modulus value according to formula (2.1) if the load curve deviates from expectations. The indirect tensile test has been recognized as an economical and practical method for quantifying the stiffness modulus of bituminous mixtures. Its appeal lies in the ability to utilize core samples, as opposed to the more complex-shaped samples often required for other stiffness modulus tests, such as the bending beam test. Additionally, laboratory-compacted specimens, such as Marshall specimens, can also be utilized. The formula employed for the calculation the load area factor is presented in Eq.(2.3).

$$k = \frac{\int_0^{t_{\max}} F(t)dt}{F_{\max}\, t_{\max}}, \text{ [6]}. \tag{2.3}$$

2.3. Uncertainty evaluation procedure

2.3.1. Parameters requiring estimation of uncertainty

The asphalt mixture's stiffness modulus S_m was calculated employing the method in question. It's important to note that the value of S_m is not directly measured; instead, it is derived from other measurements. The essential measurements, along with their respective units and symbols, are detailed in Tab.3. The equation employed to calculate the stiffness modulus is presented in Eq.(2.1).

Tab.3. Collection of measurement data.

Measurements	Symbol
Applied vertical load *N*	*F*
Horizontal displacement μ*m*	*z*
Average thickness of sample *mm*	*h*

2.3.2. Sources of uncertainty

Obtaining precise measurements of stiffness modulus using a indirect tensile testing apparatus poses numerous challenges stemming from various factors. Apart from the precision of the measuring instruments, the conformity of testing device subassemblies also significantly influences the results.

It is essential to consider all potential sources of uncertainty, which may stem from direct measurements, equipment constraints, or environmental conditions. It's important to note that the inventory of causes of uncertainty should be distinctly customized for each methodology in every laboratory [20]. This customization is necessary as it relates to factors such as the tested material, procedural methodology, technique (including the correct calculation of the correction factor k), and apparatus type.

Owing to the configuration of the measuring apparatus, the stress distribution within a specimen exhibits significant nonlinearity and is biaxial. The relationship between the load applied vertically and the strain generated and measured horizontally does not present a direct linear relationship, as is commonly seen in a uniaxial test. The most straightforward method to connect these two parameters involves assuming the material to be homogeneous, isotropic, and linearly elastic [19].

According to reference [8], the test simulates the stress state experienced at the lower position of asphalt layers, revealing the significant issue of low repeatability in stiffness modulus results obtained. The variability in results obtained from the indirect tensile test is estimated to be between *5*% and *10*% [5].
This variability represents among the lowest levels achievable across various methods of determining stiffness modulus. Moreover, the resulting stiffness modulus value is affected by other experimental variables, like the Rise-Time [7]. The presence of multiple testing procedures and the operator's capacity to adjust testing parameters introduce slight variations in the tension state observed in specimens, along with other crucial testing conditions. Nevertheless, empirical evidence indicates [5] that the scattering of results due to the variability in testing conditions is less pronounced than the dispersion attributable to material heterogeneity and instrumental inaccuracies.

Consequently, to ensure the usefulness of stiffness modulus measurement results, an examination of the measurement uncertainty associated with this analysis has been conducted. Stiffness modulus determinations were conducted using the methodology outlined in reference [18]. Table 1 provides a breakdown of the testing parameters employed. Tests were conducted on Marshall specimens, with the load curve displaying a Haversian shape.

2.3.3. Types and probability distributions of uncertainty

The guide to the expression of uncertainty in measurement [8], categorizes uncertainty components into two groups: Type A and Type B. This classification is grounded in their method of assessment and is not meant to imply a disparity in the nature of components. It's worth noting that both types of uncertainties rely on the probability density function [21]. In other words, type A uncertainty is statistically determined by analyzing the results of series of experiments. If an alternative approach is employed, the uncertainty is categorized as type B. In practice, constraints on time and financial resources compel laboratories to rely on Type B uncertainty [22]. The designated uncertainty contributors are catalogued in Tab.4.

Table 4. Assumed type of uncertainty and its distribution.

Source of uncertainty	Type	Distribution
Repeatability	A	t-distribution
Measurement of the force	B	rectangular
Deformation measurement	B	rectangular
Thickness measurement	B	rectangular
Poisson ratio	B	rectangular
Correction factor *k*	B	rectangular

2.3.4. Standard uncertainty

The standard uncertainty, often denoted as $u_A\left(S'_m\right)$, indicates the standard deviation of the results associated with type A uncertainty. To determine its value, thirty measurements of the stiffness modulus were conducted, and the standard uncertainty was determined by computing the standard deviation of the measurements [14].

$$u_A\left(S'_m\right)=\sqrt{\frac{\sum_{i=1}^{n}\left(S'_m-\overline{S'}_m\right)^2}{n-1}}\ , \tag{2.4}$$

$\overline{S}'_m$: the average of adjusted stiffness modulus MPa.

The standard uncertainties of the applied vertical load, horizontal displacement, average thickness of the sample, Poisson ratio, correction factor *k*, and caliper were calculated as type B uncertainties.

Considering the equation for the adjusted stiffness modulus S'_m, it becomes apparent that it is determined by the aforementioned quantities, expressed as follows [1]:

$$S'_m = f(F, z, h, v, k)\ . \tag{2.5}$$

With,

$$S'_m=\left(\frac{F\left(\nu+0.27\right)}{z\,h}\right)\left(1-0.322\right)\left[log\left(\frac{F\left(\nu+0.27\right)}{z\,h}\right)-1.82\right]\left(0.60-k\right), \tag{2.6}$$

$$S_m=\frac{F.\left(\nu+0.27\right)}{z.h}\ . \tag{2.7}$$

In the indirect tensile test on cylindrical bituminous specimens for stiffness modulus, four independent measurements are conducted using different instruments: a caliper for measuring thickness and diameter, thermocouples for temperature measurement in a controlled chamber, a load cell for applied load, and LVDTs for transient diametral deformation. The correlation between these variables is considered negligible.

This investigation seeks to quantify the uncertainty associated with S'$_m$ value, founded on the uncertainties in primary measurements, denoted as the stiffness modulus uncertainty u(S'_m) can be expressed as follow [23]:

$$u(S'_m)=\sqrt{\left(\frac{\partial S'_m}{\partial x_i}\right)^2 u^2(x_i)}\,, \tag{2.8}$$

$$u(S'_m)=\sqrt{\left(\frac{\partial S'_m}{\partial F}\right)^2 {u_F}^2+\left(\frac{\partial S'_m}{\partial Z}\right)^2 {u_Z}^2+\left(\frac{\partial S'_m}{\partial h}\right)^2 {u_h}^2+\left(\frac{\partial S'_m}{\partial \nu}\right)^2 {u_\nu}^2+\left(\frac{\partial S'_m}{\partial k}\right)^2 {u_k}^2}\,. \tag{2.9}$$

After derivation of the function *f* in Eq.(2.9), the equation delineates the comprehensive uncertainty in the stiffness modulus.

$$\begin{aligned} u(S'_m)=\Bigg\{&\left[\left(\frac{\nu+0.27}{zh}\right)(1-0.322(0.60-k)\left(\log\left(\frac{F(\nu+0.27)}{zh}\right)-0.82\right)\right]^2 {u_F}^2+\\ &\left[-\left(\frac{F(\nu+0.27)}{z^2h}\right)(1-0.322(0.60-k)\left(\log\left(\frac{F(\nu+0.27)}{zh}\right)-0.82\right)\right]^2 {u_Z}^2+\\ &+\left[-\left(\frac{F.(\nu+0.27)}{zh^2}\right)(1-0.322(0.60-k)\left(\log\left(\frac{F(\nu+0.27)}{zh}\right)-0.82)\right)\right]^2 {u_h}^2+\\ &+\left[\left(\frac{F}{zh}\right)(1-0.322(0.60-k)\left(\log\left(\frac{F.(\nu+0.27)}{z.h}\right)-0.82\right)\right]^2 {u_\nu}^2+\\ &+\left[\left(\frac{F(\nu+0.27)}{zh}\right)(0.322\left(\log\left(\frac{F(\nu+0.27)}{zh}\right)-1.82\right)\right]^2 {u_k}^2\Bigg\}^{\frac{1}{2}}. \end{aligned} \tag{2.10}$$

Where u_F , u_Z , u_h , u_ϑ , u_k represent the uncertainties in the independent variables from Eq.(2.5), and the partial derivatives denote sensitivity coefficients c_F , c_Z , c_h , c_ϑ , c_k respectively.

$$u(S'_m)=\sqrt{(c_F)^2 u_F^2+(c_Z)^2 u_Z^2+(c_h)^2 u_h^2+(c_\vartheta)^2\ u_\nu^2+(c_k)^2 u_k^2}\,. \tag{2.11}$$

a. Uncertainty in force measurement u_F :

Here is an alternative method to calculate uncertainty in force measurement. Significant factors influencing the total uncertainty in force measurement encompass uncertainty originating from the force sensor, zero adjustment of the force-measuring component, potential misalignment of the applied force, environmental temperature conditions during experimentation, and the rate of load application. The uncertainty in force measurement within the range of elastic deformations can be quantified by employing the ensuing equation [24]:

$$u_F = \sqrt{u_{F_{\max}}^2 + u_{F_{\min}}^2} \; . \tag{2.12}$$

Where $u_{F_{max}}$ and $u_{F_{min}}$ represent the uncertainties in force measurement nearest to the force value within the elastic deformation range. These uncertainties can be expressed as follows:

$$u_{F\max} = \frac{u_f \, F_{\max}}{\sqrt{3}} \, , \tag{2.13}$$

$$u_{F\min} = \frac{u_f . F_{\min}}{\sqrt{3}} \, . \tag{2.14}$$

Where u_f is the uncertainty in force measurement, attributable to the force sensor. The greater of two uncertainty values, whether it was obtained from the Eq.(2.15), will be utilized for subsequent calculations [1].

$$u = \sqrt{u^2{}_{ET} + u^2{}_{der} + u^2{}_{res} + u_s^2} \; , \tag{2.15}$$

where

- u_{ET} : Uncertainty associated with the calibration of the force measurement device.
- u_{der} : Uncertainty arising from drift. For the initial calibration, we will consider $u_{der} = u_{ET}$.
- u_{res} : Uncertainty associated with the resolution of the measured value.
- u_s : Uncertainty linked to the sensitivity of temperature.

To quantify these uncertainties, we reference all accessible data, encompassing observed datasets, validation outcomes, calibration certifications, specialist insights, technical specifications, and manufacturer's information. Particularly, we refer to the following guides [25].

b. Uncertainty in thickness measurement u_h :

The dimensions were quantified utilizing a digital caliper. The instrumental measurement uncertainties were calculated using the following Eq.(2.16), employing the B-type method (rectangular distribution):

$$u_h = \frac{u_{cal}}{\sqrt{3}} \tag{2.16}$$

where u_{cal} represents the measurement uncertainty associated with the digital caliper.

c. Uncertainty in deformation measurement u_z :

The uncertainties in measuring the upper $u_{z_{max}}$ and lower $u_{z_{min}}$ levels of elongation to the elongation in the elastic region are established based on the precision grade of the linear sensors LVDT (Linear Variable Differential Transformers), using the B-type method (rectangular distribution).

$$u_z = \sqrt{u_{z_{\max}}^2 + u_{z_{\min}}^2} \quad (2.17)$$

where $$u_{z_{\max}} = \frac{u_{lvdt} \cdot z_{\max}}{\sqrt{3}}, \quad (2.18)$$

$$u_{z_{\min}} = \frac{u_{lvdt} \cdot z_{\min}}{\sqrt{3}} \quad (2.19)$$

where u_{lvdt} is the uncertainty in the measurement of the linear sensors LVDT.

d. Uncertainty in Poisson's ratio u_v :

Poisson's coefficient ν, detailed in (2.20), relates axial deformation ε_h to lateral deformation ε_z [26].

$$v = -\frac{\varepsilon_z}{\varepsilon_h}. \quad (2.20)$$

- Lateral deformation is described as the ratio of the change in specimen diameter Δz to the initial diameter z_0 of the specimen at the beginning of the test.
- Axial deformation is characterized as the ratio of the change in sample diameter Δh to the initial diameter h_0 of the sample at the beginning of the test. Equation (2.21) follows from (2.20).

$$v = \frac{\Delta z\, h_0}{\Delta h\, z_0}. \quad (2.21)$$

The combined standard uncertainty u_v is determined by Eq.(2.22). In accordance with the Guide to the Measurement of Uncertainty (GUM), [1].

$$u_v = \sqrt{\left(\frac{\partial v}{\partial x_i}\right)^2 u^2(x_i)},$$

$$u_v = \sqrt{\left(\frac{\Delta z}{z_0\, \Delta h}\right)^2 u^2(h_0) + \left(-\frac{h_0\, \Delta z}{z_0\, \Delta h^2}\right)^2 u^2(\Delta h) + \left(-\frac{h_0\, \Delta z}{z_0\, \Delta h^2}\right)^2 u^2(z_0) + \left(\frac{h_0}{z_0\, \Delta h}\right)^2 u^2(\Delta z)}, \quad (2.22)$$

h_0 were measured using a digital caliper, while Δz and z_0 of the specimens were measured using linear sensors known as LVDTs (Linear Variable Differential Transformers), employing the B-type method (rectangular distribution).

Where, $$u_{\Delta h} = u_{h0} = \frac{u_{cal}}{\sqrt{3}}, \quad (2.23)$$

$$u_{\Delta z} = u_{z0} = \frac{u_{lvdt}}{\sqrt{3}}. \quad (2.24)$$

e. Uncertainty in correction factor k u_k :

In accordance with Standard EN 12697-26-2012 [6], the recommended load area factor is *0.60*, with a tolerance $u_{cf} = \pm\ 0.1$ using the B-type method (rectangular distribution).

$$u_k = \frac{u_{cf}}{\sqrt{3}}\ . \tag{2.25}$$

2.3.5. The combined uncertainty

Table 5. Uncertainty Budget for Stiffness Modulus.

Sources of uncertainty		Type	Distribution	Standard uncertainty u_i	Sensitivity coefficient	Uncertainty $(S_i\ u_i)$	Contribution (sensitivity indices) S_i en (%)
Sources	components						
u_F : Uncertainty in the measurement of the force	u_f : Force sensor Person)	B	Rectangular	*1.814854218*	/	/	/
	u_{Fmax}	B	Rectangular	*2.504257705*	/	/	/
	u_{Fmin}	B	Rectangular	*2.242306063*	/	/	/
	u_F			*3.361434684*	*7.74E-01*	*2.60E+00*	*0.773*
u_h : Uncertainty in the thickness measurement	u_{cal} : Digital caliper	B	Rectangular	*0.280165156*	/		/
	u_h	B	Rectangular	*0.161753428*	*4.06E+02*	*6.57E+01*	*19.517*
u_z : Uncertainty in the deformation measurement	u_{lvdt} : Sensors LVDT	B	Rectangular	*0.00840008*	/		/
	u_{zmax}	B	Rectangular	*2.3279E-09*	/		/
	u_{zmin}	B	Rectangular	*1.98841E-09*	/		/
	u_z			*3.06152E-09*	*9.31E-03*	*2.85E-11*	*8.47E-12*
u_ϑ : Uncertainty in the Poisson's ratio	u_{h0}	B	Rectangular	*0.280165156*	/		/
	$u_{\Delta h}$	B	Rectangular	*0.280165156*	/		/
	u_{z0}	B	Rectangular	*0.00840008*	/		/
	$u_{\Delta z}$	B	Rectangular	*0.00840008*	/		/
	u_ϑ			*1.51941E-08*	*2.83E+00*	*4.31E-08*	*1.28E-08*
u_k : Uncertainty of the correction factor	u_k	B	Rectangular	*0.057735027*	*8.79E-01*	*5.08E-02*	*0.015*
$u_A(S'_m)$: Repeatability	/	A	t-distribution	*268.4375*	*1*	*268.4375*	*79.725*

Table 5cont. Uncertainty Budget for Stiffness Modulus.

Sources of uncertainty		Type	Distribution	Standard uncertainty u_i	Sensitivity coefficient	Uncertainty $(S_i\, u_i)$	Contribution (sensitivity indices) S_i en (%)
sources	components						
The adjusted stiffness modulus $S_m^{'}$	Combined Standard Uncertainty $u_c\left(S_m'\right)$ (in *MPa*)					*276.38*	
	Expanded uncertainty of the stiffness modulus $U\left(S_m^{'}\right)$ (in *MPa*), $U\left(S_m^{'}\right)=kp\, u_c\left(S_m^{'}\right)$ with $kp=2.01$					*555.52*	

2.3.6. Expanded measurement uncertainty

The Expanded Uncertainty of measurement, denoted by $U\left(S_m^{'}\right)$ is a crucial component accompanying test results, specifying the range within which the true measurement value is anticipated to lie, with a specific level of probability. Characteristics of materials such as bituminous mixtures, particularly the stiffness modulus, typically conform to a Gaussian distribution.

According to (GUM), it is more appropriate to determine the value of the coverage factor k based on a *t*-distribution rather than a Gaussian distribution. This approach ensures that the expanded uncertainty reflects a coverage probability close to the required level, approximately *95*%.

To achieve this, the two-sided *t*-tabulated value corresponding to a *95*% confidence level and the effective degrees of freedom v_{eff}, calculated using the Welch-Satterthwaite formula [13], is used as the coverage factor. This method results in an increased expanded uncertainty.

$$\upsilon_{eff}=\frac{u_c^4(y)}{\sum_{i=1}^{N}\frac{u_i^4(y)}{\upsilon_i}} \tag{2.26}$$

where v_i represents the degrees of freedom of $u(x_i)$. The value of *k*, often denoted as k_p, where *p* is the coverage probability, provides an expanded uncertainty up that ensures the coverage probability remains approximately at the required level *p*.

Using the Welch-Satterthwaite formula, v_{eff} is calculated to be *47*. For the measurements performed $(n=8)$, the associated degrees of freedom are $\text{v}_i=n-1=7$. At a *95*% coverage probability, this corresponds to a coverage factor k_p of *2.01*.

Therefore, the expanded uncertainty is given by:

$$U\left(S_m^{'}\right)=u_c\left(S_m^{'}\right)k_p=276.38\cdot 2.01=555.52\,MPa\ . \tag{2.27}$$

After estimating the expanded uncertainty, the results should be reported in the following manner:

$$S_m'=\overline{S}_m'\pm U_{(S_m')} \tag{2.28}$$

where $\overline{S}_m'$ ’m is the mean test (or measurement) result.

Lastly, the stiffness modulus of asphalt mixtures can be presented as:

$$S'_m = (6683.75 \pm 555.52) MPa . \tag{2.29}$$

The determination of the expanded uncertainty of measurement $U(S'_m)$, involves the multiplication of the combined standard uncertainty $u_c(S'_m)$ by a coverage factor *k*, yielding a confidence level of approximately *95*%.

3. Results and discussion

3.1. Analysis of uncertainty factors

The assessment of measurement uncertainty using the indirect tensile stiffness modulus method for non-destructive testing reveals that the expanded uncertainty stands at *555.52 MPa*, constituting *8.3*% of the obtained value. Analysis demonstrates that over 80% of this uncertainty stems from material variability (Repeatability), emphasizing its significance in indirect tensile stiffness modulus testing, as depicted in Fig.4.

The second-largest contributor to uncertainty is the measurement of specimen sizes, accounting for *19.5*% of the total. Further investigation indicates that the correction factor, considering both the finite thickness of the specimen and the measurement of force, contributes approximately *0.06*% and *0.77*%, respectively, to the uncertainty budget.

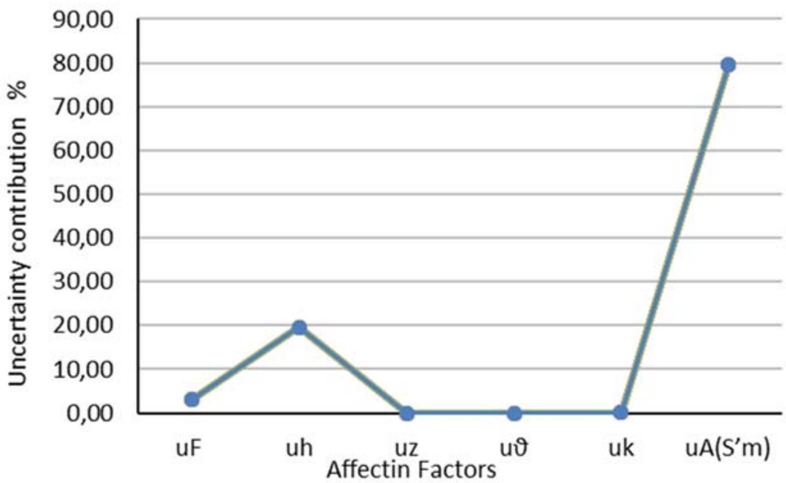

Fig.4. Effects of individual factors on combined uncertainty.

The completed uncertainty budget clearly identifies the sources of uncertainty that require improvement to significantly reduce the expanded uncertainty. The experiments outlined in this manuscript focus on addressing potential factors contributing to uncertainty, specifically variability and lack of repeatability in experiments.

To mitigate uncertainty in measurements, various variables were identified. Firstly, establishing an official protocol for assembling the utilized systems used in the indirect tensile stiffness modulus test was crucial for ensuring repeatability. This procedure necessitated attention to both the sequential tightening of bolts and the precise alignment of the interface.

Furthermore, once a systematic measuring process was established, variability between parts was substantially reduced by correcting for specimen roughness and meso-scale geometry. This suggests that the non-linear properties of the structure predominantly stem from aspects that can be controlled in the production phase of the specimen, including surface irregularities (such as roughness), meso-scale curvature, and broader scale parameters.

3.2. Diagram depicting the maximum uncertainty in the stiffness modulus: understanding the limits of result variability.

Technical and regulatory standards in civil engineering and construction materials, notably ASTM D 4123 [27], generally recommend a *10*% variability in results obtained from the indirect tension test of the stiffness modulus.

The variability limit was set at *10*%, equivalent to *668.37 MPa*, while the expanded uncertainty of measurement using the indirect tensile stiffness modulus method was $U\left(S'_m\right) = 555.52\ MPa\ \left(k_p =\ 2.01\right)$, amounting to *8.3*%. This deviates from the reference value of the elastic modulus by *1.7*%.

According to the diagram in Fig.5, we observe that the measured values of the stiffness modulus via the indirect tension test contain results outside the acceptable variability limit, indicating an unexpected dispersion of uncertainty in the elastic modulus. Specifically, results No. *02* and *03* exceed the variability limit by *3.3*% and *5.2*% respectively. This demonstrates that the measurement process is not stable over time and there is a significant risk of obtaining results outside the variability limit. Therefore, the measurement process of the stiffness modulus via the indirect tension test requires thorough improvement to ensure acceptable stability in measurements and further reduce measurement uncertainty.

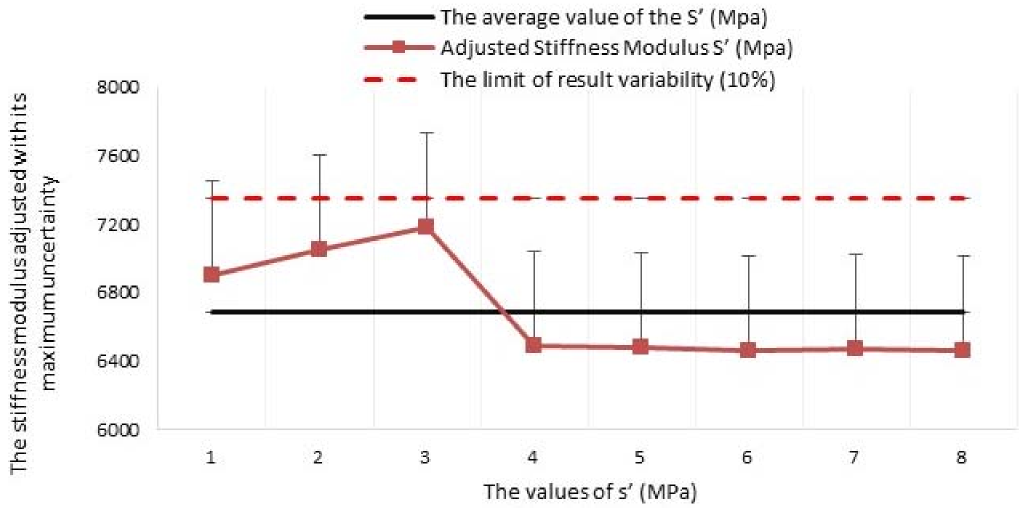

Fig.5. The maximum uncertainty on the measured values of the rigidity modulus with the limit value of variability of the results recommended by the standard ASTM D 4123, [27].

3.3. The geometric characteristics and form of the specimen

Standard test specimens (Fig.1) utilized in conventional Indirect Tensile testing devices are secured by the grips of the apparatus. Tensile load is transferred from the test sample through the upper and lower loading platens. However, when such load transfer occurs, uncertainty emerges at the interface between the loading platens and the specimen (Fig.6). Localized crushing of the sample may transpire, thereby limiting the strain experienced within the specimen bulk. Nevertheless, results have exhibited significant variation due to several experimental artifacts that compromise measurement accuracy:

– A notable error source in stiffness modulus determination arises from uncontrolled displacement occurring in the contact zone between the specimen and the loading platens (Fig.6b and c). Even though the system is in a state of static equilibrium, the shear stresses exerted on the contact surfaces can cause displacement that is directly proportional to the magnitude of the applied tensile force [28].

- Friction between the sample and the plates of the testing apparatus may restrict the Poisson effect at the sample ends and generate barrel distortion [4] (Fig.6b and c).
- Non-uniform loading of the specimen can result from misalignment of the platens in the testing machine or irregularities in specimen ends, despite efforts to apply uniform loading (Fig.6b and c). This can result in significant errors in the displacement sensor (LVDT) used to measure sample displacement [29].
- The observed variability stems from multiple factors (sample geometry, platen-sample contact, interface between the platen and the test machine, alignment of the test machine crosshead, specimen displacement in the out-of-plane direction), Emphasizing the importance of conducting measurements at multiple locations and the significance of preventing sample misalignment during preparation.

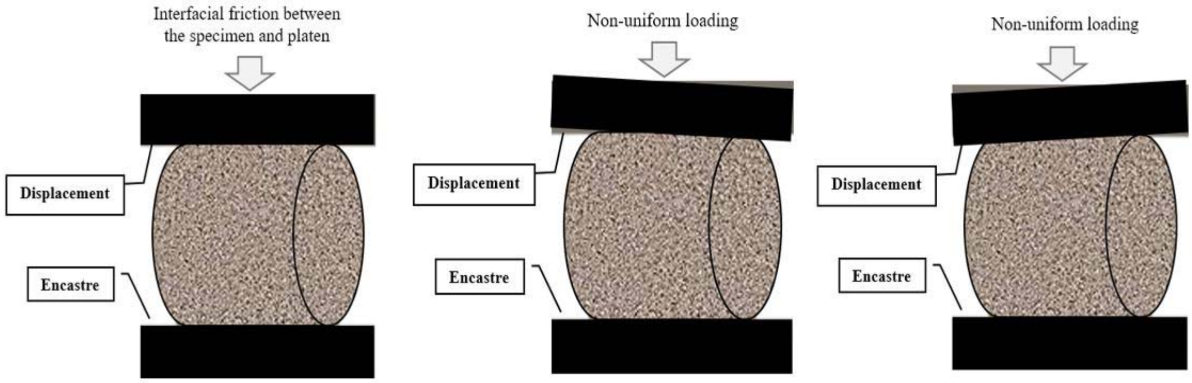

Fig.6. Factors contributing to experimental errors in indirect tensile testing. (*Note: Deformations have been magnified to demonstrate these effects*).

Additionally, we validated our methodology by comparing the calculated uncertainties with those reported in previous research. Our results show that the calculated uncertainty in the measurement of the stiffness modulus is approximately *8.3*%, at a *95*% confidence level, primarily due to imperfections in the measurement instrumentation. This observation is consistent with the findings of M. Słowik *et al.* [26], who reported that the calculated uncertainty of the stiffness modulus was *4.02*% at a *95*% confidence level. The main influential factors contributing to the measurement uncertainty were the uncertainty calculation method, random errors, and sensor resolution limits, demonstrating that limitations in measurement instruments can introduce significant biases in the stiffness test results. Similarly, studies [5], [13] have described the poor repeatability and high scatter in stiffness modulus results from the indirect tensile test, which is full agreement with our findings, emphasizing the importance of a detailed analysis of the factors influencing the measurement. According to the findings of Krystian *et al.* [9], the evaluation of the calculated uncertainty of the elastic modulus shows that over *40*% of this uncertainty arises from the repeatability of measurements. The uncertainty budget clearly identifies which sources need to be improved in order to significantly reduce the expanded uncertainty.

Our study extends this work by focusing on sensitivity analysis and the quantification of measurement uncertainty for the stiffness modulus through the indirect tensile test. Using a rigorous experimental approach, we identified test repeatability and thickness measurement as the main contributors to measurement uncertainty, accounting for *79.72*% and *19.75*% of the total uncertainty, respectively.

Furthermore, our research not only corroborates the findings of previous studies but also provides new insights into the factors influencing measurement uncertainty. These contributions are essential for refining experimental methodologies and improving the accuracy of stiffness modulus measurements in bituminous mixtures.

4. Conclusions

This study conducts a sensitivity analysis and uncertainty quantification of the stiffness modulus derived from the indirect tensile test. This paper presents numerous major conclusions derived from the examination of literature, theoretical frameworks, and empirical findings:

- The principal factor contributing to measurement uncertainty in the examined mechanical properties is the fluctuation associated with the measured, particularly the repeatability noted in stiffness modulus measurements, which constitutes *80*% of the uncertainty. The measurement of specimen sizes accounts for *19.5*% of the total error.
- Despite optimization endeavors, any measurement device is susceptible to mistakes during operation. Tensile testing procedures consistently yield data on the mechanical properties of the materials examined.
- Calibration and correction of measurement instruments must be meticulously executed to reduce uncertainty factors.
- The theoretical analysis presented in this paper highlights the significant effects of interface size and loading mode shape tolerances on the apparent nonlinear characteristics of the joint, while also addressing the impact of measuring instrument imperfections on the expected measurement uncertainty.
- Material degradation and fretting may occur under low-cycle loading conditions, particularly in areas of weak contact pressure at the interface, resulting in cumulative damage throughout the system.
- The assembly technique implemented for measurements in the indirect tensile stiffness modulus test is essential for guaranteeing result consistency. Substantial uncertainty in measurements inside the system's linear response domain, characterized by high variability or low reproducibility, requires a comprehensive evaluation of the experimental apparatus to mitigate unwanted uncertainty. If the assembly process is not meticulously regulated, measurement uncertainty may greatly exceed the frequency shifts caused by the nonlinearity of the connected interface.

Expanding research on indirect assessment tests is crucial to broaden knowledge and unlock new potential for urgent future applications in geotechnics. The integration of integrated technologies in road and environmental applications is intensifying. Consequently, it emerges as a study avenue with substantial opportunities for exploration. In future endeavors, the development of standardized methodologies to assess the uncertainty of the stiffness modulus of bituminous mixtures will be crucial. Emphasizing advanced computational models and integrating AI could significantly enhance the accuracy and reliability of predictions.

Nomenclature

ASTM – American Society for Testing Materials
F – applied vertical load (N)
GUM – Guide to the Measurement of Uncertainty
h – average thickness of the specimen (mm)
k – load area factor
LVDT – Linear Variable Differential Transformer
S_m – stiffness modulus determined through measurements (MPa)
S'_m – adjusted stiffness modulus (MPa)
$\overline{S'_m}$ – the average of adjusted stiffness modulus (MPa)
u_{der} – uncertainty arising from drift
u_{ET} – uncertainty associated with the calibration of the force measurement device
u_{res} – Uncertainty associated with the resolution of the measured value;
u_s – Uncertainty linked to the sensitivity of temperature.
z – observed lateral displacement (μm)
ϑ – Poisson's coefficient

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